Leibniz Quantum Beats Newton

Paper 0: SuperGrokTOE Rev33.2 Framework Foundations

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Abstract

Paper 0 is the entry point for the $J_3(\mathbb{O}_s)$ programme. The suite revision is **Rev33.2** (the authoritative suite label, declared here, on the Executive State Map below, and in each component's title; point releases roll all three together). Revision stamps inside bodies mark the revision at which a result *entered* the programme; titles and this sentence roll at every release, including point releases (Rev31 → Rev31.1 → Rev31.2 were point releases of the sealed Rev31); references to earlier revisions ("Rev12", "Rev19", "Rev25", ...) in the bodies below are retained as change-log stamps naming *when* a given result first entered the programme, not as the current revision label (Rev16 added Appendix I and the binding claim-language convention; Rev17 the consolidated 7/3 finite-layer result; Rev18 the in-house dynamics layer; Rev19 the engine re-stamp (m_s derived, m_e Koide value); Rev20 the structural integration layer (Appendices J/K/L); Rev21 the order-five clock and selector fences (Appendices M/N); Rev22 the honest-negatives fold; Rev23 the rest-mass program record (Appendix O); Rev24 the S188–S207 reconciliation fold; **Rev25** adds the nuclear cluster tier (Paper 9: seventeen sharp binding energies deuterium through calcium against pre-frozen bars plus one typed obstruction (^{28}Si; recount Rev27, re-adjudicated Rev28 — see the Gate 1 note in Paper 9, *Typed features and open items*), with a closed-form composition ladder), the fifth-direction theorem tier (Appendix P), the bounded golden-vacuum spectrum window (Appendix Q), the closure of the rest-mass program as ratios-complete with the framework's *one* dimensionful input registered by theorem (Appendices O/X), and the occupation selector's promotion to derived-conditional with a live public falsifier (Appendix M)).

"Occupation" is bound here at first use, and the binding holds throughout this suite. It means occupation *of golden mass-ladder rungs* — the coordinate locations $\mathcal{O} = \{0, 3, 11, 17\}$ on a logarithmic mass scale — and *not* occupation of atomic orbitals, nor of states by particles. The shared noun invites a false parity with atomic subshell filling; that parity is claimed nowhere in this suite, and reading it in was the proximate cause of a type error in an earlier internal plan, caught independently by two review lanes [A1484, A1485].

The leading mixing-sector result of the present revision is that the full mixing sector is zero-fit inside the specified correspondence map: no independently fitted PMNS or CKM angle, phase, or mixing normalization remains inside that map, with two entries (θ_{13} and δ_{CKM}) readout-conditioned rather than independently fit. The broader suite still uses explicit external inputs. Given two measured scales (m_τ and v_{EW}), one hadronic anchor (Λ_{G_2}) — inventories reconciled under the Appendix X input-count convention: *one* independent dimensionful input class after registered identifications, *two* operational calibrations, *three* named anchors — the QED input α, the external empirical Koide rule $K = 2/3$, the independent AX6 orbit selector for the Weinberg-angle branch, Route B selection, normal ordering, and a Dirac-neutrino assumption — with the working vacuum $J_{\mathrm{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1})$ now fixed (algebraically, up to permutation) by the conditional selector

theorem of Appendix E rather than carried as an external assumption — the algebraic structure $J_3(\mathbb{O}_s)$ generates the operative PMNS/CKM mixing ledger and the current charged-lepton / heavy-quark mass proxies. [P-017, P-019, P-031, P-032] The headline formulas are

$$\tan\theta_{12}^{\mathrm{PMNS}} = \frac{\sqrt{3}}{\phi^2}, \qquad \sin^2\theta_{23} = \frac{7}{16}, \qquad \sin^2\theta_{13} = \sin^4\frac{\pi}{8} = \frac{3-2\sqrt{2}}{8},$$

$$\delta_{CP} = -\frac{2\pi}{\sqrt{5}} = -160.997^\circ,$$

with the CKM entries constructed from the Route B Peirce map. This is a zero-fit claim *inside* the specified algebraic map, not a claim that the full paper suite has no measured anchors, empirical selection rules, or independent postulates. [LIB2-233] **The specified correspondence map is itself a chosen (loaded) embedding, not shown to be forced or unique by the algebra** — the selectors listed above (J_{vac}, Route B, AX6, ordering, Dirac-ν) are the loading. [P-012] So "zero-fit" is a statement about the map's internal parameter economy once that map is granted, not a first-principles prediction that the algebra alone selects these angles; the derivation of the map itself from a physical readout functor is the program's named head wall (Appendix R). The same honesty applies one level up, to the gauge content itself: the charge/gauge provenance program has reached a *settled* terminal — POSSIBLE-NOT-NECESSARY, i.e. constructible but not forced — triangulated across five independent attacks and recorded as final [A1262, A1265]. [LIB2-342] The gauge group is therefore carried as an INPUT throughout this suite (Paper 1; Appendix F), not as a derived consequence. [LIB2-342] (Rev25/26 note on the dimensionful anchors: the framework carries exactly *one* registered dimensionful input, the depth-cylinder Bargmann class, with $v_{\mathrm{EW}}/\sqrt{\sigma}$ reclassified from bare input to a conditional loaded-depth *registration*, $\varphi^{105/8}$ (registered at $\beta = 1$ from the two-element menu $\beta \in \{1, 7\}$ of Appendix N's electroweak-permanence proposition — no intrinsic selector fixes $\beta = 1$); see Appendices N/O/X.)

The algebraic core remains the cubic-invariant Yukawa structure

$$\mathcal{Y}(\Psi, \Phi_H) = \mathrm{Tr}\big[J_{\mathrm{vac}} \circ (\Psi \times \Phi_H)\big], \tag{1}$$

with $J_{\mathrm{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1})$. This projects onto the middle Peirce idempotent and fixes the ultraviolet tree-level boundary value $y_t(M_{\mathrm{Pl}}) = 1$, while standard one-loop running carries that value to the low-energy effective theory. The tree-level eigenvalue and the running coupling remain distinct objects.

Rev29 also compresses the charged-lepton mass sector into a mixed-provenance chain (algebraic anchor + structural ratio + Koide closure; see below — tiers per the Appendix O re-tier), not a derivation of all three masses from $J_3(\mathbb{O}_s)$ alone. The authoritative charged-lepton ledger is Appendix O ("The authoritative charged-lepton ledger"); every shorter form in this suite defers to it. The charged-lepton ordering from the Jordan automorphism $\exp(t^* T_{\mathrm{half}})$, with $t^* = 2\pi/\sqrt{5}$, reverses the resolvent ordering $R_1, R_2, R_3 \mapsto R_3, R_2, R_1$ and yields the assignment $d = 1 \to \tau$, $d = 2 \to \mu$, $d = 3 \to e$. The muon-to-tau ratio (STRUCTURAL tier, Appendix O — the short form is a loaded correspondence, not a zero-parameter proof) is

$$\frac{m_\mu}{m_\tau} = \left(\frac{\phi}{\sqrt{5}}\right)^{8/3} \frac{\sqrt{2}}{10} = 0.059684,$$

where $p = 8/3$ is an exact dimension ratio *read as* a topological winding ratio (target-first provenance; the derivation is not closed — Paper 7 §4) and $\sqrt{2}/10$ is fixed by the $J_2(\mathbb{O}_s)$ two-generation subalgebra. With $m_\tau = 1776.93(9)\,\mathrm{MeV}$ this gives $m_\mu^{\mathrm{tree}} = 106.05\,\mathrm{MeV}$; one-loop QED matching gives $m_{\mu,\mathrm{phys}} = 105.72\,\mathrm{MeV}$, and the Koide closure then gives

$$m_e = 0.5076\,\mathrm{MeV},$$

within -0.66% of the PDG value. The honest provenance of the charged-lepton chain is therefore mixed: m_τ is the external anchor, m_μ is STRUCTURAL (Appendix O re-tier) via the resolvent

short form, and m_e is *Koide-closed* $\star\star\star\star$ under the external empirical rule $K = 2/3$, not derived from $J_3(\mathbb{O}_s)$ alone.

The suite remains explicit about what is not yet closed. The Higgs vacuum expectation value is measured rather than derived. The weak-angle bridge requires threshold-sector completion. The heavy-quark 7/3 lepton–quark bridge is a proved *algebraic* invariant ($\dim \mathrm{Im}\,\mathbb{O}_s/\mathrm{rank}\, J_3 = 7/3$, and equivalently $\det \mathrm{Gram}(J_{\mathrm{vac}}, J^{\#}_{\mathrm{vac}})/\mathrm{rank} = 7/3$), but its promotion to a physical S-matrix residue is *open and demonstrably hard*: a dedicated literature-and-construction scout (three independent model reviews plus four in-house computations, S116) found no theorem mapping a Jordan/cubic Gram determinant to an LSZ residue, OPE coefficient, or crossing ratio, and an explicit canonical Peirce-trilinear LSZ calculation returns mass-residue ratio 1 and inclusive colour ratio 3, not 7/3. The obstruction has a sharp structural cause: the "7" is the *dimension* of $\mathrm{Im}\,\mathbb{O}_s$, not a trace of the split-signature metric (whose signed value on $\mathrm{Im}\,\mathbb{O}_s$ is -1, signature $(3,4)$); the channel coefficients $(1,1,1,-1,-1,-1,-1)$ give positive Hilbert count 7 but signed trace -1, so a residue channel-sum weighted by the physical metric does not return $+7$. The sub-percent heavy-quark residuals additionally carry a weak-isospin T_3 signature that is structural, not derived (both the carrier search and a boundary-condition revision closed negative, A715–A717). The 7/3 residual is now sharply named: it is the response of a cotangent Yukawa source $B \in J_{12}^{\vee}$ entering the (still unwritten) gauged $E_{7(7)}/J_3(\mathbb{O}_s)$ fermion action as the source term $S_{\mathrm{edge}} - \langle B, D\rangle$ — a structural source term, not a derived amplitude (Appendix I). The suite is therefore best read as a sharply structured algebraic correspondence scheme whose PMNS/CKM mixing sector carries *no fitted mixing coefficients inside the loaded Route-B correspondence* — after the Rev29 row-by-row re-tiering (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`), **two** of the eight observables stand at DERIVED-CONDITIONAL or above ($\theta_{12}^{\mathrm{PMNS}}$, δ_{CP}; the physical θ_{23} row is LOADED-CORRESPONDENCE since A1566, its internal 7/16 identity theorem-grade), the CKM first row is LOADED because its route was selected against kaon data, δ_{CKM} is COINCIDENCE-CLASS, θ_{13} is structural, $|V_{cb}|$ is LOADED-CORRESPONDENCE (physical attachment) / Structural formula (A1566), and the CKM β consistency is unresolved (Paper 3); **no row may be cited as zero-parameter** — and whose charged-lepton sector is structurally pinned with mixed provenance, while bosonic extraction, AX6, and the formal heavy-quark amplitude proof remain the principal tasks.

The post-Rev14 work (Sessions 106–108) is recorded in the new Appendix H: a structural "time layer" on the certified $E_{7(7)}$ substrate in which the past↔future moment is realised as a genuine symplectic element $T \in E_{7(7)}$ with $T^2 = -1$, the gauge-side moment dial and the compact circle K of the D_{56} grading are shown to be one $E_{7(7)}$-conjugate circle, and the clean-lepton mass residual on the golden ladder is identified (empirically) with the Koide relation. Consistent with the Definition-A posture, these are classical, finite statements on the substrate; no public observable moves.

Rev16 added Appendix I, a SPECULATIVE *probe dynamics* that addresses the "no dynamics" gap identified by external review: a classical geometric bounce (the Koecher–Vinberg cone metric places the singular $\mathrm{Det} = 0$ boundary at infinite distance from the vacuum), an *overdetermined* golden vacuum (pinned independently by the invariant selector $\det G = 7$, whose integer encodes $\sqrt{5}$ and hence ϕ, and by a grading-coupled potential at unit coupling), and an $E_{7(7)}$ clock connection whose holonomy is the time-reversal element T with $T^2 = -1$. It is fenced as a diagonal-slice probe with $\det G = 7$ and unit coupling as structural inputs and the mass scale as an INPUT; no public observable moves. Rev17 extends Appendix I with two fenced additions: a consolidated record of the 27-run 7/3 amplitude-edge probe (A843–A863), which exhausts the tested finite endpoint/self-adjoint/source layer — pinning the edge operator, its readout, its uniqueness, and its physical object (a cotangent Yukawa source $B \in J_{12}^{\vee}$) while excluding the minimizer/integrate-out classes, leaving an explicitly *non-finite* residual (the generation assignment plus the full $E_{7(7)}/J_3(\mathbb{O}_s)$ action); and a moment–annulus synthesis ($T^2 = -I$ on the **56**; a Fibonacci charge↔mass bridge; a moment-graded dual collapse whose future/past curvature asymmetry is shown *not* to be the cosmological constant). Both remain fenced and move no public observable. Rev18 folded the in-house dynamics layer (S120–S124) into Appendix I: a *derived dynamical vacuum* (golden = tachyon-free Minkowski critical point of dyonic $\omega = \pi/4$ $SO^*(8)$), the *derived σ-odd-defect selection*

of ϕ *modulo the AX1 unit* (Coleman–Weinberg shown σ-even and barred from golden at every layer), and the *partial* stabilization status (the σ-even residual lift reduced to one named field-dependent tensor $C_{iab} = \frac{1}{2}\partial_a\partial_b m_i^2$ plus a no-scale dilaton). Selection is derived modulo AX1; full stabilization is not yet closed; no public observable moves.

Notation (σ). Three unrelated objects share this letter across the suite and all three appear here. Where an **involution** is meant it is σ_{frame}, Appendix I's Jordan frame involution $\rho \mapsto 1/\rho$. $\sqrt{\sigma}$ is the **string tension**. A numeral before the letter — 2.7σ, 0.36σ — is a **standard deviation**. Two further involutions are named apart elsewhere and never appear bare here: Appendix H's throat involution σ_C and Appendix F's time-mirror $\sigma = \Omega$; the split-octonion σ_{oct} does not appear in the typeset suite.

Rev33.2 Executive State Map (one page — read this first)

Reading rule (Rev32.1; carried at every mass/mixing table): tier words are the mathematical-status axis and physical attachment is priced separately; no row may be cited as zero-parameter; cross-scheme σ are comparator distances, not tensions (A1531/T2).

How to read revision labels: the suite revision is **Rev33.2** (point releases roll this sentence and every title; the family label is Rev33). "Rev12"/"Rev19"/"Rev25" etc. inside paper bodies are historical change-log stamps (when a result entered), not the current status; current status is controlled by this page, the claim tiers, and Appendix X. Titles roll at every release, including point releases; body stamps do not.

New at Rev31 / Rev31.2 (the selector arc, S274–S280).

- *The selector arc ran end-to-end* (Appendix E, Appendix S): the parent-action closure at complete-conditional tier with the 10+1 census; the $\sqrt{6}$ clock identity proved via the golden bridge H_φ; the $26 \oplus 10$ split grammar-forced at zero price; the rank-character variational representation theorem upgraded DERIVED-IN-CLASS; and the selection obstruction WIN-OBSTRUCTED: the four-relation principle Φ_{diag} is **adopted as a declared structural principle**, not derived. [P-007]
- *The two tens are proved inequivalent* (Casimirs 1/4 vs 1/6) and the intertwiner numeral $1/\sqrt{6}$ is derived as the selector's own Casimir.
- *Point releases* Rev31.1/Rev31.2 (S280): the explicit-destination link rewrite (the S274 link audit), suite hygiene, and the frozen Rev31.2 seal that the S281 cold round reviewed. No public observable moved anywhere in the Rev31 family.

New at Rev32 (the hostile-review + verification fold, S281–S288).

- *The A-1 hostile review ran 3/3 lanes against the sealed Rev31.2* and every confirmed defect is repaired in this revision: the revision-label sentence you are reading (point-release semantics), the Appendix E Route-B monotonicity proof replaced with the verified gap proof (conclusion unchanged), stale tier language re-tiered, and the referee dictionary restated at current status.

- *Comparators refreshed to PDG 2026 / NuFIT 6.1 with the cross-scheme adjudication*: percent offsets are primary and scheme-labelled; bare cross-scheme σ are retyped as naive comparator distances that set no status (Appendix X, Papers 2/3/5). The $d = 1$ reading of the Δm^2 ratio remains the one genuine falsified row, now printed with its dataset; the suite's own $d = 2$ rung (R_2) is printed beside it as a $\Delta m^2_{21}/\Delta m^2_{31}$ ratio prediction asserting $m_1 = 0$ — an assertion of the framework, not a derivation (Paper 5, Rev33.0).
- *The clause-27 carrier grammar is printed*: three tens named (physical $\cong$ selector at Casimir 1/6 $\neq$ boundary ad K_{dyn} at 1/4), the adjoint wall and quadratic-bridge census (Appendix S), and the dynamics-seed reconciliation beside the no-energy-order fence (Appendix X).
- *Selection frontier*: the C_0 six-coefficient independence theorem is banked at countermodel grade (scoped); **F-SEL(c) closes negative (scoped)** — Casimir-naturality, the lead refined-family candidate, *excludes* Φ_{diag} rather than deriving it, so Φ_{diag} remains a priced adoption and the live selection frontier is the G_7 motivation (Appendix E).
- *One independent dimensionful input class* — the three printed counts are reconciled under the input-count convention (Appendix X): one input class, two operational calibrations, three named anchors; the noun after the number changed, no count disappeared.

New at Rev28 (the twenty-round fold, D1126–D1146).

- *The boundary question is closed at its tier* (Appendix S): the ladder is the affine universal cover, the unpointed interface is obstructed, the affine support theorem recovers the finite projector from the continuum, and **AX-MG is proved a genuine independent boundary axiom in both clauses** (coefficient-one and degree-one) by exact countermodel. Nothing derivation-shaped remains there. The 0.0331562% row is dead twice over with located cause; 0.2744490308% stands as a legal miss.
- *The geometry spine is separated and closed* (Appendix T): "W" was two operators (W_{sig} preserves the KK grading, S_{shell} reverses it); there is no third cascade and no A_4 completion; one shared radius; the transition cocycle is derived at **zero continuous parameters**; the two signature sheets are *not* glued at the F_4 tier (Sylvester obstruction) but are two real structures in one complexified Albert fibre.
- *A four-dimensional Cartan carrier* closes inside the Albert signature sector ($\mathfrak{so}(5)$, an S^4 Cartan geometry), with metric and orientation *trace-derived* and triality acting as the imaginary unit; a MacDowell–Mansouri parent action for that sector follows as a CANDIDATE, explicitly not the full particle/gauge/mass action.
- *A reflection-positive state exists at finite regulator*, together with a positive nonlinear simplicity measure — so gravity-measure *existence* is closed at that tier, while the pure MM/EH weight is proved not to be a positive coercive covariance. [P-014] **Einstein universality** is the remaining named object.
- *The observation functor is built* (Appendix T, $\mathcal{O}_{\mathrm{MOS}}$): modular score $K_{\mathrm{rel}} = D_\varphi$ (the log leg a theorem, not an axiom), unital complete positivity fixing the normalization, an exact POVM, a unique minimal dilation, and Pin/OS evenization — unique under seven premises with verified teeth, and carrying **no continuous modulus**. What the microscopic theory has not done is *select* it. $\mathcal{O}_{\mathrm{MOS}}$ is the record-quotient functor only; the mass/generation observation map $\mathcal{O}_{\mathrm{gen}}$ is a distinct, unbuilt object (Appendix T).

- *Actuality splits.* The record type is derived (history $\iff$ character $\iff$ boundary record are one object); the realized value is proved to be **sample data, not a parameter**. Appendix L's firewall stays for the specific realized outcome.
- *Freedom is retyped.* The bits-plus-dimensions headline is **withdrawn pending typed rebuild** (its 17-dimensional decomposition is internally inconsistent as typed) and replaced by a typed ledger with no total — because the observation-functor faithfulness wall has no computed kernel. What can be said: *exactly one continuous modulus (ρ) survives anywhere in the end-state ledger.*
- *A laboratory notebook is added* (Appendix Y): all ninety adversarial rounds from S228 to S253 — dispatch, session, headline result, and the component that carries it — plus a reverse index from printed object to certifying round, the casualty register, and a disclosure. **The disclosure:** before Rev28 this suite cited no assessment above A1357, so the Rev26 and Rev27 folds had entered the tex without their citations. Nothing printed was unsupported; much of it was unfindable. Appendix Y is the audit trail, and four rounds (A1414, A1416–A1418) that had no ledger row anywhere are named in it.
- *Unchanged by all of it*: KILL-NODE 2 fired, now off shell and for the full microscopic loading; χ VALUE-OPEN; the engine unchanged; α and colour fenced; $V_{\mathrm{CKM}} = I$; no public observable moved.

Promoted at Rev25.

- *Nuclear cluster tier* $\to$ Paper 9: **seventeen** sharp binding energies deuterium$\to$calcium at pre-frozen bars (worst untyped 0.77%, ^{3}H; 0.37% is the worst of the $A \geq 5$ non-silicon continuation) *plus one typed obstruction* (^{28}Si at 0.334%: pre-answer bar 1% (D1060) PASSED; entered through the frozen disjunctive refinement gate (D1061 3(b), pre-send sha `85c69f1f...`) on its obstruction branch (A1357) — neither a sharp nor a miss at a pre-answer bar), the Hoyle pass, eleven correct sign calls, two glueball bands; closed-form composition ladder $\mathcal{T}_n = (n-2) + \mathrm{tr}_\varphi(R_n)$ with two blind vault matches (φ^2 at 26 ppm; $\mathcal{T}_9$ at 0.10%). Known soft flank stated in the Paper 9 posture box: charge magnitudes open.
- *The scale input, registered*: exactly **one** dimensionful input by theorem ($\dim H^2 = 1$, unconditional census) — the depth-cylinder Bargmann class, $\sqrt{\sigma}/m_e = 870.84$ ($\pm 1.51\%$); sealed null guard (Appendices O/X). Appendix O closed *ratios-complete.*
- *Occupation selector* $\to$ DERIVED-CONDITIONAL: three-layer derivation (mod-2/5/7), Appendix M. [LIB2-029]
- *Fifth-direction theorem tier* (Appendix P): 5D-necessity two-sided; tick = fifth momentum mod 4 (exact); flicker = NS spin structure.
- *Anomaly audit closed exactly* (T3-P; Paper 1/Paper 7/Appendix F) and the *E_7-closure theorem* (Paper 1).

Added, deliberately bounded. Appendix Q: the golden vacuum's classical spectrum closes at machine zero — *conditional tier throughout* (κ_{phys} = structural candidate; one recorded blind loss kept on the books); the live dynamics lane stays out until Rev26.

Promoted at Rev26 (the dynamics-frontier fold, S228–S240).

- *The interacting vertex* $\to$ Appendix R: the head wall ("no parent action") attacked directly and recounted as a package — one merged loaded import ($[\sigma_{\mathrm{CD}}]$ holonomy welded to its induced Cartan metric η_+), three continuous data $\{g_3, C_{\mathrm{sub}}, r_C\}$, and one loaded non-equivariant placement kernel

$\Pi_{5\leftarrow 6}$ — carrying the program's **first derived, parameter-free (discrete, derived) relative vertex sign**, $\mathcal{A}(4,5;1)/\mathcal{A}(5,4;1) = -1$ (structurally a trilinear antisymmetry; amputated, F4-fenced, not an amplitude). Conditional/candidate tier.

- *The σ-bridge scale registration* → Appendix O: the electroweak hierarchy is *paid* at conditional/loaded-depth tier, $\boxed{v_{\rm EW}/\sqrt{\sigma} = \varphi^{105/8} = 553.30}$ (framed-depth operator $13+\frac{1}{8}$) (registered at $\beta = 1$ from the two-element menu $\beta \in \{1,7\}$ of Appendix N's electroweak-permanence proposition — no intrinsic selector fixes $\beta = 1$), the suite's first foregrounded dimensionful result — a *framed registration relative to the marked compact-positive carrier*, not a frame-free prediction (the frame-free route is closed negative, Appendix F) — with $m_\tau/\sqrt{\sigma} = 4$ its exact dimensionless companion.
- *Theorem-tier upgrades:* the frame-free Higgs-line no-go now *derived* in-house (Appendix F, $E_{7(7)}$); the Marked Carrier Theorem (Paper 4, selection level); the 7/3 finite-carrier theorem (Appendix K).

Re-priced / added at Rev27 (the priced-selector fold, S242–S245).

- *The discrete selector wall, closed as a priced ledger* → Appendix S: the D1116–D1121 arc types the missing parent-action selector as $[j_A] \in \mathbb{R}\mathrm{P}^2$ (plus two magnitudes, a relative-phase/lift datum, and the standing orientation bit), with *six independence theorems* certifying that no banked principle selects it. One conditional deduction is new — the V-side quadrature $z_- = \pm i\, z_+$, forced by the candidate shape action (itself unpromoted). Conditional/candidate tier; no observable moves.
- *The exact φ-ladder mass completion is REFUTED at the ratio tier* (Appendix O): no single-χ additive closure reproduces the occupancy-support ratios (0/24 bijections); the ordered φ-depth *skeleton* and the few-percent *shape* survive, the exact completion does not. The support $\{3, 11, 17\}$ is independently corroborated as the adjacent-integer minimax (Appendix M).
- *Four in-house over-claims caught and corrected* (casualties #4–#7, Appendices S/X): the independent-channel doorway (retired for the paired-source theorem); "the action votes for the root line" (now common-lift-conditional); the blanket $\leq$ 2-parameter mass no-go (withdrawn — a fitted period-70 harmonic passes; watch item banked); and the depth-exclusion cut (retired).

Fenced / input (unchanged). α (five-legged fence, Appendix N); the compact colour real form; the gauge group at group level (two proven walls + uniqueness-of-completion, Appendix F); the generation count (input, with a forced-conditional skeleton); the external Koide rule $K = 2/3$. [P-011]

Still open (named). The 7/3 S-matrix/LSZ bridge (Paper 7); the charge-*magnitude* sector (Paper 9, §8 — signs and counts derived, magnitudes open); the two-circulation sector (the typed ^{28}Si residual); the continuum/Yang–Mills functor; the θ_{13} readout forcing and the $\delta_{\rm CKM}$ $5\varphi^{-5}$ derivation; the σ-bridge is *paid* at conditional/loaded-depth tier ($v_{\rm EW}/\sqrt{\sigma} = \varphi^{105/8}$ (registered at $\beta = 1$ from the two-element menu $\beta \in \{1,7\}$ of Appendix N's electroweak-permanence proposition — no intrinsic selector fixes $\beta = 1$), promoted above, Appendix O) — what stays open upstream is the *origin* of the loaded boundary $\mathfrak{B}_{\rm phys}$, and the loaded placement kernel $\Pi_{5\leftarrow 6}$ of Appendix R.

Live kill conditions. $\{31, 39\}$ (hash-stamped occupation falsifier, Appendix M); a second underivable dimensionful ratio (falsifies the one-input theorem chain, Appendix X); DUNE-era $\geq 3\sigma$ exclusion of δ_{CP} or the 7/16 octant branch (Paper 7); any scored miss at a *pre-answer* frozen bar, or any *untyped* miss at any frozen gate, or a broken sign call in the Paper 9 protocol; a confirmed non-hadronic a_μ anomaly (associator null, Paper 7).

Claim-language convention (binding across the suite)

Every load-bearing statement in this suite carries exactly one of five provenance tiers, and the language used for it is constrained by that tier. This convention is *binding suite-wide*: it governs the title pages, the abstracts, the section prose, and the status stars. The full per-quantity provenance ledger is Appendix X; the table below fixes the *words* each tier may and may not use, so that a claim's wording cannot outrun its evidence.

Tier	Meaning	Allowed words	Banned words
PROVED	Forced by the Jordan/Freudenthal algebra with no fit to particle data; a uniqueness/selection theorem closes it.	proved, theorem, forced, derived (from the algebra), closed	—
CERTIFIED	Machine-verified computation on the certified $E_{7(7)}/J_3$ substrate, reproducible from the bundled kernels; a *computational* result (often conditional on a named substrate/axiom), not an analytic uniqueness theorem.	computed, certified, "Theorem/Proposition (certified computation)", reproducible	proved (analytically), closed, "derived from nothing"
STRUCTURAL	Value correct and algebraically motivated, but *permitted* rather than *selected*; no uniqueness theorem closes it.	structural, algebraically motivated, permitted, target, correspondence, conditional (under a named axiom)	proved, theorem, derived, closed, "100/100", "zero free parameters", emergence
INPUT	Genuinely external: a measured dimensionful scale, an empirical selection rule, or a discrete datum the algebra does not fix.	input, external, measured, assumed, empirical, declared	derived, predicted, emergent, forced, "from nothing"
HISTORICAL	Retained legacy record, superseded by later work; kept for provenance only.	historical, legacy, superseded, retained for record	current, holds, established, the present state

Enforcement rule (S112). A claim may use PROVED-tier language only when a stated theorem closes it with no external input. A *conditional theorem* must name its load-bearing axiom inline (e.g. "conditional theorem under the channel-expectation readout axiom"); it is a STRUCTURAL claim with a derivation path, not a PROVED one. Phrasings such as "100/100," "all green," "zero free parameters" (for the suite as a whole), "closed," and "emergence" are retired except where a PROVED-tier theorem licenses them. [LIB2-233, LIB2-234] The dimensionful mass scale is INPUT throughout (the Honest Boundary; Appendix X), and no relabelling of J_{vac} from input to output is asserted until the full-J_3 dynamical lift succeeds (Appendix I).

Two-axis provenance (Rev27): mathematical status × physical attachment

A single provenance axis conflates two independent questions: (i) *is the algebraic or computational statement established?* and (ii) *is its identification with a physical observable derived?* A quantity can be theorem-grade on (i) while its physical attachment on (ii) remains loaded. From Rev27 every load-bearing claim therefore carries *both* a **mathematical status** — the five tiers above (PROVED / CERTIFIED / STRUCTURAL / INPUT / HISTORICAL) — *and* an orthogonal **physical attachment**: DERIVED-ATTACHMENT (the map to the observable is proved), CONDITIONAL-READOUT (attaches only under a named readout/axiom), LOADED-CORRESPONDENCE (true inside the chosen loaded map, whose boundary is a declared input), FITTED (matched to data), or EXTERNAL-INPUT (measured/empirical). *Physical attachment never inherits the mathematical tier*: an exact theorem about the algebra is not, by itself, a physical prediction until its attachment is stated. [LIB2-322]

Internal statement	Mathematical status	Physical attachment
$\det G = 7$	exact algebraic theorem	LOADED-CORRESPONDENCE: does not by itself give $\sin^2\theta_{23} = 7/16$ (kinematic normalization, Paper 3)
$7/3 = \mathrm{Tr}(\mathbf{1}\oplus\mathbf{3}\oplus\bar{\mathbf{3}})/\mathrm{Tr}(\mathbf{3})$	exact trace/index theorem	CONDITIONAL-READOUT as an *index*; *not* an LSZ residue — the metric-weighted-residue route is closed-negative (`s902`, App. I)
$8/3 = \dim \mathbb{O}_s/\mathrm{rank}\, J_3$	exact dimension ratio	LOADED-CORRESPONDENCE: not, alone, a mass exponent
T_{half} frequency $\sqrt{5}/2$	exact computation	CONDITIONAL-READOUT: identifying the orbit parameter with the measured Dirac phase needs the readout map
$z_- = \pm i z_+$	exact conditional extremum eq.	CONDITIONAL-READOUT: conditional on S_{shape}, an unpromoted candidate (App. S)

Crosswalk to the scorecard tier column (Rev32.9, S304.5). The scorecard and several paper tables carry a single *tier* column with a different vocabulary. It is a compressed reading of the two axes above, not a third system; each row's own two-axis statement governs where they differ.

Scorecard tier	Mathematical status	Physical attachment
Axiom	INPUT (a stated premise)	not applicable
Theorem	PROVED or CERTIFIED	DERIVED-ATTACHMENT only where the map to the observable is proved; otherwise the row also names its attachment
Derived-conditional	PROVED or CERTIFIED	CONDITIONAL-READOUT
Structural	STRUCTURAL	stated per row (CONDITIONAL-READOUT or LOADED-CORRESPONDENCE)
Loaded	any	a discrete choice made against data (a route, an octant, an ordering): FITTED in the discrete sense
Loaded-correspondence	any	LOADED-CORRESPONDENCE
Reproduced	STRUCTURAL or CERTIFIED	LOADED-CORRESPONDENCE: the construction reproduces data it was built toward
Coincidence-class	STRUCTURAL (candidate)	none established: a numeral agreement without a derivation
REFUTED	HISTORICAL	withdrawn

This axis is what makes the "zero-fit" statements precise: they are LOADED-CORRESPONDENCE claims (fit-free *interior*, declared-input boundary; Appendix X), not DERIVED-ATTACHMENT ones. The per-quantity mathematical tier remains as tagged throughout the suite; the physical attachment is stated in the Appendix X evidence column and, where load-bearing, inline.

Definition-A Boundary Ledger (Rev25)

So that no claim's wording can outrun its evidence, the load-bearing claims are ledgered here by tier before the reader enters the suite. Every use of "derived," "selected," "barred," or "closed" in the body points to a row below; a row marked OPEN or CONDITIONAL is never written as closed. The dynamics rows (I.1–I.6) refer to Appendix I; their reproduction artifacts ship in `machinery/` and are checked by `verify_rev19.sh`.

Claim	Tier	Status / inputs / repro artifact
Derived (no external selector)		
I.1 Golden = tachyon-free Minkowski critical point of dyonic $\omega = \pi/4$ $SO^*(8)$	CERTIFIED (cond. on the E_7 substrate)	reproducible: `s120c` + shipped `e7_substrate/`; Hessian $(0_-, 34_0, 36_+)$. Obs. moved: **no**
I.2 Coleman–Weinberg cannot *uniquely* select golden in the tested σ-equivariant layers	CERTIFIED no-go	`s123b`, `s124b`; bein-independent ($\mathrm{Aut}(J_3)$-equivariance forced). Obs.: **no**
I.6 $N_g = 3 = \chi(\mathbb{O}P^2)$ as a Morse count	CERTIFIED $\chi(\mathbb{O}P^2) = 3$; $= N_g$ as a correspondence (App. X: N_g an Input)	`s124c`; cell indices $\{0, 8, 16\}$. The generation→family / A1→Peirce *assignment* is OPEN. Obs.: **no**
Derived modulo the AX1 unit		
I.3/I.4 σ-odd defect: $F(\rho) = 1 \Leftrightarrow$ AX1; $+\frac{1}{2}\|B\|^2$ completion; radial $m^2 = 7/4$	CERTIFIED modulo AX1	`s123c`, `s124a`; unit magnitude $\gamma = 1 \equiv$ AX1 (named unit, not derived from nothing). Obs.: **no**
Named structural / source term (amplitude open)		
7/3 heavy-quark bridge = cotangent Yukawa source $B \in J_{12}^{\vee}$	STRUCTURAL / named source	`HeavyQuark_T3`; proved algebraic invariant ($\dim \mathrm{Im}\,\mathbb{O}_s/\mathrm{rank}$); amplitude promotion OPEN (unwritten $E_{7(7)}/J_3$ action). Obs.: **no**
Computed / open		
I.5 One-loop lift of the golden point's near-flat directions	COMPUTED (Appendix Q, Gate H: inertia $(9, 0, 1)$); a certified one-loop stationary point OPEN	tree: Morse–Bott minimum on a 6-dim flat branch reaching an unstable origin (`s1283`, `s1361`, `s1249`); the remaining flat direction is the no-scale INPUT. Obs.: **no**
Declared inputs (external / named)		
AX1 $\rho^2 - \rho - 1 = 0$ (the golden unit)	INPUT (named axiom)	assumed, not derived from nothing. Obs.: **no**
Mass scale + anchors: m_τ, v_{EW}, Λ_{G_2}, α, Koide $K = 2/3$, $N_g = 3$-as-engine-input	INPUT	declared in the `INPUTS` sheet / Appendix X (Honest Boundary). Obs.: **no**

Rev19 / S136 release boundary. Relative to Rev18 the engine advanced: m_s is now *derived* ($\Lambda_{G_2}\varphi\, 3^{1/4}$, S130) so $m_c = (7/3)m_s$ reads 1291.9 MeV (+1.54% → +1.48%); the electron Koide value reads $m_e = 0.5076$ MeV (S134); and the Outputs workbook gains five landscape explainer tabs (S133). The `SGTOE_Machine.xlsx` hash *at Rev19* was `sha256 7aef7f20...` (prior `3d2c7a0a`, `437ce51b`, `1fc4757e`). These are the only displayed-observable changes. The Appendix I dynamics fold (introduced Rev18) is unchanged: the golden critical point is *computed* on the dyonic $SO^*(8)$ substrate; σ-even Coleman–Weinberg cannot uniquely select golden within the tested σ-equivariant point-spectrum layers; the σ-odd defect selection is derived *modulo the AX1 unit*; and full stabilization is *not* closed — the one-loop inertia on the golden point's near-flat directions is computed ($(9, 0, 1)$, Appendix Q, Gate H), no certified one-loop stationary point exists yet, and the remaining flat direction is the no-scale scale/lift input. AX1 is named, not hidden.

How to read this suite: four layers

This suite is not one document but four, bound together and now separated in the PDF's bookmark panel. Knowing which layer a page belongs to tells you what kind of claim you are reading and how much weight it carries.

1. **Authoritative papers (Papers 0–9).** The claims. Anything the framework asserts about physics is stated here, with its tier and its premises attached. If a statement is not in this layer, it is not being claimed.
2. **Theorem and derivation appendices (Appendices A–T).** The work behind the claims: constructions, proofs, no-go theorems and the fences that scope them. A paper cites this layer; this layer does not enlarge on what the paper claims.
3. **Input and price ledger (Appendix X).** What the framework *assumes*. Every declared input, every unpriced clause, and for each one what would end the argument. A reader checking whether a result is genuinely zero-parameter should start here and not in the papers.
4. **Historical notebook (Appendix Y).** What was actually done, round by round, including the rounds that failed — currently A1300–A1572 (its span is printed in its own title; through Rev32.4 it stopped at A1500, sixty-two rounds short, and said so only there; Rev32.6 adds A1563–A1572). It is a locator and an audit trail, not a second statement of the physics. Its "Carried by" column names the component that carries each round's content — or records that nothing carries it yet.

Two things this separation deliberately does not yet do (Rev30). Paper 8 is the *junkyard* — deleted content, dead ends and junked ideas — so by function it belongs to the historical layer, but it is numbered and ordered as a paper and is left there rather than moved. Likewise a "computational supplement" would gather Appendices B, D and Q, which sit inside the theorem run. Grouping either would make the bookmark tree jump backwards through the page order, which is worse navigation rather than better; completing the separation for those two requires changing the canonical component order, and that is a papering decision, not a formatting one. Both are recorded as open.

The picture in three layers (Rev32.6; A1567 T3, house-edited). The suite carries two strong geometries that are not yet one object. The first is a *real hyperbolic scaling spine*: the reciprocal Jordan vacuum generates Peirce spectra, resolvent ratios, logarithmic mass depths, boost rungs, braid iterates and the shell-wide moiré law, connected by exact identities and changes of basis — while the same spectral law on different carriers is, by the suite's own rule, not a carrier identification. The second is a *compact phase spine*: on the live **56** the generator K gives a bounded E_7 clock circle and the half-turn $T^2 = -I$; on V_3, T_{half} gives the leptonic rotation with $\Omega = \sqrt{5}/2 = \phi - \frac{1}{2}$, an equality specific to the golden $(1, 3)$ eigenvalue that does not turn a real rung multiplier into a complex one. The third layer is *missing*: no theory-selected normal operator combines a real rung step with a nonzero compact phase (Appendix N, Part N.E), and the physical readout/basepoint layer remains loaded or model-only. The older wormhole/black-hole–moiré and "beating" language survives only with the verbs *models*, *carries* and *is compatible with* (Appendices O, H, E); the obstruction is primarily in the physics, and this paragraph exists so the prose no longer hides the separation.

1 Introduction

Rev29 marked the first point in the programme where the full mixing sector could be presented as closed rather than merely promising; **Rev29 withdraws the word "closed"**. Sessions 22–30 fixed the CKM sector, the PMNS reactor angle, and the leptonic Dirac phase inside the specified algebraic correspondence map, so the suite carries eight mixing observables with no fitted mixing coefficients *inside* that map. After the Rev29 row-by-row re-tiering (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`) two of the eight stand at DERIVED-CONDITIONAL or above ($\theta_{12}^{\mathrm{PMNS}}$, δ_{CP}; the physical θ_{23} row and $|V_{cb}|$ are LOADED-CORRESPONDENCE since A1566/D1244, Rev32.6, the internal 7/16 mismatch identity staying theorem-grade) and **none may be cited as zero-parameter**. The purpose of Paper 0 is to state that algebraic core cleanly, keep the measured-versus-derived boundary explicit, and give the reader one place where the current status of the entire bundle can be read without navigating the specialist papers first.

The companion manuscripts remain internal working papers rather than external review articles. Their job is to expose the algebra, the bookkeeping, the benchmark comparisons, and the still-open bridges in a form that can be audited and revised. Rev29 therefore uses a deliberately explicit vocabulary: some quantities are theorems, some are structural algebraic outputs, some are measured anchors, and some remain open. The new Appendix X, "Input Ledger" (titled "Zero-Parameter Input Ledger" through Rev32.8; renamed Rev32.9, R-13), lists the algebraic inputs of the zero-fit mixing sector and separates them from measured anchors in one place.

2 Operational axioms and current reading

AX1: Algebraic vacuum. The working vacuum is

$$J_{\mathrm{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1}), \qquad \phi = \frac{1+\sqrt{5}}{2}. \tag{2}$$

It fixes the Peirce spectrum $\lambda_1 = \phi$, $\lambda_2 = 1$, and $\lambda_3 = \phi^{-1}$.

AX2: Vacuum selector. The preferred vacuum $J_{\mathrm{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1})$ is fixed (algebraically, up to permutation) by the conditional selector theorem of Appendix E: unit determinant, inversion symmetry, and the DET-7 condition force $\rho^2 + \rho^{-2} = 3$, hence $\rho = \phi$ (equivalently, the golden-field units of $\mathbb{Z}[\phi]$ together with DET-7 give $\{1, 0, -1\}$ uniquely). The vacuum ratio ϕ is therefore an algebraically forced structural feature *given* the Appendix E premises (modulo the AX1 unit) — not a free parameter, but not premise-free either: Appendix E types DET-7 as a structural postulate and its selector theorem as conditional (Rev32.9; through Rev32.8 this said "not … an externally carried assumption").

AX3: Minkowski balance. The operational scheme uses the current vacuum-balance condition as part of the structural framework and not as a free fit parameter.

AX4: Non-associative CP source. The basic CP source is

$$[e_1, e_2, e_3] \equiv i, \tag{3}$$

so CP violation is treated as a structural consequence of the split-octonion sector rather than an added phase convention.

Current reading (entered Rev29). AX1, AX3, and AX4 remain active operational inputs. The vacuum selector is no longer carried as a free parameter in the working bundle. The suite still does *not* present the electroweak scale, the weak-angle threshold bridge, or the full rest-mass programme as solved. Those bridges remain named explicitly and are carried forward as the next research tasks.

3 Peirce decomposition and the cubic invariant

Relative to the Jordan frame $\{e_1, e_2, e_3\}$, the algebra splits into three diagonal idempotent lines and three off-diagonal Peirce blocks. The identity

$$\phi^2 + \phi^{-2} = 3 \tag{4}$$

remains a structural through-line. It gives the transparent form $\tan^2\theta_{12} = 3/\phi^4$, it underlies several resolvent-family ratios, and it is the simplest exact algebraic identity linking the outer Peirce weights.

The Yukawa sector has two layers. The same-block Peirce *carrier* invariant (the unit-eigenvalue theorem of A674) is

$$\mathcal{Y}_{\text{carrier}}(\Psi, \Phi_H) = \mathrm{Tr}\big[J_{\text{vac}} \circ (\Psi \times \Phi_H)\big], \tag{5}$$

while the literal one-generation SM Yukawa is the *cross-block* trilinear $\{J_{12}(H), J_{13}(\Psi_L), J_{23}(\Psi_R^c)\}$ with the physical Higgs in the vector $\mathbf{10} = J_{12} \oplus \mathbb{R}c_1 \oplus \mathbb{R}c_2$ (A686–A689). Placing both Ψ and Φ_H in the $(1,3)$ Peirce block (the carrier reading) forces

$$\Psi \times \Phi_H = -2\,\mathrm{Re}(\psi\bar{h})\, e_2, \tag{6}$$

so contraction with J_{vac} extracts the middle eigenvalue $\lambda_2 = 1$ and therefore fixes

$$y_t(M_{\text{Pl}}) = 1 \tag{7}$$

at tree level.

Peirce-to-physics dictionary (C3)

For a referee unfamiliar with the Jordan-algebra language, the following one-to-one map between Peirce sectors and physical observables summarises the mixing-sector ledger. All eight carry no fitted mixing coefficients *inside* the correspondence map.

Rev29 supersession — read before the table. As of Rev32 the status markings in the third column are *restated at the current two-axis status* (the pre-Rev29 "THEOREM ⋆100" markings survive only in sealed revisions Rev31.2 and earlier); the authoritative source remains the row-by-row re-tiering (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`). After propagation and the A1566 retype (Rev32.6), **two** of the eight sit at DERIVED-CONDITIONAL or above ($\theta_{12}^{\text{PMNS}}$, δ_{CP}, each with its premises printed; the internal 7/16 mismatch identity is theorem-grade and its physical θ_{23} attachment, with $|V_{cb}|$, is LOADED-CORRESPONDENCE); the CKM first row is LOADED because its route was selected against kaon data; δ_{CKM} is COINCIDENCE-CLASS; θ_{13} is structural and $|V_{cb}|$ is LOADED-CORRESPONDENCE (physical attachment) / Structural formula (A1566). **No row below may be cited as zero-parameter.** No number in the table changes.

Algebraic object	**Physical observable**	**Formula and status**	
$(2,3)$ Peirce block (8 off-diagonal degrees of freedom)	Solar mixing angle θ_{12}	$\tan\theta_{12} = \sqrt{3}/\phi^2$; DERIVED-CONDITIONAL (Rev29 re-tier; premises in Paper 3); $\|J_{\text{vac}}\|^2 = \phi^2 + 1 + \phi^{-2} = 3$	
$(1,2)$ Peirce block DET-7 Gram metric, $\det G = 7$	Atmospheric angle θ_{23}	$\sin^2\theta_{23} = 7/16$; internal mismatch identity theorem-grade; physical octant attachment LOADED-CORRESPONDENCE (A1566/D1244, Rev32.6; `s501` a candidate map, not a selector); $n_{23} = 7$ from $\det G(\text{diag}(\phi, 1, \phi^{-1})) = 7$ (DET-7 theorem; Gram matrix criterion); AX2 eliminated	
$(1,3)$ Peirce block D_4 triality $8_v \leftrightarrow 8_s$	Reactor angle θ_{13}	$\sin^2\theta_{13} = \sin^4(\pi/8) = (3 - 2\sqrt{2})/8$; STRUCTURAL $\star\star\star$ (round-trip framing T^2, exponent-2; forcing pending [s502/A1275]); J_{vac} at $\pi/4$ in 8_v, triality maps $\pi/4 \to \pi/8$	*Stamps*
V_3 rotation orbit $T_{\text{half}}\|_{V_3}$, frequency $\Omega = \sqrt{5}/2$	Leptonic CP phase δ_{CP}	$\delta_{CP} = -2\pi/\sqrt{5} = -160.997°$; DERIVED-CONDITIONAL (Rev29 re-tier; structural support; full eigenvalue–angular-velocity equivalence lemma forthcoming, App B.7); 0.36σ central-value, NuFIT 6.1 (NH), err $+26/-36°$	
Peirce block nesting $W_{13} \subset W_{12} \oplus W_{23}$	CKM magnitudes $\|V_{us}\|, \|V_{cb}\|, \|V_{ub}\|$	$\|V_{us}\| = \sin\theta_{12}/\sqrt{6}$; $\|V_{cb}\| = 1/(9\sqrt{7})$; $\|V_{ub}\| = \|V_{us}\|\|V_{cb}\|/\sqrt{6}$; pre-Rev29 marking $\star$100, superseded — the CKM first row is LOADED (route selected against kaon data) and $\|V_{cb}\|$ is LOADED-CORRESPONDENCE (physical attachment) / Structural formula (A1566)	
Sub-leading J_{vac} deformation	CKM Dirac phase δ_{CKM}	$\delta_{\text{CKM}} = \arctan\sqrt{3(\phi + 5\phi^{-5})} = 68.1297°$; COINCIDENCE-CLASS (rating pending the G_7 motivation; number retained); 0.8σ γ-fit / 1.6σ direct	
Split-octonion $(4,4)$ coupling weights			

in this table ($\star$100*,* THEOREM*,* Proved*) are the Rev29-era marking, superseded en bloc by the Rev29 re-tier (Paper 3 §"Row-by-row re-tiering"); the current tier of each row is the re-tier table's. Footer added Rev32.5.*

The three named operational anchors v_{EW}, m_τ, and Λ_{G_2} fix the electroweak scale, the lepton-tower

anchor, and the hadronic scale respectively (anchor inventory per the Appendix X input-count convention; α_s is a measured constant, not an anchor); the remaining entries are $J_3(\mathbb{O}_s)$-algebraic correspondences *at the tiers stated in each cell* (Rev29 two-axis status; no row here is citable as zero-parameter).

4 Technical clarifications

Tree-level vs. running Yukawa

Rev8 derives a tree-level eigenvalue and nothing stronger than that. Below the ultraviolet matching scale, the top Yukawa obeys the standard one-loop Standard Model equation

$$\frac{dy_t}{d\ln\mu} = \frac{y_t}{16\pi^2}\,\beta_t(y_t, g_1, g_2, g_3, \ldots), \tag{8}$$

so that the operational reading is

$$y_t(M_{\rm Pl}) = 1 \tag{9}$$

as an ultraviolet boundary value. The $y_t(M_Z) \approx 0.967$ printed beside it before Rev33.2 was not this value's running endpoint: the measured-side $\overline{\rm MS}$ coupling at M_Z is $y_t(M_Z) = 0.970 \pm 0.002$ — the SL-54 extraction run down from M_t (SL-57; `s1337`) — a data comparator, and the suite does not claim that $y_t(M_{\rm Pl}) = 1$ runs to it. The tree-level eigenvalue is exact inside the algebra; the loop evolution is the ordinary Standard Model bridge.

Split-octonion algebra and verification

All products are taken in the split-octonion algebra $\mathbb{O}_s$ with the Cayley–Dickson construction $(p,q)(r,s) = (pr + s^*q,\ sp + qr^*)$, the split inner product $\langle x, y\rangle_{CD} = \sum_{k<4} x_k y_k - \sum_{k\geq 4} x_k y_k$, and the standard Jordan product $x \circ y = \frac{1}{2}(xy + yx)$. The Freudenthal cross product is

$$x \times y = x \circ y - \tfrac{1}{2}\mathrm{Tr}(x)\, y - \tfrac{1}{2}\mathrm{Tr}(y)\, x + \tfrac{1}{2}[\mathrm{Tr}(x)\mathrm{Tr}(y) - \mathrm{Tr}(x \circ y)]\, I. \tag{10}$$

The suite uses these definitions consistently in the prose, in the appendices, and in the local verification kernels. The generator $T_{\rm half}$ of the V_3-rotation has been confirmed to lie in $\mathrm{Der}(J_3(\mathbb{O}_s)) = \mathfrak{f}_{4(4)}$ with derivation error 3.4×10^{-15} and rotation residual $\|\exp(t^* T_{\rm half}) \cdot J_{\rm vac} - J_{\rm vac}^{\#}\| = 2.1 \times 10^{-11}$ (kernel v3.9 E121, Session 33 — the kernel of that era; the shipped kernel is named in Appendix X).

Algebra selection: metric signatures and $\mathbb{O}_s$ (C new)

A natural question is why $J_3(\mathbb{O}_s)$ specifically, rather than some other Jordan algebra. The answer is a short chain of physical requirements standing on one *adopted premise* at its head (*Rev29*). Earlier revisions of this section described the chain as following "with no free choices"; that phrasing is withdrawn. It is not true of Step 1, and Step 1 is where the load sits. Paper 1 (§"Why $J_3(\mathbb{O}_s)$?" and abstract) already carries the corrected statement; this paper now matches it verbatim.

*Step 1 (*adopted premise*, not a derivation) — both Lorentz signatures are physically equivalent, and we* choose *a convention-neutral fibre.* [P-001] The two metric conventions $(1,3)$ and $(3,1)$ describe the same physical spacetime; no experiment distinguishes them. *We adopt* a sign-convention-neutral fibre:

placing both conventions on equal footing inside a single composition algebra *motivates* an inner product of signature $(1,3)+(3,1)=(4,4)$. *Rev29 correction.* The neutral $(4,4)$ signature is a *model choice* that places both conventions on equal footing, not a consequence forced by the convention equivalence alone; this is stated as an adopted premise, not a derivation. (Same wording, Paper 1 abstract and Paper 1 §"Why $J_3(\mathbb{O}_s)$?" item (2).) The equivalence of the two conventions does *not* by itself require an algebra carrying both at once: either convention alone is a consistent choice, and a theory built on one of them is not thereby refuted. What the premise buys is neutrality, not necessity.

Step 2 — the only real normed composition algebra with split signature $(4,4)$ *is* $\mathbb{O}_s$. Among the four normed division algebras $(\mathbb{R},\mathbb{C},\mathbb{H},\mathbb{O})$ and their split counterparts, the unique 8-dimensional algebra whose norm form has neutral signature $(4,4)$ is the split octonion algebra $\mathbb{O}_s$. The compact octonion algebra $\mathbb{O}$ has positive-definite norm $(+\cdots+)$ and is excluded because it does not accommodate the split signature required in Step 1.

Step 3 — three generations fix J_3 *(under the three-generation condition).* The cohomology $H^*(\mathbb{O}P^2;\mathbb{Z})$ of the octonionic projective plane is non-trivial in exactly three degrees $(0,8,16)$, and the Atiyah–Hirzebruch spectral sequence yields exactly three non-trivial generators, fixing the matrix size at 3×3. The unique 3×3 Hermitian matrix algebra over $\mathbb{O}_s$ is $J_3(\mathbb{O}_s)$.

Kernel confirmation (Session 33). The requirement is not merely conceptual: using the Cayley–Dickson split-octonion product with a Euclidean inner product (signature $(8,0)$) in the Jordan product gives $\dim \mathrm{Der}(J)=12$, not the expected $\dim \mathfrak{f}_{4(4)}=52$. Replacing the inner product with the $(4,4)$-signature split inner product $\langle x,y\rangle_{CD}=\sum_{k=0}^{3}x_k y_k-\sum_{k=4}^{7}x_k y_k$ restores $\dim \mathrm{Der}(J)=52$ exactly, with maximum residual $|M\cdot \mathrm{NS}|<2\times 10^{-15}$. The algebra itself rejects the wrong signature.

Summary. Physical equivalence of $(1,3)$ and $(3,1)$ $\rightsquigarrow$ *adopted premise*: a convention-neutral fibre of signature $(4,4)$ $\Rightarrow$ $\mathbb{O}_s$ uniquely (under the added requirement of an eight-dimensional real normed composition algebra) $\Rightarrow$ $J_3(\mathbb{O}_s)$ uniquely (three-generation condition). Only the second and third links are implications; the first is written $\rightsquigarrow$ because it is the adopted premise, not an inference. The argument is therefore *premise-plus-uniqueness* — a stated model choice followed by two genuine uniqueness steps — rather than a selection principle derived from experiment. So read, it remains independent of, and separate from, the numerical agreement of the mixing-sector predictions.

Rev29 re-tiering: the $(1,3)\cong(3,1)$ **premise.** The claim that convention equivalence *forces* the $(4,4)$ fibre, and the accompanying phrase "no free choices," are WITHDRAWN. The corrected statement, held in common with Paper 1 and Paper 4: *the neutral* $(4,4)$ *signature is a model choice that places both conventions on equal footing, not a consequence forced by the convention equivalence alone; this is stated as an adopted premise, not a derivation.* Steps 2 and 3 are unaffected — given the $(4,4)$ premise and an eight-dimensional real normed composition algebra, $\mathbb{O}_s$ is unique, and given the three-generation condition, $J_3(\mathbb{O}_s)$ is unique. The kernel confirmation below is likewise unaffected: it tests the algebra's response to the signature, not the provenance of the signature. Nothing downstream of the fibre changes tier; what changes is that the entry point no longer advertises the fibre as free of choices.

The $(4,4)$ signature is not merely a static convention choice; it has a dynamical reading. The two metric conventions $(1,3)$ and $(3,1)$ each correspond to a distinct vacuum-collapse cycle; their simultaneous presence creates an interference pattern — a moire of the two cycles — whose combinatorial structure is precisely the $(4,4)$ split-octonionic algebra. The SO(8) triality automorphism $\tau: 8_v\to 8_s\to 8_c$ of $\mathfrak{so}(4,4)$ is the order-3 twist connecting the two sectors (it is *not* an automorphism of $(\mathbb{O}_s,\times)$); its stabilizer $G_2^*=G_{2(2)}\subset SO(4,4)$ acts as the band-gap symmetry at the moire interface. Bosons

have zero winding in this moire ($V_{\mathrm{CW}}^{\mathrm{bos}} \equiv 0$, proved); fermions carry a Möbius half-twist ($V_{\mathrm{CW}}^{\mathrm{ferm}} \neq 0$, proved), with the vacuum minimum at $r^\star = \varphi^2$ selecting the golden-ratio hierarchy. The three Peirce idempotents $\{f_1, f_2, f_3\}$ are the three topological winding sectors; the three fermion generations are *conjectured* to be three winding levels of the moire. This is physical intuition, *not a theorem.* In particular, algebraic replication of three generations inside a single $J_3(\mathbb{O}_s)$ is *excluded*: three families need $3 \times 16 = 48$ fermionic dimensions, which do not fit in the 27 (8+8+8 = 24 off-diagonal, and triality maps $8_+ \to 8_- \to 8_v$, so the blocks are not three gauge-identical copies) [A691]. [LIB2-298] The natural alternative — a *continuous* topological realization in which the three generations are Dirac zero-modes over a moire background glued by the order-3 triality τ — is also *excluded* [A693]: the τ-invariant zero-mode $\Psi_0 = \frac{1}{3}(x + \tau x + \tau^2 x)$ is forced to carry an equal 1/3 weight in the vector block $8_v = J_{12}$ (kernel-verified), mixing a boson into the fermionic mode and ruining its Standard Model charges; moreover a one-cycle holonomy gives $\dim\ker(1-\tau) \leq 16$ and cannot multiply a 16-dimensional fiber into the 48 states three generations require. The only direction not excluded is a *discrete* (noncommutative-geometry) reading in which $\{f_1, f_2, f_3\}$ define a three-point internal space and the Dirac operator is a finite matrix; there the number three is an input, not yet derived. That discrete branch is now *built* (Rev32.3; lane A1549, house `s1133/s1134`): the replication map $\iota : C^*(|M|) \to C^*(J_{\mathrm{vac}})$, the unique unital $*$-isomorphism with $\iota(P_\lambda) = f_\lambda$, realises the discrete Kan extension $T \oplus T \oplus T$ with a common diagonal factor — aligned by construction, $V = I$ — so the moire story's surviving branch is a construction, not a to-do; it picks no branch, scale or scheme, and the number three remains an input. [LIB2-298] The open object is the *noncentral defect* (A1550/A1551): a registered off-diagonal source inducing $[Y_u, Y_d] \neq 0$, located by three coordinates — a staged intertwiner, the E_6/F_4 basepoint (Appendix T, a declared placeholder), and one Appendix-T door. The moire picture is therefore a *span narrative* whose discrete branch is built and whose continuous branch is refuted; it does *not* establish three generations by any continuous-topology or algebraic mechanism.

Remark (the shell-wide moire law; A1551 rider, Rev32.3). On the reciprocal $\mathbf{27} \oplus \mathbf{27'}$ shells, $S_d = \exp(d \ln\varphi\, D_{KK})$ carries the eigenvalue pair $(\varphi^{2d}, \varphi^{-2d})$ on every symplectically paired direction, and the resolvent pair per direction is $(R_d,\ -(1+R_d))$ with one conserved -1 per pair, $\mathrm{Tr}_{56} = -28$. The resolvent family, the S197 knot anchor and the moire two-cycle thus carry the same reciprocal spectral law, lifted to all 28 pairs of the **56** — one law seen on three carriers, not an identification of the carriers.

Lorentz-signature category (T4-H, partial resolution). The neutral $(4,4)$ signature is carried by the split-octonion/Clifford fibre, not by an eight-dimensional physical spacetime. The ghost-safe physical readout is a *frame restriction*:

$$V_{4,4} = W_{1,3} \oplus W'_{3,1}, \qquad \Pi_{\mathrm{phys}} : V_{4,4} \to W_{1,3}, \qquad \Pi_{\mathrm{phys}}^2 = \Pi_{\mathrm{phys}}.$$

Only $W_{1,3}$ is assigned physical Lorentz propagation; the complement $W'_{3,1}$ is an algebraic dual-frame sector, not a tower of propagating extra-time fields. The two-time ghost problem is thus avoided by category: the extra directions are never quantized as physical coordinates. The subgroup preserving the split is

$$\mathrm{Spin}(4,4) \supset \mathrm{Spin}(1,3)_{\mathrm{phys}} \times \mathrm{Spin}(3,1)_{\mathrm{alg}}, \qquad \mathfrak{so}(4,4) = \mathfrak{so}(1,3) \oplus \mathfrak{so}(3,1) \oplus (4_{\mathrm{phys}} \otimes 4_{\mathrm{alg}}),$$

with $28 = 6 + 6 + 16$ (kernel-verified). The off-block 16 is a representation-theoretic complement only (the vector–vector coset of [A714]); it is *not* identified with the $42 - 26$ scalar-count gap and is not a physical tower. This category was adjudicated by three independent assessments

(two cold) with unanimous convergence [A727–A727c]; the survivor is two-layered — the fibre-level $\mathrm{Spin}(1,3) \times \mathrm{Spin}(3,1)$ frame group and, at the Jordan layer, the compact rank-four Standard-Model input (which cannot embed in $\mathfrak{so}(4,4)$ itself: the maximal compact is $SU(2)^4$, all A_1 factors). The compact Standard Model gauge group thus remains the independent rank-four input used elsewhere in the suite; $B - L$ is constructible but not gauged at this stage [A710]. A dynamical reduction (gauge-quotient) route would require a constraint algebra removing three extra timelike directions and is not constructed; the holographic/interface reading remains physical intuition. The external frame selector leaves the Peirce blocks, J_{vac}, and the proved Coleman–Weinberg dichotomy untouched [A728], so the suite's proved results are insulated from this category choice.

A decorative combinatorial illustration (the Boolean-hypercube / I Ching trigram indexing of the $\mathbb{O}_s$ basis, formerly Box C.1) has been moved to Paper 8 §Curios; it is an observation on shared combinatorics [A626], not load-bearing structure.

Cubic invariant terminology

The phrase "cubic invariant" is used in two related senses in the literature. The global Jordan cubic norm $\mathrm{Det}(Q)$ classifies orbits. The local trilinear form $\mathrm{Tr}[J_{\mathrm{vac}} \circ (\Psi \times \Phi_H)]$ is the unique cubic object used here to close the Yukawa sector while preserving the off-diagonal Higgs geometry. Rev8 separates those two roles explicitly.

5 Charged-lepton and rest-mass mechanism: current Rev29 status

The electron baseline inherited from Rev8,

$$m_e^{(0)} = M_{\mathrm{Pl}} \exp\left[-\frac{\pi}{8\alpha}\right] \phi^4 \sqrt{2} = 0.5036 \text{ MeV}, \tag{11}$$

now functions as a historical semiclassical cross-check rather than as the operative closure mechanism. The current Rev29 charged-lepton chain is the QED-corrected Koide closure:

$$\delta_{\mathrm{QED}} = \frac{3\alpha}{4\pi} \ln \frac{\Lambda_{G_2}^2}{m_\mu^2} = +0.3125\%, \tag{12}$$

$$m_{\mu,\mathrm{phys}} = \frac{106.05}{1.003125} = 105.72 \text{ MeV}, \tag{13}$$

followed by

$$(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = \frac{3}{2}(m_e + m_\mu + m_\tau), \tag{14}$$

which yields

$$m_e = 0.5076 \text{ MeV} \qquad (-0.66\% \text{ PDG}). \tag{15}$$

Thus m_τ is the single charged-lepton anchor, m_μ is fixed by the resolvent short form (Structural tier, Appendix O) plus one-loop QED matching, and m_e is fixed by Koide using the QED-corrected muon mass. No phenomenological electron phase dressing remains in the current charged-lepton sector.

6 Operational Rev29 ledger

Table 1: Operational Rev29 ledger. **Rev32.1 reading rule: tiers name mathematical provenance; physical attachments are stated separately; no row may be cited as zero-parameter; displayed σ values are named comparator distances, not profile likelihoods.**

Quantity	Expression or source	Status	Comment
y_t	$\mathrm{Tr}[J_{\mathrm{vac}} \circ (\Psi \times \Phi_H)]$	ALGEBRAIC	Tree-level eigenvalue 1 at M_{Pl} (a boundary value); the measured-side $y_t(M_Z) = 0.970 \pm 0.002$ (SL-57) is a data comparator, not its endpoint.
V_{us}, V_{ud}, V_{ub}	Peirce J_{12} block and hierarchy relation	ALGEBRAIC	First-row CKM relations carried structurally inside the current map.
δ_{CP}	AX4 non-associator phase map	ALGEBRAIC	Geometric phase target used in the PMNS sector.
$\sin^2\theta_W$	tree-level exceptional ratio	ALGEBRAIC	RG bridge to the measured Z-pole quantity remains a separate QFT step.
$SU(3)_c \times SU(2)_L \times U(1)_Y$	Peirce decomposition and slot reduction	ALGEBRAIC	Working gauge structure of the suite.
m_e	QED-corrected Koide chain	KOIDE-CONSISTENT	$m_e = 0.5076\,\mathrm{MeV}$ (-0.66% PDG); depends on external empirical $K = 2/3$; no electron phase dressing remains.
$v = 246\,\mathrm{GeV}$	electroweak scale	MEASURED	Measured, not derived, in Rev8.
AX2 vacuum selector	structural fixed-point theorem for ϕ	THEOREM	Working vacuum $\mathrm{diag}(\phi, 1, \phi^{-1})$ no longer treated as a free parameter in Rev29.

7 Roman surface model of generation mixing

The off-diagonal Peirce closure $P_{ij} \circ P_{jk} \subset P_{ik}$ in $J_3(\mathbb{O}_s)$ has a concrete geometric realisation as the *Roman surface* (Steiner surface, 1840) [3], a degree-two map $f : \mathbb{RP}^2 \to \mathbb{R}^3$:

$$(x', y', z') \longmapsto (y'z',\, z'x',\, x'y'). \tag{16}$$

This replaces the earlier intuition picture with a mathematically precise object. The structural correspondences are:

- Three double-point lines of the Roman surface $\leftrightarrow$ three off-diagonal Peirce blocks ($\mathbb{Z}_2^2$ grading).
- Six pinch points (at $(\pm\frac{1}{2}, 0, 0)$ and cyclic permutations) $\leftrightarrow$ rank-drop points of the diagonal Jordan adjoint; the determinant-zero locus maps to the Roman double-line skeleton [A680].
- Non-orientability $\leftrightarrow$ Möbius boundary condition on $(1,3)$-representation spinors (Theorem A.7.4).

Setting diagonal anchors $x' \sim \varphi$, $y' \sim 1$, $z' \sim \varphi^{-1}$ for Generations 3, 2, 1 respectively, the cross-product inverts the hierarchy in the quark sector: Gen 1 quarks map to the largest Peirce block P_{23} ($\lambda_{23} = \varphi^{1/2}$), acquiring the largest confinement-driven constituent mass. Leptons follow the primary anchor hierarchy; quarks follow the inverted cross-product hierarchy. This lepton/quark duality is a rigid algebraic consequence of the embedding, not a phenomenological assumption. [A632]

8 Known gaps, deferrals, and tensions

Known gaps and deferrals.

(a) **Vacuum theorem carried.** The working bundle no longer treats ϕ as a free input, and the derivational presentation is printed: Appendix E gives *two* explicit proofs — Route A (printed and valid at the conditional-selector tier) and Route B (printed, with the uniqueness step repaired at Rev32 via the verified order-gap argument; conclusion unchanged).

(b) **Higgs VEV.** The electroweak value $v = 246\,\text{GeV}$ is measured rather than derived. The algebra fixes dimensionless structure; the weak scale remains an external anchor.

(c) **Weak-angle RG bridge.** The tree-level weak-angle output requires renormalization-group running, threshold corrections, and scheme choice before comparison to the measured Z-pole quantity.

(d) **Electron mass: Koide-closed under external $K = 2/3$.** The electron mass is *Koide-closed* $\star\star\star\star$ via the QED-corrected Koide chain [A636, A637]: $m_e = 0.5076\,\text{MeV}$ (-0.66% PDG). This result is *conditional on the external empirical Koide rule* $K = 2/3$, not derived from $J_3(\mathbb{O}_s)$; an in-framework derivation of the *value* $K = 2/3$ is not yet available. The *shape* relation $Q = 2/3 \Leftrightarrow r = \sqrt{2}$ *is* derived (Appendix O, "The Koide shape"): the shape is derived, the value is an external selector, and the two are not the same claim. The Rev8 instanton-phase ansatz and associated bookkeeping are superseded and retired.

(e) **Charged-lepton chain.** The charged-lepton sector is an advanced compressed chain with mixed provenance: m_τ is the algebraic anchor, $m_\mu = 106.05\,\text{MeV}$ is STRUCTURAL (Appendix O re-tier) via the resolvent short form, and $m_e = 0.5076\,\text{MeV}$ is *Koide-closed* $\star\star\star\star$ under external $K = 2/3$. It is not a derivation of all three charged-lepton masses from $J_3(\mathbb{O}_s)$ alone.

9 Path to falsifiability

Rev29 turns the remaining rest-mass validation steps into an explicit protocol rather than a future aspiration.

(1) Use the m_μ/m_τ structural formula (Appendix O tier) as the charged-lepton family anchor for the next mass-operator kernel.

(2) The electron mass $m_e = 0.5076\,\text{MeV}$ (-0.66% PDG) is *Koide-closed* $\star\star\star\star$ under external $K = 2/3$ via the QED-corrected Koide chain [A636, A637] — conditional on Koide, not a $J_3(\mathbb{O}_s)$-internal derivation. The next mass-sector task is the formal S-matrix proof of the 7/3 amplitude bridge for m_b and m_c.

(3) Construct the formal S-matrix proof that the Peirce-closure amplitude equals $n_{23}/\text{rank}(J_3) = 7/3$ for the heavy-quark bridge.

(4) Treat failure of the 7/3 S-matrix amplitude derivation as a failure of the heavy-quark bridge mechanism, not of the fit-free PMNS/CKM mixing sector or the algebraic m_μ/m_τ ratio.

10 Conclusion

Paper 0 states the Rev29 programme in its current honest form. The cubic invariant, the Peirce decomposition, and the benchmark ledger remain the algebraic core. The weak-angle AX6 selector and the heavy-quark S-matrix amplitude bridge remain the principal open items, while the PMNS/CKM mixing sector carries *no fitted mixing coefficients inside the loaded Route-B correspondence* (six quantities take exact algebraic formulae there, of which **two** — θ_{12} and δ_{CP} — survive the Rev29 row-by-row re-tiering and the A1566 retype (Rev32.9: through Rev32.8 this said three) (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`) at DERIVED-CONDITIONAL or above; the CKM first row is LOADED, δ_{CKM} is COINCIDENCE-CLASS, θ_{13} is structural, $|V_{cb}|$ is LOADED-CORRESPONDENCE (physical attachment) / Structural formula (A1566), and **no row may be cited as zero-parameter**; and the CKM $\beta = 22.5°$ vs. reconstructed $23.44°$ consistency is unresolved, Paper 3) and the charged-lepton mass sector is structurally pinned with mixed provenance (anchor m_τ + algebraic m_μ/m_τ + Koide-closed m_e under external $K = 2/3$; the compact ratio's relation to the two-gap operator of Appendix O is a pending dual-route reconciliation, not a completed derivation). [LIB2-235] The framework is presented as a sharply structured algebraic correspondence scheme. The two principal open items—the weak-angle AX6 selector and the formal S-matrix promotion of the 7/3 lepton–quark bridge—are now *characterized* rather than merely outstanding: the latter is established as an algebraic invariant and shown to require genuinely new exceptional-field-theory machinery for its amplitude form (Appendix I). Neither blocks publication of the fit-free PMNS/CKM mixing sector or the structurally pinned charged-lepton sector; both are stated as falsifiable, well-posed problems rather than completed results.

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Leibniz Quantum Beats Newton

Paper 1 (Rev33.2): Peirce Decomposition, Algebra Selection, and Gauge-Group Assignment

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Abstract

This paper isolates the Peirce decomposition of the split exceptional Jordan algebra and states the gauge-group *assignment* — the input label identified (not produced) by the Peirce frame — in its updated form. The central point is that the working algebra is not chosen ad hoc: the physical equivalence of the $(1,3)$ and $(3,1)$ Lorentz-signature conventions *motivates adopting* a sign-convention-neutral $(4,4)$ fibre, and *under the added requirement* of an eight-dimensional real normed composition algebra this selects the split octonion algebra $\mathbb{O}_s$. (The neutral $(4,4)$ signature is a *model choice* that places both conventions on equal footing, not a consequence forced by the convention equivalence alone; this is stated as an adopted premise, not a derivation.) The three-generation condition then fixes the 3×3 Hermitian Jordan algebra $J_3(\mathbb{O}_s)$. All local split-octonion products are understood in Cayley–Dickson form with the SplitCD inner product; the older Fano/sign-flip surrogate is not used. Once the vacuum $J_{\text{vac}} = \text{diag}(\phi, 1, \phi^{-1})$ is fixed, the three Peirce blocks carry inequivalent algebraic roles that organize colour, weak, and Yukawa structure. Legacy Route A flavour identifications are retired; the live bundle uses the Route B canonical CKM entry carried in Paper 3, which Paper 3 types LOADED (selected against kaon data; Rev32.9 — through Rev32.8 this said "theorem-level").

1 Introduction

Paper 1 states the gauge-sector claim as cleanly as possible. Within the working $J_3(\mathbb{O}_s)$ map used by the suite, the Peirce decomposition of the chosen vacuum *identifies* the operative gauge assignment

$$SU(3)_c \times SU(2)_L \times U(1)_Y \tag{1}$$

without any additional gauge-group-selection axiom. The gauge group is thus *identified* by the algebra as a group-level *input* — not *produced* by it: its emergence as a maximal-supergravity gauging is obstructed by two representation-theoretic walls, and its low-energy completion is unique only within a stated restricted class, up to $U(1)_{B-L}$ and global form (Appendix F). The orbit selector AX6_{pol} ($\Lambda_\star = 3P/Q(Y) = 6\phi - 1$ on the exact-colour orbit; the degree-six Det^2 object $\cos(6\psi) = \frac{1}{2}$ once printed under the same name is a distinct, retired selector — Paper 4, Rev32.2) is an independent postulate required to fix the Weinberg-angle branch within the identified group, and is discussed separately in Paper 4. In Rev29 this claim is strengthened by an upstream algebra-selection argument: $J_3(\mathbb{O}_s)$ is the unique 3×3 Hermitian Jordan algebra over the unique real normed composition algebra compatible with a neutral $(4,4)$ norm. The same corrected Cayley–Dickson algebra now underlies the whole bundle.

2 Why $J_3(\mathbb{O}_s)$?

The algebraic core is fixed by a short chain of physical requirements.

(1) The metric conventions $(1,3)$ and $(3,1)$ are physically equivalent.

(2) *We adopt* a sign-convention-neutral fibre; placing both conventions on equal footing in a single composition algebra *motivates* a neutral $(4,4)$ norm (a model choice, not forced by the equivalence alone).

(3) *Under the added requirement* of an eight-dimensional real normed composition algebra, the unique signature-$(4,4)$ realization is $\mathbb{O}_s$.

(4) The three-generation condition fixes the 3×3 Hermitian Jordan algebra over $\mathbb{O}_s$.

Kernel verification sharpens the same statement: with the wrong (Euclidean) inner product the derivation algebra has dimension 12, whereas the SplitCD inner product gives $\dim \mathrm{Der}(J_3(\mathbb{O}_s)) = 52 = \dim \mathfrak{f}_{4(4)}$.

3 Peirce decomposition at the golden-ratio vacuum

Let

$$J_{\mathrm{vac}} = \phi e_1 + 1 \cdot e_2 + \phi^{-1} e_3 \tag{2}$$

in a Jordan frame $\{e_1, e_2, e_3\}$. Then

$$J_3(\mathbb{O}_s) = \bigoplus_{i=1}^{3} \mathbb{R} e_i \oplus J_{12} \oplus J_{13} \oplus J_{23}. \tag{3}$$

Each off-diagonal block is eight-real-dimensional and carries split-octonion data; the diagonal lines carry the spectral weights $\lambda_1 = \phi$, $\lambda_2 = 1$, and $\lambda_3 = \phi^{-1}$.

The $E_{6(6)} \to SO(5,5) \times SO(1,1)$ branching $27 \to 16 + 10 + 1$ has three conjugate realizations, one per primitive idempotent c_k (A684): the suite uses the c_3-selected branch, $16 = J_{13} \oplus J_{23}$, $10 = J_{12} \oplus \mathbb{R}c_1 \oplus \mathbb{R}c_2$, $1 = \mathbb{R}c_3$, consistent with $t_R \in J_{13}$ (A674) and J_{12} as the CKM sector. The three branches are related by the S_3 outer automorphism (triality) of D_4. The Jordan adjoint map $X^{\#} = J_{\mathrm{vac}}^{-1}$ (Weyl reflection $c_3 \leftrightarrow c_1$, A680) sends the active vacuum to the conjugate branch while leaving the spinor sector $J_{13} \oplus J_{23}$ invariant: J_{12} is the vector (bosonic) block in both the c_3 and c_1 branches.

3.1 Roman surface and the diagonal Jordan adjoint [proved, A680]

Let $D = \{\mathrm{diag}(x,y,z) : x,y,z \in \mathbb{R}\} \subset J_3(\mathbb{O}_s)$ be the diagonal cubic Jordan subalgebra. For $X = \mathrm{diag}(x,y,z)$, the quadratic Jordan adjoint is

$$X^{\#} = \mathrm{diag}(yz,\, xz,\, xy). \tag{4}$$

Thus the classical Roman-surface map

$$T : S^2 \to \mathbb{R}^3, \qquad T(x,y,z) = (yz, xz, xy),$$

is precisely the diagonal coordinate form of the Jordan adjoint $X^{\#}$, restricted to the unit sphere: the homogeneous map $X \mapsto X^{\#}$ is quadratic, so on S^2 it acts on $X/\|X\|$ (domain stated Rev32.9, A1587). Since $T(-x,-y,-z) = T(x,y,z)$, the map descends through the antipodal quotient $S^2/\{\pm 1\} = \mathbb{RP}^2$, and its image is the compact Roman (Steiner) surface — the bounded parametrized image. That image is a proper subset of the real zero locus of the Steiner quartic $y^2z^2 + x^2z^2 + x^2y^2 - xyz = 0$, which also contains the unbounded coordinate axes; verifying the quartic therefore does not by itself establish equality with the image (Rev32.9, A1587).

The three Roman coordinates carry a direct Peirce-triality reading: under S_3 permutation of (x,y,z), the output coordinates (yz, xz, xy) are permuted equivariantly, mirroring the three pairwise Peirce channels J_{23}, J_{13}, J_{12}. (Note: S_3 is the equivariance group of the map, not the quotient group producing $\mathbb{RP}^2$; the projective topology arises from the antipodal $\mathbb{Z}_2$.)

The Jordan determinant $N(X) = xyz$ vanishes on the zero-divisor locus. Under T, this locus maps to the three self-intersection double-line axes of the Roman surface. [LIB2-333] The six pinch points (at $(\pm\frac{1}{2}, 0, 0)$ and cyclic permutations) arise from the rank-drop points of $\mathrm{d}T$.

The vacuum $J_{\mathrm{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1})$ satisfies $J_{\mathrm{vac}}^{\#} = \mathrm{diag}(\phi^{-1}, 1, \phi) = J_{\mathrm{vac}}^{-1}$ (Jordan inverse), consistent with $\det J_{\mathrm{vac}} = 1$. The homogeneous adjoint thus sends the golden-ratio vacuum to its Weyl reflection; the unit-sphere Roman map sends $J_{\mathrm{vac}}/2$ (since $\|J_{\mathrm{vac}}\|^2 = 4$) to $J_{\mathrm{vac}}^{\#}/4$ — the same direction at a quarter of the scale (Rev32.9).

Since T is quadratic (degree 2), the antipodal invariance $T(-u) = T(u)$ mirrors the fundamental property of spin-$\frac{1}{2}$ states: physical observables (mass terms, currents) are quadratic in the spinor amplitude and hence invariant under $\psi \to -\psi$, requiring a 4π rotation to return the spinor to itself. *Rev29 correction.* The antipodal $\mathbb{Z}_2$ on the Peirce diagonal *models* the spinor double-cover parity. The $\mathbb{Z}_2$ producing $\mathbb{RP}^2 = S^2/\{\pm 1\}$ and the $\mathbb{Z}_2$ generating the spinor double cover $\mathrm{Spin}(3) \to SO(3)$ are abstractly isomorphic and carry the same sign-parity bookkeeping; that is an *interface-type* statement — a correspondence between two $\mathbb{Z}_2$ actions on two different spaces — and not an identity of the two $\mathbb{Z}_2$'s. Earlier revisions asserted that they *are* "the same $\mathbb{Z}_2$"; that assertion is withdrawn. The antipodal involution here acts on the diagonal S^2 of Jordan eigenvalue directions, not on a spinor space, and no spinor-bundle map realizing the correspondence is constructed (the explicit Möbius spinor bundle for the Peirce blocks remains [OPEN] below). The half-spinor 8_+ representation of $\mathrm{Spin}(4,4)$ carrying t_R (with $C_X(8_+) = 4$ [PROVED, A670]) exhibits the 720° property independently; the diagonal Jordan adjoint provides a geometric model *of* that $\mathbb{Z}_2$ structure at the level of the Peirce diagonal, and nothing beyond a model is claimed.

Roman–Jordan adjoint: [PROVED]. S_3 equivariance of T: [STRUCTURAL]. Antipodal $\mathbb{Z}_2$ *models* (does not equal) the spinorial double cover parity: [STRUCTURAL], interface-type (*Rev29*). Explicit Möbius spinor bundle for Peirce blocks: [OPEN]. LQG monad-cell interpretation: [SPECULATIVE].

4 Gauge-group assignment (the input label and its two-sided boundary)

Gauge-group reading (claim superseded; see boundary box below). The Peirce decomposition *identifies and constrains* the working gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ through the reduction of the slot symmetry from S_3 to a colour-preserving branch — a *labeling* of derived structure, not an emergence or selection mechanism. An earlier formulation of this box could be read as an emergence claim; that reading is superseded by the theorem-grade gauge boundary below (and Appendix F): the gauge group is an INPUT, identified by the Peirce frame but not produced by it. No extra gauge-group-selection mechanism is used in the Rev29 observable map.

Charge/gauge provenance: the terminal is possible-not-necessary, and it is final. The gauge content is *algebraically constructible and physically realizable, but not forced.* Two halves. (i) *Algebra — closed:* the full FTS conformal $\mathfrak{sl}(2)$ acting on $56 = J_3 \oplus \mathbb{R} \oplus \mathbb{R} \oplus J_3$ *is* the spin-3/2 charge clock ($56 \downarrow = V_{3/2} \oplus 26\, V_{1/2}$), so the clock target is canonical *once the full FTS local system is admitted*; the scalar-pair-only $\mathfrak{sl}(2)$ is a distinct class carrying no $V_{3/2}$ and cannot export the relevant structure [A1262, verified in-house]. (ii) *Physical realization — contingent:* the bare Möbius throat carries only the trace-0 half-twist S, whereas the clock is the trace-3 figure-eight $M_3 = T^3 S$; these are disjoint $SL(2,\mathbb{Z})$ conjugacy classes (elliptic versus hyperbolic), so no change of basis bridges them. The missing datum is the T^3 trefoil / three-generation winding, which enters as framing-blind homotopy *input.* Whether that input is forced was attacked five independent ways — static forcing principles, necessity audit, worldline composition coherence, algebraic reachability, and framed-ribbon geometry — and all five returned *not forced* [A1260, A1261, A1265, A1267, A1266]. This is a *settled negative*, not an open question, and it is the formal reason the gauge group is carried as an INPUT in the box above rather than derived.

The logic is operational. The three slots begin with an S_3 permutation symmetry; the chosen vacuum breaks that symmetry spectrally because $\phi \neq 1 \neq \phi^{-1}$. Colour acts on the block triplet, the weak doublet is read from the off-diagonal Higgs/Yukawa block, and hypercharge is the residual diagonal weighting compatible with the same frame. No legacy Route A flavour mechanism is invoked here; the current bundle treats Route A as dead and uses the Route B normalization of the mixing paper, which that paper types LOADED (selected against kaon data).

Gauge-boundary update (S75–S80; full statements and proofs in Appendix F). The status of the gauge group as an INPUT is now bounded on both sides by theorem-grade results obtained on the certified $E_{7(7)}$ embedding-tensor substrate. *(i) Obstruction (two walls):* no gauging of maximal N=8, D=4 supergravity, in any symplectic frame, has a gauge algebra containing a compactly embedded $\mathfrak{su}(3) \oplus \mathfrak{su}(2)$ — the branching-wall theorem — and no gauging contains a *unitary-class* $\mathfrak{su}(3)$ (the matter-colour embedding $\mathbf{8} = \mathbf{3} \oplus 5 \cdot \mathbf{1}$) — the charge-wall theorem; only vector- and adjoint-class $\mathfrak{su}(3)$'s are ever gauging-realizable. Gauge-group *emergence as a gauging* is closed negative; $\mathfrak{su}(2)_L$ is exactly the piece that can never fit, and the right algebra, where it fits, provably never carries the right representation. *(ii) Uniqueness of completion:* among compact low-energy completions gauging the full derived colour + weak action on exactly the derived sixteen fermions with the derived hypercharges and 10_H-only scalar closure, $G_{\rm SM}$ is unique up to $U(1)_{B-L}$ (extra Cartan computed unique, $3(B-L)$) and up to global structure $\Gamma \in \{1, \mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_6\}$. Uniqueness-of-completion is NOT emergence; the input label stands. *(iii) Shadow selection:* the maximal derivable remnant, an $\mathfrak{su}(3) \oplus \mathfrak{u}(1) \oplus \mathfrak{u}(1)$ of shadow type, is selected *dynamically* — vacuum stability of the missing-16 scalar sector fires uniquely at $SO(6,2)$ within the scanned family, whose dyonic completion is a published Minkowski vacuum with residual $SO(6) \times SO(2)$; the SM charge *labels* for the two $\mathfrak{u}(1)$'s failed their test at the origin point (S78, rep-level; pre-registered demotion applied), were excluded for the entire 36-parameter vacuum family by counting (S78b), and are excluded for *every* gauging by the charge wall (S80): the shadow is structural, not the SM in disguise. None of this changes the observable map of this paper; it fixes the epistemic status of its gauge input.

5 Structure-algebra viewpoint

The reduced structure algebra and derivation algebra are

$$\mathrm{Str}_0(J_3(\mathbb{O}_s)) \cong \mathfrak{e}_{6(6)}, \qquad \mathrm{Der}(J_3(\mathbb{O}_s)) \cong \mathfrak{f}_{4(4)}. \tag{5}$$

At the physical-map level the sector-reduction chain is

$$\mathfrak{e}_{6(6)} \supset \mathfrak{f}_{4(4)} \supset \mathfrak{g}_{2(2)}. \tag{6}$$

The gauged colour subgroup $SU(3)_c$ is the compact real form inside the maximal compact $K = Sp(1) \times Sp(3)$ of $F_{4(4)}$ (the stabilizer of the trace), following the A513 chain

$$K \supset 1 \times Sp(3) \supset U(3) \supset SU(3). \tag{7}$$

The colour, weak, and hypercharge labels are the low-energy names attached to that reduction.

6 SM field-content bookkeeping

The representation-level dictionary is now fixed for one generation by the explicit B-map (A688/A689); it was only partial through Session 49. The low-energy field content is the Standard Model set

$$Q_L, \quad u_R, \quad d_R, \quad L_L, \quad e_R, \tag{8}$$

with a Dirac-neutrino extension ν_R assumed in the neutrino paper. [P-032]

The one-generation Peirce-block assignments under the explicit B-map are (Session 50; see Paper 6 §4.4 and A685–A689 for derivations):

Peirce block	D_4 rep	Standard Model content	Status
$J_{13} = 8_+$	half-spinor	left-handed doublets $Q_L = (t_L, b_L)$, $L_L = (\nu_L, e_L)$	PROVED under B [A685/A689]
$J_{23} = 8_-$	half-spinor	right-handed conjugates u^c, d^c, e^c, ν^c	PROVED under B [A685/A689]
$J_{12} \subset \mathbf{10}$	vector	Higgs doublets H_u, H_d (in $\mathbf{10} = J_{12} \oplus \mathbb{R}c_1 \oplus \mathbb{R}c_2$)	PROVED under B [A686/A688]

Notes: (i) The left–right split $8_+ = Q \oplus L$ and $8_- = u^c \oplus d^c \oplus e^c \oplus \nu^c$ is fixed by the SM-commuting chirality $\Gamma_{QL} = \Gamma_3\Gamma_4\Gamma_8\Gamma_9$ [A685]. (ii) The Higgs lives in the *vector* **10**, not in J_{13}; the physical one-generation Yukawas are the cross-block trilinear $\{J_{12}(H), J_{13}(\Psi_L), J_{23}(\Psi_R^c)\}$ [A686–A689], with J_{13} in the middle slot. The earlier "$t_R, b_R^c \in J_{13}$" placement (A674/A675) is the pre-B Peirce same-block *carrier* convention, not the literal SM Yukawa container; it survives only as the unit-eigenvalue carrier theorem. (iii) Three Peirce eigenvalues $(\phi, 1, \phi^{-1})$ map to tree-level carrier strengths [A674 Peirce–Yukawa theorem].

Status under B: all one-generation quark and lepton placements and all sixteen hypercharges are PROVED under the explicit B-map (A689). [LIB2-108] The remaining open items are: the canonical selection of B from $J_3(\mathbb{O}_s)$ alone (D690); the explicit three-generation embedding; and the algebraic anomaly check — the matter content fills one $SO(10)$ **16** per generation, which is anomaly-free, but the in-framework calculation is not yet written. *(Superseded, S64 note: the canonical-B question was closed by the A707/A708 audits — the continuous boost is excluded by compact gauge and the exhaustive signed-permutation enumeration leaves one frame component after quotienting, so A_B is discharged relative to the compact SM gauge embedding — and the one-generation anomaly calculation is now written and kernel-verified [A696]; see the later status updates in this paper.)* [LIB2-108]

A678/A679 lepton finding (the obstruction the B-map later resolved): For all $q \in \{\frac{1}{6}, \frac{1}{3}, \frac{1}{2}\}$, the $\mathbf{26}_{F_4}$ branch via $USp(6) \times USp(2)$ contains no $(1,2)_Y$ lepton doublet and no $(1,1)_{-1}$ charged-lepton singlet (kernel-verified, A678). The structure group $\mathrm{Str}_0(J_3(\mathbb{O}_s)) \simeq E_{6(6)}$ acts on the full $\mathbf{27} = J_3(\mathbb{O}_s)$; the $27 \downarrow F_{4(4)} = 26 \oplus 1$ adds only a neutral singlet $(1,1)_0$ (A679). At the complexified E_6 GUT level, the **27** does contain lepton slots (inside the $SO(10)$ spinor **16**), but the compact $SO(10)$ chain is not a real subgroup chain of the split $E_{6(6)}$. The next candidate is $SO(5,5) \times SO(1,1) \subset E_{6(6)}$, under which $27 \to 16 + 10 + 1$ with a split-real spinor (A681). This route is the one that succeeded: under the explicit B-map the leptons L_L and e^c, ν^c are placed in $J_{13} \oplus J_{23}$ and all lepton Yukawas and hypercharges are reproduced (A688/A689). The simple-**26** obstruction above is therefore resolved; the only residual freedom is the canonical selection of the B-map orientation (D690).

7 Cubic invariant and the Peirce carrier theorem

The same-block cubic

$$\mathcal{L}_{\text{carrier}} \propto \mathrm{Tr}\big[J_{\text{vac}} \circ (\Psi \times \Phi_H)\big], \tag{9}$$

with Ψ, Φ_H in the same off-diagonal Peirce block, is the Peirce *unit-eigenvalue carrier theorem*, *not* the literal SM Yukawa: the Freudenthal cross product isolates the middle diagonal idempotent and contraction with the vacuum extracts $\lambda_2 = 1$, yielding the tree-level carrier eigenvalue $y_t(M_{\text{Pl}}) = 1$ [A674]. The literal one-generation SM Yukawa is instead the *cross-block* trilinear

$$\{J_{12}(H),\ J_{13}(\Psi_L),\ J_{23}(\Psi_R^c)\} \ =\ \mathrm{Re}\big((\bar{x}y)\bar{z}\big)\, c_2 \tag{10}$$

with the Higgs $H_{u,d}$ in the vector $\mathbf{10} = J_{12} \oplus \mathbb{R}c_1 \oplus \mathbb{R}c_2$, verified component-by-component for $QH_u u^c$, $QH_d d^c$ and $LH_d e^c$ under the explicit B-map (A687–A689). In this way the gauge and (carrier) Yukawa readouts share one Jordan frame, while the physical $16 \times 10 \times 16$ Yukawa is the cross-block triple.

8 Anomaly-cancellation status

With the explicit $B + SU(5)$ embedding, one generation of fermions is

$$Q_L = (3,2)_{1/6}, \quad u^c = (\bar{3},1)_{-2/3}, \quad d^c = (\bar{3},1)_{1/3}, \quad L = (1,2)_{-1/2}, \quad e^c = (1,1)_1, \quad \nu^c = (1,1)_0,$$

i.e. the $SO(10)$-spinor branching $16 \to 10 \oplus \bar{5} \oplus 1$ under $SU(5)$, with the same index hypercharge rule used by the B-map ($Y_{0,1,2} = -\frac{1}{3}$, $Y_{3,4} = +\frac{1}{2}$). In the left-handed Weyl convention the local anomaly coefficients all vanish (kernel-verified, cross-checked in house, A696):

$$SU(3)^3 :\ 2 - 1 - 1 = 0, \qquad SU(3)^2U(1) :\ \tfrac{1}{6} - \tfrac{1}{3} + \tfrac{1}{6} = 0, \qquad SU(2)^2U(1) :\ \tfrac{1}{4} - \tfrac{1}{4} = 0,$$

$$U(1)^3 :\ \tfrac{1}{36} - \tfrac{8}{9} + \tfrac{1}{9} - \tfrac{1}{4} + 1 = 0, \qquad U(1)\text{-gravity} :\ 1 - 2 + 1 - 1 + 1 = 0.$$

The Witten $SU(2)$ global anomaly is absent: three colour quark doublets plus one lepton doublet give 4 left-handed $SU(2)$ doublets per generation, an even number. [LIB2-109] The singlet ν^c contributes nothing to the Standard Model gauge anomalies; it completes the $SO(10)$ 16 and supplies the right-handed-neutrino slot. The split-real origin ($\mathrm{Spin}(4,4)$, $\mathrm{Spin}(5,5)$, $E_{6(6)}$) does not affect this calculation, which depends only on the compact $SU(3)_c \times SU(2)_L \times U(1)_Y$ acting on the chiral content. The identified one-generation matter content is therefore anomaly-free [PROVED under explicit $B + SU(5)$]. Anomaly cancellation holds for any number of complete generations and does *not* by itself fix the generation count, which remains an independent input. [LIB2-109]

Rev25 update: the complete *exact* audit (kernel `s588`, S214, upgrading the structural check `s572`) verified that all SM gauge and gravitational anomaly sums cancel exactly from the framework's own derived hypercharges, formally closing the hostile-board item T3-P; see Paper 7 (Rev25 additions) and Appendix F. [LIB2-109]

9 Rev25 structural additions (fold of S209/S223 results)

Novelty label: new specialization proved here.

Theorem 9.1 (E_7-closure of the congruence dressing; `s679`, S223; independently re-proved by the hostile lane, A1343)**.** *Let* $\mathfrak{p} = \mathrm{span}(S_{70}) \subset \mathrm{Sym}(56)$, $\mathfrak{k} = [\mathfrak{p},\mathfrak{p}]$, $\mathfrak{e}_7 = \mathfrak{k} \oplus \mathfrak{p}$ *(Cartan pair, machine-verified).* [LIB2-197] *Then* $\mathrm{SPD} \cap E_{7(7)} = \exp(\mathfrak{p})$ *exactly, and for any two* E_7*-Grams* $A = \exp(2S_A)$, $\mathcal{M} = \exp(2\Phi)$ *with* $S_A, \Phi \in \mathfrak{p}$*, the unique SPD solution* B *of* $BAB = \mathcal{M}$ *satisfies* $\log B \in \mathfrak{p}$ *— i.e.* $B \in E_{7(7)}$ *identically.* [LIB2-197] *(Five-line proof, each assumption machine-verified; membership residuals* $\leq 3.6 \times 10^{-14}$ *at all eleven measured points; the control test has teeth: a random grade-compatible symmetric matrix has* $\mathfrak{p}$*-residual* 0.982*.)*

This upgrades the measured grade anatomy of the dictionary program from observed to *derived*: the dressing lives on the $E_7/SU(8)$ coset, always — a frame-free kinematic reduction from 56×56 functions to 70.

Conditional remark (labeled; `s520/s521/s523`, S209). An independent algebraic chain reproduces the one-generation content from division-algebra data: one generation from the Witt ideal of Cl(6); the anomaly-free 16; and $\mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ (the Dixon algebra) yielding the full SM gauge group with the correct hypercharges. The algebra is robust and kernel-verified, but its *physical* attachment to the suite is **conditional** on the κ_{phys} candidate mechanism and the ι_ψ gluing — both open (Appendix Q posture). Recorded as a convergence signal, not banked as a derivation of the field content.

10 Status and caveat

The gauge-group claim is algebraic inside the Rev29 map, but it does not yet amount to a universal selector theorem excluding every rival subgroup assignment in every extension. What the present paper claims is narrower and precise: (a) the Peirce decomposition fixes the working gauge assignment used by the suite; (b) no extra gauge-group-selection axiom is added on top of the Jordan frame; (c) the independent AX6 orbit selector fixes the Weinberg-angle branch within the already identified gauge group, not the gauge group itself; and (d) the identification with the Standard Model remains part of the programme's falsifiable correspondence map.

11 Conclusion

Paper 1 is the shortest route from the abstract algebra to the familiar gauge labels of particle physics. In Rev29 it also makes explicit why the underlying algebra is $J_3(\mathbb{O}_s)$ rather than some other Jordan algebra: the same Lorentz-signature equivalence argument that selects $\mathbb{O}_s$ also selects the non-compact real form $\mathfrak{f}_{4(4)}$ needed by the rest of the suite. An independent published precedent for the $F_4 + J_3(\mathbb{O})$ route to Standard Model quantum-number classification is provided by Todorov and Drenska [1].

References

[1] I. Todorov and M. Drenska, "Octonions, exceptional Jordan algebra and the role of the group F_4 in particle physics," *Adv. Appl. Clifford Algebras* **28** (2018) 82, `arXiv:1805.06739`.

Leibniz Quantum Beats Newton

Paper 2 (Rev33.2): Mass-Matrix Hierarchy and Resolvent Quintics

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Abstract

This paper reorganises the fermion-mass sector around the Rev29 resolvent family, the cubic-invariant Yukawa coupling, and the clarified renormalization-group story. The central statement is that the hierarchy law

$$R_d = \frac{1}{\phi^{2d} - 1} \tag{1}$$

acts as the common algebraic kernel behind the second-to-third generation mass ratios, while the local cubic invariant fixes the tree-level top-Yukawa eigenvalue at $y_t = 1$. The d-index assignment is *selected, not derived* (Paper 5; ruling R-23, Rev32.9): no printed theorem fixes which index goes with which ratio, and the two assignments this paper prints are different objects, related in §3.1.

1 Introduction

Rev29 keeps the mass sector compact while adding the Sprint 3 blind-drop charged-lepton update. The local cubic invariant fixes the top Yukawa. The resolvent family organises the hierarchy ratios. The remaining issue is not whether the dimensionless structure is present, but how the measured electroweak scale and low-energy running convert that structure into GeV numbers.

2 Cubic-invariant Yukawa closure

The same-block Peirce *carrier* coupling (unit-eigenvalue theorem, A674; the physical SM Higgs $H_{u,d}$ sits in the vector **10** and the literal Yukawa is the cross-block triple, A686–A689) is

$$\mathcal{L}_{\text{carrier}} \propto \mathrm{Tr}\big[J_{\text{vac}} \circ (\Psi \times \Phi_H)\big]. \tag{2}$$

With Ψ and Φ_H placed in the $(1,3)$ Peirce block, the Freudenthal cross product projects onto e_2 and the contraction with $J_{\text{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1})$ extracts $\lambda_2 = 1$. Therefore

$$y_t(M_{\text{Pl}}) = 1. \tag{3}$$

At low energy this boundary value runs under the ordinary Standard Model equations, so the tree/loop separation is explicit rather than implicit; the measured-side $y_t(M_Z) = 0.970 \pm 0.002$ (SL-57, `s1337`) is a data comparator, not its endpoint (Rev33.2: the 0.967 printed here before was that comparator, not a running result).

3 Resolvent family

The common hierarchy kernel is

$$R_d = \frac{1}{\phi^{2d} - 1}. \tag{4}$$

The first four members are

$$R_1 = \frac{1}{\phi^2 - 1} = \phi^{-1}, \tag{5}$$

$$R_2 = \frac{1}{\phi^4 - 1} = \frac{1}{\phi(\phi + 2)}, \tag{6}$$

$$R_3 = \frac{1}{\phi^6 - 1} = \frac{1}{4\phi^3}, \tag{7}$$

$$R_4 = \frac{1}{\phi^8 - 1}. \tag{8}$$

Numerically these are $R_1 \approx 0.6180$, $R_2 \approx 0.1708$, $R_3 \approx 0.0590$, and $R_4 \approx 0.0217$. These are normalized mass ratios inside the resolvent family, not the raw resolvent denominators.

3.1 RGE Preservation and Structural Stability (A3 closure)

The resolvent family is built on the Peirce eigenvalue spectrum $\{\lambda_1 = \phi, \lambda_2 = 1, \lambda_3 = \phi^{-1}\}$. The operational d-index assignment used in this paper is (*Rev32.9, ruling R-23*: through Rev32.8 this paragraph said the spectrum "structurally fixes" the assignment; it does not — the assignment is a selected, post-hoc feature rule, as Paper 5 types $d(\nu) = 2$):

$$\begin{aligned} &d = 1 \to \text{CKM transfer sector}, \\ &d = 2 \to m_{\nu_2}/m_{\nu_3}, \\ &d = 3 \to m_\mu/m_\tau, \\ &d = 4 \to m_s/m_b. \end{aligned} \tag{9}$$

The A3 closure result is that these dimensionless ratios preserve to sub-percent accuracy under standard one-loop running from the ultraviolet matching scale to M_Z. The reason is representation-theoretic: same-representation fermions carry identical leading Casimirs, so the anomalous-dimension corrections cancel in the ratios. That protects the ratios under running; it does not select the assignment. *How the two printed assignments relate (Rev32.9, H202).* The table above pairs $d = 3$ with the ratio m_μ/m_τ as a *label*: the scored value 0.059684 is the monomial of Eq. (17), not the resolvent member $R_3 = 1/(\phi^6 - 1) = 0.059017$. The reversal theorem of §3.2 pairs *leptons* with resolvent indices ($d = 1 \to \tau$, $d = 2 \to \mu$, $d = 3 \to e$). The two are different objects; neither selects the other.

3.2 Sprint 3: Jordan-automorphism reversal theorem and charged-lepton blind drop

Session 30 supplies the first nontrivial charged-lepton blind drop built from the full 27×27 Albert/Freudenthal machinery. Let

$$D_R = \mathrm{diag}(R_1, R_2, R_3) \tag{10}$$

and let L_{D_R} denote Jordan multiplication by D_R on $J_3(\mathbb{O}_s)$. The Sprint 3 kernel constructs the Jordan automorphism

$$U_{\text{Thalf}} = \exp(t^* T_{\text{half}}), \qquad t^* = \frac{2\pi}{\sqrt{5}}, \tag{11}$$

and verifies the conjugated operator

$$L_{M_{\text{lepton}}} = U_{\text{Thalf}}\, L_{D_R}\, U_{\text{Thalf}}^{-1}. \tag{12}$$

Two structural results follow.

First, $L_{M_{\text{lepton}}}$ is isospectral to L_{D_R}, with the Albert-Freudenthal spectrum

$$\{R_1, R_2, R_3, \tfrac{1}{2}(R_i + R_j) \times 8\}, \tag{13}$$

verified numerically to a maximum eigenvalue mismatch of 9.01×10^{-14}. Second, the action of U_{Thalf} reverses the diagonal ordering

$$(R_1, R_2, R_3) \longmapsto (R_3, R_2, R_1), \tag{14}$$

so the charged-lepton assignment is forced algebraically to be

$$d = 1 \to \tau, \qquad d = 2 \to \mu, \qquad d = 3 \to e. \tag{15}$$

This is the U_{Thalf} *reversal theorem*: the heaviest lepton is paired with the smallest resolvent index because the Jordan automorphism literally reverses the diagonal ordering set by J_{vac}.

With the single external anchor

$$m_\tau = 1776.93(9)\ \text{MeV}, \tag{16}$$

the exact tree-level muon-to-tau ratio is

$$\frac{m_\mu}{m_\tau} = \left(\frac{\phi}{\sqrt{5}}\right)^{8/3} \times \frac{\sqrt{2}}{10} = 0.059684 \qquad (\text{PDG: } 0.059461,\ \text{miss } +0.37\%), \tag{17}$$

giving

$$m_\mu = 106.05\ \text{MeV} \qquad (+0.37\%\ \text{PDG}), \tag{18}$$

where the exponent $p = 8/3 = \dim(\mathbb{O}_s)/\text{rank}(J_3)$ is an exact dimension ratio [A616], read as a topological winding ratio (8 octonionic dimensions per 3 generation winding levels), and the normalisation constant $C = \sqrt{2}/10$ is read off the two-generation sub-algebra $J_2(\mathbb{O}_s)$ [A618]: $\dim(J_2) = 10$ and off-diagonal trace norm $= \sqrt{2}$.

Rev29 correction. Both ingredients were previously typed PROVED, and the formula was cited as carrying *zero free parameters*. That typing is withdrawn. [LIB2-247] Paper 0's tier/attachment table already prints the honest row — the exact dimension ratio $8/3 = \dim \mathbb{O}_s/\text{rank}\, J_3$ is LOADED-CORRESPONDENCE: "not, alone, a mass exponent" — and the same holds of C. The rows of this subsection therefore stand at **Structural** (mathematical tier), **Loaded-correspondence** (physical attachment) and **Reproduced** (numerical status): the ratio is reproduced, not derived. [LIB2-122, LIB2-247, LIB2-265, LIB2-266] *Physical attachment never inherits the mathematical tier*; an exact dimension count is not by itself a mass exponent or a mass normalisation.

Provenance: target-first, and the derivation was never closed. The record runs backwards from the datum, not forwards from the algebra. [D616:99–102] computes $0.0595/0.4220 = 0.141$ from PDG and *asks* whether the number is algebraic in φ. [A616:50–63] then offers $\sqrt{2}/10$ at 0.37% while stating in the same breath that "the algebraic origin is not yet proved." [A618:47–51] supplies $\dim J_2 = 10$

afterward, i.e. the dimension is matched to a coefficient already in hand. [A618:134–140] concedes that the divide-by-dimension step "is argued by analogy." [A618:147] lists the formal derivation as **BLOCKING** — and that item is closed nowhere in the corpus. [LIB2-122]

*The +0.37% residual **is** the fit gap.* The coefficient the data require is $C_{\text{req}} = 0.140894$; the algebra offers $\sqrt{2}/10 = 0.141421$. C enters the ratio *multiplicatively*, so $\sqrt{2}/10 \,/\, C_{\text{req}} = 1.00374$ (recomputed Rev32.9 with $m_\tau = 1776.93$, $m_\mu = 105.6583755\,\text{MeV}$; the pair 0.140900/1.00370 printed through Rev32.8 reproduces the superseded $m_\tau = 1776.86$) — the +0.37% miss in Eq. (17) is precisely the distance between the required coefficient and the offered one. It is not an independent residual sitting on top of a closed derivation, and any downstream absorption of +0.37% (next paragraph) is an absorption of the coefficient gap itself and must be read as such. [LIB2-123]

Rev29 re-tiering: m_μ/m_τ. PROVED / zero-parameter is WITHDRAWN. New typing: **Structural** · **Loaded-correspondence** · **Reproduced**. Grounds: (i) provenance is target-first (D616 asks the question from the PDG number; A616 supplies $\sqrt{2}/10$ while recording that its algebraic origin "is not yet proved"; A618 supplies $\dim J_2 = 10$ afterward and concedes the divide-by-dimension step "is argued by analogy"); (ii) A618:147 lists the formal derivation as BLOCKING and no component of the corpus closes it; (iii) the +0.37% residual is the multiplicative gap between the required $C = 0.140894$ and the offered $\sqrt{2}/10 = 0.141421$, i.e. the fit gap itself. No number is deleted — $p = 8/3$ and $C = \sqrt{2}/10$ stand exactly as printed; what changes is what may be claimed for them. [LIB2-122] This row may not be cited as zero-parameter.

[Retracted, Rev32:] an earlier sentence here claimed the +0.37% residual is "fully accounted for by a one-loop QED correction [A637]". That claim contradicts the authoritative ruling sixteen lines above (and Appendix O): the +0.37% residual *is* the fit gap on C, and the QED correction below is a separate, smaller matching effect (+0.3125%) applied to the tree value — it does not account for the coefficient gap. [LIB2-123, LIB2-126] The ledger ruling wins. The complete five-step electron derivation is:

(1) $m_\mu^{\text{tree}} = 106.05\,\text{MeV}$ from the re-tiered ratio formula above (STRUCTURAL / LOADED-CORRESPONDENCE / REPRODUCED; *Rev29*). [LIB2-125]

(2) QED running: $\delta_{\text{QED}} = \frac{3\alpha}{4\pi} \ln \frac{\Lambda_{G_2}^2}{m_{\mu,\text{phys}}^2} = +0.3125\%$.

(3) Physical muon mass: $m_{\mu,\text{phys}} = m_\mu^{\text{tree}}/(1 + \delta_{\text{QED}}) = 105.7233\,\text{MeV}$ (+0.061% PDG; Appendix X carries the rounded +0.06%). [LIB2-126] *Rev32.9:* the four digits are needed — the Koide root from the rounded 105.72 displays as 0.5078, while the chain's own value gives the 0.5076 below.

(4) The Koide sum rule ($K = 2/3$) is applied to the physical muon mass at the Λ_{G_2} boundary [A637]; the current attachment carries the Koide value and its branch — the smaller positive root for $\sqrt{m_e}$; the larger gives $m_e \approx 43.7$ GeV and moves only this row (`s1207`) — as *external* (Appendix O). *Rev32.9:* through Rev32.8 this item said the UV boundary condition "forces" the rule.

(5) $m_e = 0.5076\,\text{MeV}$ $\quad$ (-0.66% PDG; PDG: $0.5110\,\text{MeV}$).

Koide branch note. Two positive roots satisfy $K = 2/3$ for the $\{m_\tau, m_\mu, m_e\}$ system. [LIB2-127] The Rev29 mass chain selects the light root $m_e = 0.5076\,\text{MeV}$, matching PDG at the -0.66% level. [LIB2-127] The heavy root ($\approx 43.7\,\text{GeV}$) is excluded as outside the charged-lepton mass spectrum.

The Artin bridge (Appendix A §A.7.8) guarantees that the 1-loop QED self-energy is associative (two-generator subalgebra) and hence exact at this order [A633]. The QED running direction is unambiguous: mass grows UV in QED, so dividing m_μ^{tree} by $(1+\delta)$ gives the lower physical mass.

The status of the charged-lepton mass sector is now split honestly (authoritative ledger: Appendix O, "The authoritative charged-lepton ledger"): m_τ is the external anchor; m_μ follows from the algebraic ratio formula [A616, A618], re-typed STRUCTURAL / LOADED-CORRESPONDENCE / REPRODUCED above and *not* citable as zero-parameter (*Rev29*); and m_e is KOIDE-CONSISTENT ⋆ ⋆ ⋆⋆. [LIB2-265, LIB2-266, LIB2-268, LIB2-269] The electron mass is determined by the external empirical relation $K = 2/3$ acting on that m_μ chain and the measured m_τ anchor [A636, A637]. [LIB2-127, LIB2-267]

3.3 Generation structure as topological winding sectors

The three Peirce idempotents $\{f_1, f_2, f_3\}$ of $J_3(\mathbb{O}_s)$ have a topological interpretation consistent with the mass ratio formula above. Each idempotent f_i is a winding sector of the $(4,4)$ moire vacuum: the three generations are three distinct topological winding levels in the split-octonionic fibre. The exponent $p = 8/3$ then reads as: 8 split-octonionic dimensions per 3 winding levels. The normalisation $C = \sqrt{2}/10$ is read as the 2-generation transition amplitude: $J_2(\mathbb{O}_s) = \mathbb{R}f_1 \oplus \mathbb{R}f_2 \oplus P_{12}$, of dimension 10, with off-diagonal trace norm $\sqrt{2}$. Rest mass is therefore the topological energy of a winding that cannot be removed by continuous deformation: fermions carry a Möbius half-twist in the $(4,4)$ vacuum (bosons do not), and this half-twist generates $V_{\text{CW}}^{\text{ferm}} \neq 0$ with minimum at $r^\star = \varphi^2$. This picture is a *physical narrative* consistent with all proved results (see Appendix A §A.7.4); it is not itself a theorem at the present programme state.

3.4 Light quark constituent masses and the confinement scale

The Quark Gauge-Protection Theorem (Appendix A, Theorem A.7.5) forces the light quarks (u, d, s) to acquire mass exclusively through the G_2 confinement potential ΔV_{G_2}, not through Coleman–Weinberg radiative corrections. The mechanism that produces this G_2 confinement scale at weak coupling — a spectral singularity of the cubic-norm cone rather than a strong-coupling limit — is given in Appendix I (*Confinement as the spectral geometry of the cubic norm*). The constituent mass formula derived from the chiral gap equation driven by ΔV_{G_2} is:

$$m_u^{\text{const}} \approx m_d^{\text{const}} \approx \Lambda_{G_2}\, \varphi^{1/2} \approx 331\,\text{MeV}, \tag{19}$$

with $\Lambda_{G_2} \approx 260\,\text{MeV}$ ($260 \pm 0.5\,\text{MeV}$, Appendix X) the framework's hadronic calibration — an external dimensional calibration in the sense of the Appendix X input-count box, its value taken from hadronic phenomenology; not a derived scale, and not a measured Λ_{QCD} [A618] (Rev32.9, R84-h). The PDG constituent mass targets are $m_u \approx 336\,\text{MeV}$ (-1.57%) and $m_d \approx 340\,\text{MeV}$ (-2.73%; both at the full-precision value 330.73 MeV, Rev32.9), consistent with the formula at leading order.

The approximate degeneracy $m_d \approx m_u$ is structural: the $(4,4)$ sign flip $\eta = -1$ for down-type quarks represents a phase alignment in the pseudo-Riemannian vacuum, not a magnitude suppression. The physical isospin splitting $m_d - m_u \approx 4\,\text{MeV}$ (current masses) is of QED and electroweak loop origin, outside the tree-level $J_3(\mathbb{O}_s)$ geometry.

For the strange quark, incorporating the G_2 colour Casimir modifier $K_{G_2}/K_{SU(3)} = 3$ [A631] via the G_2 Langlands-self-dual geometric mean of the long-/short-root scales (the geometric mean

$m_s^2 = m_{\text{short}}\, m_{\text{long}}$ is exact under G_2 root-system self-duality, s194) yields:

$$m_s^{\text{const}} = \Lambda_{G_2}\, \varphi \times \left(\frac{K_{G_2}}{K_{SU(3)}} \right)^{1/4} = 553.7\,\text{MeV}, \tag{20}$$

placing the strange constituent mass at the threshold of the G_2 versus $SU(3)$ colour strength transition. [A631, A632]

3.5 Top quark: fit-free *construction* from the Jordan idempotent, carrying a recorded cross-scheme comparator distance ($d_{\text{cmp}} = +5.57$, no status — Rev32.1, A1531/T2)

Rev29 correction. This subsection was formerly headed "zero-parameter prediction." It is a fit-free *construction*: the boundary condition $y_t(M_{\text{Pl}}) = 1$ is fixed without a fitted parameter, but the resulting tree value sits +0.87% above the PDG 2026 direct-measurement (MC) mass ($172{,}600 \pm 270\,\text{MeV}$; the cross-section pole mass is a distinct object — Appendix X, Rev32.9), a naive cross-scheme comparator distance $d_{\text{cmp}} = +5.57$ — experimental-error units only, not a tension and not status-setting (Rev32.1 retype, A1531/T2; the honesty-ledger BLACK typing of this row is retyped accordingly). A construction whose scheme-matched significance is not yet computable is not a successful prediction and is not reported as one here.

The top quark Yukawa coupling is fixed to $y_t(M_{\text{Pl}}) = 1$ by the cubic-invariant Yukawa theorem (Appendix A); the Peirce idempotent derivation in the remark following Theorem A.6 provides a complementary proof [A632]. What the theorem fixes is the *carrier* eigenvalue $y_t = 1$; its reading as the physical top Yukawa at the Planck scale is the declared attachment W01 (Appendix A, interface marker W01) — an attachment, not a derivation (Rev32.5, A1564). [LIB2-139, P-027] This is a UV boundary condition. Standard RG evolution carries the coupling into the low-energy effective theory; the measured-side $\overline{\text{MS}}$ coupling there, $y_t(M_Z) = 0.970 \pm 0.002$ (SL-57, the SL-54 extraction run down by `s1337`), is a data comparator and not a consequence of the boundary value. The tree-level relation

$$m_t = \frac{v_{\text{EW}}}{\sqrt{2}} = \frac{246.22\,\text{GeV}}{\sqrt{2}} \approx 174{,}100\,\text{MeV}, \tag{21}$$

should therefore be read as the electroweak-scale tree estimate after inserting the measured Higgs VEV, not as simultaneous equality of $y_t(M_{\text{Pl}})$ and $y_t(M_Z)$. [LIB2-121, LIB2-139, P-028] The resulting offset from the PDG 2026 direct-measurement (MC) mass ($172{,}600 \pm 270\,\text{MeV}$; +0.87%; naive cross-scheme $d_{\text{cmp}} = +5.57$, no status — Rev32.1 refresh, A1531/T2) is recorded in Appendix X. [LIB2-121] The $\sqrt{2}$ factor is structurally identical to the off-diagonal norm of $J_2(\mathbb{O}_s)$ [A618]: both arise from the Peirce decomposition of the same algebra.

The suppression of $y_b \approx 0.024$ and $y_c \approx 0.007$ relative to $y_t = 1$ is explained by the complement rule $f_k \circ x = 0$ for $x \in P_{ij}$ $(k \neq i, j)$: the bare coupling of δj_3 to P_{12} is exactly zero. Physical y_b and y_c are generated by the Peirce closure mixing insertion $P_{ij} \circ P_{jk} \subset P_{ik}$, which suppresses them by the mixing amplitude. Their precise values require $\overline{\text{MS}}$ matching in the low-energy field theory. Artin's theorem ensures algebraic closure inside the subalgebra generated by any two octonionic elements; this is an algebraic protection statement, not a theorem about loop diagrams. RG running and renormalisation are addressed separately in Paper 6 and Appendix D.

3.6 Heavy-quark unification: the 7/3 bridge

The DET-7 invariant $n_{23} = \det(\mathrm{Gram}(J_{\mathrm{vac}}, J^{\#}_{\mathrm{vac}})) = 7$ and the Jordan rank $\mathrm{rank}(J_3) = 3$ together force the ratio $n_{23}/\mathrm{rank} = 7/3$ [A635, A637]. Applied to the heavy-quark sector, this ratio acts as a generation-independent bridge between the lepton and quark mass scales:

$$m_b = \tfrac{7}{3}\, m_\tau = \tfrac{7}{3} \times 1776.93\,\mathrm{MeV} = 4146\,\mathrm{MeV} \quad (-0.95\%\ \text{vs PDG 2026};\ d_{\mathrm{cmp}} = -6.64,\ \text{no status; full precision 4146} \tag{22}$$

$$m_c = \tfrac{7}{3}\, m_s^{\mathrm{const}} = \tfrac{7}{3} \times 553.7\,\mathrm{MeV} = 1291.9\,\mathrm{MeV} \quad (+1.49\%\ \text{vs PDG 2026};\ d_{\mathrm{cmp}} = +4.22\ (+3.69\ \text{with the}\ \Lambda_{G_2}\ \text{round} \tag{23}$$

Rev29 correction, retyped Rev32.1 (A1531/T2) — report the comparator distance, and type it honestly. The m_b row must be quoted with its offset: $m_b = (7/3)m_\tau = 4146\,\mathrm{MeV}$ against PDG 2026 $m_b(m_b) = 4186 \pm 6\,\mathrm{MeV}$ is -0.95% *and* $d_{\mathrm{cmp}} = -6.64$ (cross-scheme, no status; full precision, Rev32.9). [LIB2-130, LIB2-270, LIB2-271, LIB2-272] Rev29 typed this row **BLACK** from the bare σ; the honesty ledger has since *retyped* those rows as comparator-distance records (the Rev32 adjudication below), and Rev32.1 propagates that retype to this box — the earlier text here cited the ledger for a BLACK status the ledger no longer carries, and compared against the superseded PDG 4183 ± 7. Earlier revisions of this paper printed only the percentage; the corrected house style is: percent primary, both schemes stated, d_{cmp} as non-status-setting audit metadata, at this and every other m_b site in this paper. Appendix X states the governing rule: *percentage agreement and σ-level tension are not equivalent when PDG measurements are sub-percent precise.* At $\pm 6\,\mathrm{MeV}$ on the PDG 2026 value $4186\,\mathrm{MeV}$ the bottom mass is a 0.14% measurement, so the -0.95% bridge offset corresponds to a naive cross-scheme comparator distance $d_{\mathrm{cmp}} = -6.64$ — experimental-error units only, cross-scheme, not a tension and not status-setting (Rev32 retype, A1531/T2). The charm bridge value $m_c = 1291.9\,\mathrm{MeV}$ lies $+1.49\%$ above the PDG 2026 $\overline{\mathrm{MS}}$ comparator $m_c(m_c) = 1272.9 \pm 4.5\,\mathrm{MeV}$. [LIB2-131, LIB2-273, LIB2-274, LIB2-275, LIB2-276] Dividing by the comparator's experimental error gives a naive cross-scheme comparator distance of $+4.22$ ($+3.69$ with the Λ_{G_2} rounding band folded in, `s1211`), but the bridge is constituent-anchored and no constituent-to-$\overline{\mathrm{MS}}$ conversion or scheme uncertainty is supplied; the number is therefore not a physics tension and does not carry a BLACK classification. [LIB2-131, LIB2-273, LIB2-274, LIB2-276, LIB2-314] A scheme-matched significance awaits an explicit conversion with uncertainty.

[LIB2-314]

Both values land in the $< 3\%$ band without additional parameters. Under the Rev32 cross-scheme adjudication the percent offsets are the primary statement; the d_{cmp} figures above are secondary audit numbers that set no status, and the earlier "5.3σ BLACK" framing of the bottom row is retyped accordingly (a scheme-matched Z_{matched} is not yet computable for either row — Appendix X, scheme table). [LIB2-313] The DET-7 construction supplies the 7/3 bridge as a declared boundary value at the self-consistent threshold $\mu = m_Q$; reading that value as an $\overline{\mathrm{MS}}$ mass at $\mu = m_Q$ needs the unresolved matching/scheme step; no QCD running from Λ_{G_2} is applied (naive running from Λ_{G_2} to m_Q gives $\sim 1057\,\mathrm{MeV}$, a factor ~ 4 below the PDG value [A635]).

Generation-independence of the Peirce closure amplitude is *proved* ⋆⋆⋆⋆⋆ via Peirce idempotent permutation symmetry: the three idempotents $\{f_1, f_2, f_3\}$ are algebraically permutation-equivalent, so no generation-dependent factor can appear in the bare coupling amplitude [A637/Q1B].

Status of the 7/3 bridge: **Structural** ⋆⋆⋆★. The formal S-matrix proof connecting the Gram-determinant ratio to the physical heavy-quark coupling amplitude is the one open gap in the mass

sector; it does not block publication and is stated explicitly as an open programme item.

Reconciliation with Paper 7 (framing). The numerical m_b, m_c values above are the *boundary-value* (threshold-matched) realization of the 7/3 ratio. Paper 7 treats the *same* ratio as an exact finite-layer (bare-vertex) matching boundary ($K_B = I_7$ from the cubic Jordan norm; $I_8 \to I_7$ under SM covariance) whose *low-energy survival* is a falsifiable 4321/twin-Pati–Salam prediction rather than a closed low-energy mass theorem. The two statements are the same open object viewed at the two ends of the running: 7/3 is exact at the matching scale, and its persistence to $\mu = m_Q$ as a clean MS-bar relation is precisely the S-matrix/threshold question named above. The m_b, m_c entries are therefore reported as boundary-value predictions, not as a derived low-energy theorem; the STRUCTURAL ⋆⋆⋆⋆ tier reflects exactly this. [LIB2-130, LIB2-270, LIB2-271, LIB2-272, LIB2-275, LIB2-313]

Declared-scheme cells (Rev33.2). Under the PI's blind declarations (Appendix X, T5-A) the top boundary is $y_t(M_{\rm Pl}) = 1$ in $\overline{\rm MS}$, which runs to $y_t(172.6\,{\rm GeV}) = 1.211$ (conversion price 0.020) against the data value 0.9330 ± 0.0017 — excluded at the declared scheme ($Z = +14.0$) — and the 7/3 bridge is read as $m_b(m_b) = (7/3)\, m_\tau(m_b)$ in $\overline{\rm MS}$, 4124.4 MeV (-1.47%, $Z = -9.65$, excluded at the declared scheme); charm is incomparable by scheme (`s1336`). The percentages printed above are the Rev33.1 cross-scheme readings and remain audit numbers.

Heavy-quark RG/matching audit. The heavy-quark boundary values $m_t^{(0)} = v_{\rm EW}/\sqrt{2}$, $m_b^{(0)} = \frac{7}{3} m_\tau$ and $m_c^{(0)} = \frac{7}{3} m_s^{\rm const}$ were tested against *standard* SM/QCD running and matching with *no* tuned matching scale. [LIB2-132] The result is negative. [LIB2-132] For the top, treating $y_t(v_{\rm EW}) = 1$ as an $\overline{\rm MS}$ Yukawa and applying the standard pole/$\overline{\rm MS}$ conversion moves the prediction *up* (the pole mass exceeds the running mass), the wrong direction; treating $v_{\rm EW}/\sqrt{2}$ as a pole-like estimate leaves the $+0.9\%$ residual. For the bottom, the self-scale reading leaves the -0.9% residual while running the boundary value from $\mu = m_\tau$ to m_b worsens it; for charm, running from the strange constituent scale overcorrects downward. [LIB2-132, LIB2-314] Each tension lies on a running curve only for a *tuned* scale not fixed by the algebra. [LIB2-132, LIB2-312] Hence the m_t, m_b, m_c tensions are *genuine* threshold problems, not ordinary running artifacts (A713), with the required corrections in the top-*down*/bottom-*up* sense. (The originally proposed 16-Goldstone source was subsequently excluded as a dimensional coincidence, A714; see Paper 4 §4.2.)

Pre-registered fixed-scale test (R190, Rev33.2). With the menu fixed before the run and identical on c and b, the 7/3 bridge read as an $\overline{\rm MS}$ mass $m_q(\mu)$ at its own scale, at 3 GeV and at 2 GeV and run to $m_q(m_q)$ at four loops gives, against $m_c(m_c) = 1272.9 \pm 4.5$ MeV and $m_b(m_b) = 4186 \pm 6$ MeV (every cell printed; `s1339`):

Quark	own scale	$\mu = 3\,{\rm GeV}$	$\mu = 2\,{\rm GeV}$
c	$+1.49\%$ ($Z = +4.2$)	$+21.6\%$ ($Z = +31.0$)	$+12.8\%$ ($Z = +24.1$)
b	-0.95% ($Z = -6.6$)	-6.2% ($Z = -36.0$)	-13.3% ($Z = -47.2$)

No fixed scale rescues either quark; fixing the scale below the quark mass moves both away, and A713's tuned scale lies on the far side ($\mu^* = 1.228$ GeV for c, 4.410 GeV for b — printed for orientation, never a row).

Weak-isospin signature and the structural threshold term (S61–S62, A715–A717). The residual pattern itself is structured. The needed corrections (m_t -0.864%, m_b $+0.961\%$, m_c -1.476%;

Rev32.11, PDG 2026 $m_t = 172.60$, $m_b = 4186 \pm 6$ MeV) satisfy $|\delta m_t/m_t| = |\delta m_b/m_b|$ to 0.67σ of the PDG uncertainty on m_b ($|\kappa_t - \kappa_b| = 0.194$ pp against $\sigma_{\kappa_b} = 0.289$ pp, ratio 0.67; at the superseded comparators $m_t = 172.56$ and PDG 2025 $m_b = 4183 \pm 7$ the two agreed to three significant figures, 0.003 pp — that agreement and the 0.148 pp printed through Rev32.10 both rest on that superseded m_t, `s1190`) with *opposite* sign, which is reproduced by a single weak-isospin coupling

$$\frac{\delta m_f}{m_f} \;=\; -\,\kappa\, T_3(f), \qquad \kappa \simeq 1.824\%, \tag{24}$$

the odd combination $\kappa_b - \kappa_t$ carrying a 0.212 pp spread from the PDG errors (`s1182`), and by no other single standard-model charge: electric charge predicts a 2:1 top/bottom ratio (data: 1:1) and any universal (isospin-blind) coupling gives no sign flip; both are excluded (A715, confirmed by two independent assessments). Charm, being up-type ($T_3 = +\frac{1}{2}$), requires an additional generation factor χ_c on top of the T_3 weighting: 1.761 in the engine's normalisation (κ_t from the top alone and the engine's own charm need), 1.618 ± 0.382 under `s1182`'s $\kappa_{\rm odd}$ and the charm need printed above (`s1218`). The two readings sit inside one error bar, and the closeness of either to $1 + \varphi^{-1}$ is not evidence (a look-elsewhere control, not a derivation); χ_c also moves with Λ_{G_2} (`s1211`). This signature is *structural, not derived*: the gauge-invariant operator named here as carrying it, the weak-adjoint Higgs spurion $(H^\dagger \tau^a H)\, \mathcal{O}^a_{\rm Yukawa}$ built from the verified cross-block Yukawa trilinear, is writable and charge-correct. Its τ^3 weighting is carried by a registered object — the slot-norm involution $P = 2Q - (B - L)$ on the **16** — and a leg-local P deformation of the one trilinear forces the relative up/down sign $\delta_t = -\delta_b$ (`s1218`); the conjugation $H_d = \bar{H}_u$, the vacuum grading $J_{\rm vac} = {\rm diag}(\varphi, 1, \varphi^{-1})$ and the Peirce structure constants, the objects A716 tested (verified in the exact A688/A689 component map), do not supply it. Which leg carries the deformation is a choice of operator, not a free sign: the singlet leg is a dimension-four reweighting that agrees with this weak-adjoint spurion on every neutral entry and differs on every charged entry. With the spurion named, what is inserted by hand is the deformation's existence, one sign — fixed by no registered symmetry (all 384 valid frames and the gauge family K commute with P), and taken from the data (doublet leg, $\kappa = -0.912\%$) — and $|\kappa|$ (`s1349`, Rev33.2). A boundary-condition revision fares no better (A717): the top boundary $m_t = v_{\rm EW}/\sqrt{2}$ ($y_t = 1$) is rigid and its residual is not ordinary pole/$\overline{\rm MS}$ matching, while the soft down-type bridge revision $\frac{7}{3} m_\tau\,(1 + \kappa/2)$ — numerically excellent, landing on $m_b(m_b)$ to 0.002% with the *same* κ fixed by the top — reduces no parameter, since no existing algebraic quantity fixes $1 + \kappa/2$. The honest ledger is therefore: bare algebraic boundary conditions plus a phenomenological structural T_3 threshold term, with κ and χ_c open. The equal-and-opposite t/b pattern is falsifiable: improved PDG precision that breaks $|\delta m_t/m_t| = |\delta m_b/m_b|$ would remove the T_3 reading (Paper 7). A separate environmental scan (five independent analyses, one with explicitly computed one-loop self-energies; A719/A719b, S63) finds that known electroweak pole–$\overline{\rm MS}$/Yukawa matching effects are percent-level and scheme-dependent in precisely this sector, with defensible scheme choices spanning T_3-odd shifts of $[-0.8, +3.8]$ pp around the observed $\kappa \simeq 1.824\%$; the $-\kappa T_3$ reading is therefore scheme-conditional — neither firm evidence of structure beyond the boundary conditions nor cleanly absorbed by standard matching — and the operative falsification target remains the breaking of the t/b equality at improved precision.

The custodial-odd one-loop matching shift, priced with its own scale ambiguity. The scheme dependence just bounded can be computed rather than bounded. In the gaugeless limit the custodial-odd combination $\Delta_{\rm odd}(\mu) := \delta_t - \delta_b$ of the one-loop Yukawa-sector self-energies is the T_3-odd shift this sector supplies at one loop, and the tadpole and Higgs wave-function pieces cancel identically within it (`s1186`). Evaluated at the registered electroweak anchor $\mu = v$ it gives $\Delta_{\rm odd}(v) = -1.4820\,\% \pm 0.6467\,\%$, the quoted error being the one-loop scale ambiguity across

$\mu \in [v/\sqrt{2}, v\sqrt{2}]$ at $d\Delta_{\rm odd}/d\ln\mu^2 = -0.933\,\%$ per unit. Against the measured odd residual — $\kappa_{\rm odd} \simeq 1.824\,\%$, carrying the 0.212 pp spread from the PDG errors printed above (`s1182`) — the two agree at $+0.50\sigma$, the σ dividing by the single combined denominator $\sqrt{0.2121^2 + 0.6467^2} = 0.6806\,\%$ that carries both errors (`s1193`). *No σ for this comparison is printed on the experimental error alone.* The theory error is 3.05× the experimental one, so this is a consistency statement and not a test; it becomes a test when the matching is carried to two loops. The scale is declared, not derived: $-\kappa_{\rm odd}$ is reached exactly at $\mu^\star = 295.8$ GeV, a 20 % shift off v, and $\mu = v$ is not the best-fitting choice — $\mu = 2m_t$ agrees comparably, at -0.42σ on that same denominator, so the preference rests on the distance to exactness and not on a σ gap. Charm is not covered by this combination, χ_c above remaining open, and the reading leaves the weak-isospin signature typed a MECHANISM CANDIDATE rather than a derivation.[1]

Independent algebraic derivation (S44, A641): An alternative route to the factor 7 bypasses the Gram calculation entirely. Each off-diagonal Peirce block $P_{ij} \cong \mathbb{O}_s$ has dimension 8. The real part ($\dim = 1$) decouples into diagonal shifts; the 7 imaginary units of $\mathbb{O}_s$ (the Fano-plane generators of the off-diagonal interactions) carry the independent degrees of freedom. Therefore:

$$\frac{7}{3} = \frac{\dim(\mathrm{Im}(\mathbb{O}_s))}{\mathrm{rank}(J_3)} = \frac{7}{3}. \tag{25}$$

This derivation requires no reference to $J_{\rm vac}$ or the Gram matrix. The DET-7 theorem then confirms $n_{23} = 7$ for $J_{\rm vac} = \mathrm{diag}(\phi, 1, \phi^{-1})$ specifically [A641/Q2A], making the two routes mutually consistent.

Uniqueness of the golden vacuum (A641/Q5A): For general $\mathrm{diag}(a, b, c)$ with $abc = 1$ the Gram determinant evaluates to $\det(G) = (a^2 + b^2 + c^2)(a^{-2} + b^{-2} + c^{-2}) - 9$. Setting $\det(G) = 7$ forces $(a^2 + b^2 + c^2)(a^{-2} + b^{-2} + c^{-2}) = 16$. Among diagonal elements of $J_3(\mathbb{O}_s)$ with *algebraic-integer entries over* $\mathbb{Q}(\sqrt{5})$ and $abc = 1$, this condition is satisfied *uniquely* (up to permutation) by $\{a, b, c\} = \{\phi, 1, \phi^{-1}\}$. Equivalently, the condition selects *uniquely* the diagonal whose characteristic polynomial is palindromic, i.e. satisfies $T_1 = T_2 = 1 + \sqrt{5}$ (self-adjugate symmetry of the golden vacuum). No other algebraic-integer norm-1 diagonal element of $J_3(\mathbb{O}_s)$ satisfies both conditions simultaneously.

Freudenthal complementarity (E49): $R_{12} + R_{23} = 1$ exactly, with $R_{12} = \phi^{-4}$ and $R_{23} = 1 - \phi^{-4}$. This identity follows from the DET-7 Gram structure and Peirce orthogonality; it supplies the geometric anchor for the 7/3 lepton–quark bridge.

Cross-sector envelope (scope note). The 7/3 bridge and the resolvent family above are *within-sector* relations: they organize ratios inside the charged-lepton and inside the quark sectors. The much larger *cross-sector* gap (e.g. $m_t/m_e \sim e^{12.7}$) is a separate question. The octonion-index decomposition $\mathbb{O}_s \to 1(e_0) \oplus 1(e_7) \oplus 3 \oplus \bar{3}$ under $SU(3) \subset G_2$ supplies a rigorous lepton/quark band *label* (leptons and the Higgs on the colour singlets, quarks on the colour triplets), and a Casimir-weighted localization $c_f = c_0 + \alpha\, C_2^{SU(3)}(f)$ would compress the cross-sector envelope to one coefficient. An explicit $SU(3)$-structure Dirac-operator analysis shows this *form* is geometrically natural only under a homogeneous adjoint-isotropy condition (generic curvature splits weights and the band law fails), while the coefficient

[1] `s1186` evaluates the matching at a *constant* y_t, where a complete matching calculation wants $y_t(\mu)$; both the pole reading $y_t = \sqrt{2}\, m_t/v$ and $y_t = 0.967$ (the suite's former row-15 value, now read as the measured-side coupling at M_Z, SL-57: 0.970 ± 0.002) are carried and the two differ in the third digit of $\Delta_{\rm odd}$. The gaugeless limit drops the hypercharge odd running and the transverse-gauge finite pieces that carry no m_t^2 enhancement.

and sign remain free moduli. The cross-sector envelope is therefore REDUCED TO ONE GEOMETRIC MODULUS under that assumption, *not derived*; the within-sector mechanisms of this section are unaffected. See Appendix X (Color/singlet localization envelope) for the full status.

3.7 The clean-lepton residual: the golden-ladder skeleton and the Koide fine-structure

Write each charged lepton's golden beat-depth above the electron anchor as

$$n_f = \log_\varphi(m_f/m_e), \qquad \varphi = \tfrac{1+\sqrt{5}}{2}, \tag{26}$$

so the electron is depth 0 (the pure-charge anchor) and $m_f = m_e\,\varphi^{n_f}$. Empirically the three charged leptons sit close to the *integer* golden ladder $\{0, 11, 17\}$: $n_\mu = 11.08$, $n_\tau = 16.94$. But the integer ladder alone is good only to a few percent, and the residual is *structured* — it changes sign between generations: the pure integers predict $m_\mu = m_e\varphi^{11}$ some 3.75% *low* and $m_\tau = m_e\varphi^{17}$ some 2.70% *high* (opposite signs), so the deviation is not a single uniform warp of the ladder.

The structured residual is the Koide relation. With

$$Q = \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2}, \tag{27}$$

the PDG pole masses give $Q = 0.666661$, i.e. $Q - \frac{2}{3} = -6.2 \times 10^{-6}$: the cleanest known charged-lepton mass relation. Fed the measured electron and muon masses, the constraint $Q = \frac{2}{3}$ *predicts* the tau,

$$m_\tau^{\text{pred}} = 1776.97 \text{ MeV} \quad (\text{vs. PDG 2026 } 1776.93 \pm 0.09 \text{ MeV},\ +0.002\%), \tag{28}$$

which in golden-depth units is $n_\tau = 16.9448$ versus the observed 16.9447 — the Koide constraint reproduces the tau residual to $\sim 10^{-4}$ in depth, parameter-free, where the pure-integer and half-integer ladders miss by ~ 0.055. So the gross hierarchy is the integer golden ladder (the charge/winding skeleton of §3.3); the *fine* residual on top of it is Koide.

Geometrically Koide is a statement about the *half-depth* ladder. Writing $\sqrt{m_f} = \sqrt{m_e}\,\varphi^{n_f/2}$, the vector $(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$ makes a fixed angle with the diagonal $(1, 1, 1)$:

$$\cos^2\theta = \frac{(\sum \sqrt{m_f})^2}{3\sum m_f} = \frac{1}{3Q} = \frac{1}{2} \quad \Longleftrightarrow \quad \theta = 45^\circ \text{ (measured } 44.9997^\circ), \tag{29}$$

which holds exactly when $Q = \frac{2}{3}$. The relevant ladder is therefore the half-depth $n/2$ — the same spinor "half" singled out by the live time-bridge $T^2 = -1$ (Appendix H, Theorem H.1): the lepton mass fine-structure lives at the half-turn level.

Significance. The prediction is unambiguous: Koide$(m_e, m_\mu) = \frac{2}{3}$ has two tau roots, the heavy one 1776.97 MeV (matches PDG) and a light one 3.32 MeV (excluded). It is not generic: among random triples whose μ, τ depths lie in the same near-integer band as the data (± 0.15 of $\{11, 17\}$), only $\sim 0.05\%$ — about 1 in 2,000 — reach $|Q - \frac{2}{3}| < 6 \times 10^{-6}$, so the observed agreement is a thin coincidence *on top of* the golden ladder, not a by-product of being near integers. Input count: given m_e, the two free lepton masses carry two integer-ladder residuals; the ladder is a 0-parameter skeleton and Koide is one equation that removes the tau residual exactly — 1 of 2 explained, no overfitting.

Honest limits. This is an *empirical confrontation*: the PDG pole masses and the Koide relation are external; Koide is not (yet) derived from the substrate. The agreement is recorded as a structural coincidence on the golden half-depth ladder, not a theorem. Koide removes one of the two lepton residuals exactly (the tau, given e and μ); the remaining muon integer-offset $n_\mu - 11 = +0.0795$ is the one number Koide does not fix and is left *open* ($\mu_{\text{offset}}/\kappa \approx \frac{1}{3}$ is suggestive only, not claimed — three data points are not overfit). No public observable moves to theorem grade; this strengthens the lepton rest-mass row of the scorecard (electron: Koide-consistent; muon: STRUCTURAL/LOADED-CORRESPONDENCE/REPRODUCED after the Rev29 re-tiering, §3.2, not PROVED; tau: anchor) without changing the declared input status of the dimensionful scale. Evidence: `s108a_lepton_residual.py` + `_S108a_lepton_residual.json` (keys `a2_koide_Q`, `a3_depth_pred_minus_obs`, `a4_angle_deg`); significance from `s108e`; `_S108_manifest`.

Tiering note (S245). A separate in-house test closes off the cheapest hoped-for completion of this ladder: an exact single-χ closure of the mass *ratios* on the occupancy depth support $\{0, 3, 11, 17\}$ is *refuted at the ratio tier* (no admissible bijection; Appendix O, the additive-χ closed-negative, and the closed-selector ledger of Appendix S). The ordered golden-depth *skeleton* and the few-percent *shape* on it are unaffected — the integer order is structural, the exact completion is not.

4 Resolvent quintics

For publication purposes the suite packages each sectoral mass as the positive real root of a monic quintic

$$\mathcal{Q}_d(\mu) = \mu^5 - R_d = 0, \qquad \mu = \frac{m}{m_\star}, \tag{30}$$

where $m_\star$ is the relevant sectoral anchor. The quintic language is a bookkeeping device: it keeps the common algebraic coefficient explicit without pretending that a unique ultraviolet dynamical quintic has already been constructed.

5 Mass-sector assignments

Table 1: Operational mass assignments in Rev29. **Rev32.1 reading rule: tiers name mathematical provenance; physical attachments are stated separately; no row may be cited as zero-parameter; displayed σ values are named comparator distances, not profile likelihoods.**

Sector	Representative	Algebraic factor	Comment
Top	m_t	$y_t = 1$; $m_t = v_{\mathrm{EW}}/\sqrt{2}$	Fit-free *construction*, not a prediction (*Rev29*); +0.87% vs PDG 2026, $d_{\mathrm{cmp}} = +5.57$ cross-scheme, no status (Rev32.1 retype, A1531/T2); pole/$\overline{\mathrm{MS}}$ running moves the wrong way — an open threshold problem (§3.5, §3.6, App. X), not a clean pass. This row reads $y_t = 1$ at the electroweak scale; Appendix A and Paper 0 read $y_t(M_{\mathrm{Pl}}) = 1$ — two different statements, not one (Rev32.9, A1642 O2). (Withdrawn at Rev33.2: the $m_t = 168.36\,\mathrm{GeV}$ (-2.46%) once printed here as the second reading's endpoint was $0.967\,v_{\mathrm{EW}}/\sqrt{2}$, the tree reading of the measured coupling at M_Z, not a theory output — `s1337`.)
Strange (constituent)	m_s^{const}	$\Lambda_{G_2}\varphi(K_{G_2}/K_{SU(3)})^{1/4}$	$\approx 554\,\mathrm{MeV}$; $G_2/SU(3)$ threshold.
Up/Down (constituent)	$m_{u,d}^{\mathrm{const}}$	$\Lambda_{G_2}\varphi^{1/2}$	$\approx 331\,\mathrm{MeV}$; $-1.57\%/-2.73\%$ PDG (full precision 330.73).
Bottom	m_b	$\frac{7}{3}m_\tau = 4146\,\mathrm{MeV}$	7/3 unification; -0.95% vs PDG 2026 ($m_b(m_b) = 4186 \pm 6\,\mathrm{MeV}$), $d_{\mathrm{cmp}} = -6.64$ cross-scheme, no status (Rev32.1 retype, A1531/T2; §3.6, App. X). DET-7 threshold. STRUCTURAL $\star\star\star\star$

Sector	Representative	Algebraic factor		Comment
Charm	m_c	$\frac{7}{3}m_s^{\text{const}}$ 1291.9 MeV	=	7/3 unification; +1.49% vs. PDG 2026 $m_c(m_c) = 1272.9 \pm 4.5$ MeV; $d_{\text{cmp}} = +4.22$ (+3.69 with the Λ_{G_2} rounding band folded in, `s1211`) is cross-scheme audit metadata, not status. DET-7 threshold. STRUCTURAL ⋆⋆⋆⋆
Neutrino hierarchy	m_{ν_2}/m_{ν_3}	R_2		Solar-to-atmospheric ratio channel: $R_2^2 = 0.02918$ vs. 0.03002 (NuFIT 6.1 IC24 NO, $\Delta m^2_{21}/\Delta m^2_{3\ell}$); now typed as a $\Delta m^2_{21}/\Delta m^2_{31}$ ratio prediction asserting $m_1 = 0$ — an assertion of the framework, not a derivation (Paper 5, Rev33.0; `s1104`/A1546 for the one-family result). $d(\nu) = 2$ SELECTED, not derived. STRUCTURAL (ladder tier).
Charged leptons	m_μ/m_τ	$(\phi/\sqrt{5})^{8/3}\sqrt{2}/10$		STRUCTURAL / LOADED-CORRESPONDENCE / REPRODUCED (*Rev29*; PROVED/zero-parameter withdrawn — provenance target-first, formal derivation BLOCKING at A618:147, and the +0.37% miss is the multiplicative gap $C_{\text{req}} = 0.140894$ vs. $\sqrt{2}/10 = 0.141421$, §3.2). Gives $m_\mu = 106.05$ MeV from $m_\tau = 1776.93(9)$ MeV.
d-index assignment	selected, not derived (R-23, Rev32.9)	$R_d = 1/(\phi^{2d} - 1)$		ratios RGE-preserved to $< 1\sigma$; the assignment itself is a post-hoc feature rule (Paper 5), and the $d = 3$ entry for m_μ/m_τ is a label, not R_3 (§3.1).

Mass schemes and comparison scales

For all mass comparisons the scheme is part of the prediction. Rev29 therefore uses the following comparison convention.

Mass	Predicted value	Scheme	Reference scale	PDG comparison
m_e	0.5076 MeV	pole / physical charged-lepton mass	on shell	−0.66%
m_μ	105.7233 MeV physical after QED matching	pole / physical charged-lepton mass	on shell	+0.061%
m_τ	1776.93(9) MeV	measured anchor; pole / physical charged-lepton mass	on shell	input
m_t	174.10 GeV	tree-level Yukawa/EW-boundary value (not pole, not $\overline{\text{MS}}$; App. X scheme table)	electroweak scale	+0.87% vs. PDG 2026; d_{cmp}=+5.57, no status
m_b	4.146 GeV	tau-pole-anchored bridge (compared to $\overline{\text{MS}}$, cross-scheme, declared)	$\mu = m_b$	−0.95% vs. PDG 2026; d_{cmp}=−6.64, no status
m_c	1.292 GeV	constituent-anchored bridge (compared to $\overline{\text{MS}}$, cross-scheme, declared)	$\mu = m_c$	+1.49% vs. PDG 2026; d_{cmp}=+4.22 (+3.69 with the Λ_{G_2} band, `s1211`), no status
m_s^{const}	554 MeV	constituent mass proxy	Λ_{G_2} hadronic scale	structural proxy

6 Conclusion

The Rev29 mass sector structurally pins the charged-lepton sector with mixed provenance (not a derivation of all three masses from $J_3(\mathbb{O}_s)$ alone) and establishes the 7/3 lepton–quark bridge. The charged-lepton status is: m_τ is the external anchor, m_μ follows from the algebraic ratio formula at Structural / Loaded-correspondence / Reproduced (*Rev29*; not zero-parameter — the +0.37% miss is the coefficient fit gap, §3.2), and $m_e = 0.5076$ MeV is Koide-consistent ⋆⋆⋆⋆ from the QED-corrected external Koide chain (−0.66% PDG). The top mass is fixed by the parameter-free idempotent *construction* (an algebraic identity of the carrier, not a zero-parameter claim about the observable) $m_t = v_{\text{EW}}/\sqrt{2}$, which sits +0.87% above the PDG 2026 value ($d_{\text{cmp}} = +5.57$, cross-scheme, no status — Rev32.1 retype, A1531/T2) — not a successful prediction (*Rev29*; §3.5, App. X). Bottom and charm masses are unified via the 7/3 DET-7 bridge at STRUCTURAL ⋆⋆⋆⋆ accuracy. The resolvent family is a structural Peirce consequence; the light-quark constituent sector is tied to the G_2 confinement scale. The heavy-quark sub-percent residuals are genuine (A713), carry a weak-isospin

T_3 signature, and are carried honestly as a phenomenological structural threshold term — neither a derived carrier (A715/A716) nor a principled boundary-condition revision (A717) is available, and both searches are recorded as closed negatives. The single remaining formal gap is the S-matrix proof connecting the Gram-determinant ratio to the physical heavy-quark Peirce closure amplitude; this does not block publication and is stated explicitly as an open programme item. The surviving open work is concentrated in the bosonic extraction bridge, the AX6 axiom, and the not-derived $\{\kappa, \chi_c\}$ threshold numbers of the heavy-quark sector.

Leibniz Quantum Beats Newton

Paper 3 (Rev33.2): CKM and PMNS Mixing in the $J_3(\mathbb{O}_s)$ Framework

Tom O'Sieg

August 2026

Abstract

Read this paper on two layers — a Rev29 fold notice. This paper contains two bodies of work that are easy to mistake for one, and the Rev29 fold prints the distinction rather than leaving a reader to infer it.

The Rev29 mixing ledger (Sections 2–5) is a *readout-conditioned* construction. Its angle relations, its first-row Route B weight $1/\sqrt{6} = 1/\sqrt{2} \cdot 1/\sqrt{3}$, its $\beta = \pi/8$ Shilov *conjecture* (printed as a theorem through Rev32.8 and retyped at Rev32.9: the printed proof computes γ, not β; §2.2) and its completion map all live downstream of the loaded Route B correspondence. Where a row in that ledger is marked THEOREM it is a theorem *given that correspondence and its named readout rule* — not a derivation from structure with no measured input anywhere in the chain. The full 3×3 matrix is a completion map: after the re-tiering of §5 and the A1566 retype, *two* of eight parameters stand at DERIVED-CONDITIONAL ($\theta_{12}^{\mathrm{PMNS}}$, δ_{CP}), each with its premise printed; the physical θ_{23} row is LOADED-CORRESPONDENCE (its internal 7/16 identity theorem-grade); θ_{13} is structural, $|V_{cb}|$ is LOADED-CORRESPONDENCE (physical attachment) / Structural formula (A1566), the CKM first row is LOADED, and the construction completes the rest. It is not an eight-parameter prediction and was never claimed as one.

The structural layer (Section 8, folded at Rev29 from A1471) is upstream of all of that. There, module naturality forbids generation mixing outright, the framework's structural value is $V_{\mathrm{CKM}} = I$, and any observed departure is typed as a controlled defect. This layer makes no numerical mixing claim at all.

These do not contradict each other, but they are not the same claim, and the suite has not previously said so in print. A structural layer that gives I and a loaded readout layer that reproduces measured angles are compatible exactly to the extent that the loading is honestly declared — which is what the A654 supersession in Section 8 exists to do, and what the priced-clause register in Appendix X now enumerates.

What the re-tiering settled. That row-by-row audit has now been performed (§5, kernel `s959`; A1566 retype, Rev32.6). Ten of eleven ledger rows carry a marking different from their Rev29 column (the Rev29 hand count was six, treating the CKM first row as one row and Algebraic as a non-tier). The CKM first row is LOADED, because this paper selected Route B over Route A by comparing both to kaon data; δ_{CKM} is COINCIDENCE-CLASS while the motivation for G_7 is missing; the physical θ_{23} row and $|V_{cb}|$ are LOADED-CORRESPONDENCE (A1566: the internal 7/16 mismatch identity stays theorem-grade; its octant registration is unselected); and two rows survive at DERIVED-CONDITIONAL with their premises printed ($\theta_{12}^{\mathrm{PMNS}}$, δ_{CP}). **No row in Sections 2–5 may be cited as zero-parameter**, and the agreement figures quoted there (1.3%, 1.8%, 0.8σ, 0.62σ) are agreement of a partly loaded construction with data — not evidence for the structure.

This paper collects the Rev29 mixing-sector ledger in a single place. The quark matrix is organised by Peirce-sector transition amplitudes; the quark Dirac phase is attached to the non-associative sector; and the leptonic matrix is organised by spectral, Casimir, and Schur-type projections on the same exceptional-Jordan background. The purpose of the paper is not to blur the difference between exact algebraic structure and measured benchmark values. It is to state both clearly.

The Rev29 update has three parts. First, the CKM first row is written in its canonical Route B form with $|V_{us}| = \sin\theta_{12}^{\mathrm{PMNS}}/\sqrt{6}$ and $|V_{ud}|$ fixed by first-row unitarity. The Route-B weight $1/\sqrt{6}$ factorises as $1/\sqrt{2}\cdot 1/\sqrt{3} = 1/\sqrt{3!}$ (s499): the $1/\sqrt{2}$ trace-form block times the $1/\sqrt{3}$ three-generation normalisation — a derived product, independent of any kaon fit. [LIB2-050] Second, the full 3×3 CKM matrix is carried in standard form and all six unitarity relations are recorded explicitly. Third, the PMNS matrix is written with the exact angle set $\tan\theta_{12}^{\mathrm{PMNS}} = \sqrt{3}/\phi^2$ (PMNS solar angle; proved as theorem), $\sin^2\theta_{23} = 7/16$ (F_4 mismatch geometry; internal $\sin^2\theta_{\mathrm{mismatch}} = 7/16$ theorem-grade; physical octant LOADED-CORRESPONDENCE (A1566); the pre-Rev29 stamp was THEOREM ⋆100, D575), $\sin^2\theta_{13} = (3-2\sqrt{2})/8$ (double $\beta{=}\pi/8$; D572), and the authoritative Rev29 V_3-rotation phase theorem for δ_{CP}. The Jordan-adjoint mixing carrier — called the neutrino mass operator through Rev28, renamed at Rev29 because the literal mass reading is disproved in Paper 5 — is defined as $X_\nu = J_{\mathrm{vac}} \times J_{\mathrm{vac}} = J^{\#}_{\mathrm{vac}} = \mathrm{diag}(\phi^{-1}, 1, \phi)$ (Freudenthal cross-product, Rev9 Session 11).

A closing reconciliation (§6, kernels `s270`–`s282`) records a hostile in-house audit of this sector and its in-house adjudication: the parity-graded *occupation* clockwork and the *dynamical* cross-generation spurion are both fenced, but the *kinematic* geometric derivation of the angles below is verified to survive that audit; the theorem-grade internal constructions are unchanged (physical tiers per §5).

1 Introduction

The Rev29 mixing story is a clean separation of source maps. The CKM matrix is the quark-sector shadow of Peirce-sector transition amplitudes. The PMNS matrix is the lepton-sector shadow of spectral, Casimir, and Schur-type projections on the same vacuum. The algebraic inputs are common; the physical maps are not. The use of F_4 and $J_3(\mathbb{O})$ for Standard Model classification has independent published precedent in Todorov and Drenska [1].

2 CKM sector

The CKM first row is canonically fixed by the Peirce J_{12} block through Route B:

$$|V_{us}| = \frac{\sin\theta_{12}^{\mathrm{PMNS}}}{\sqrt{6}} \approx 0.22526. \tag{1}$$

With

$$V_{cb} = \frac{1}{9\sqrt{7}} \approx 0.041996, \qquad V_{ub} = \frac{|V_{us}|\,|V_{cb}|}{\sqrt{6}} \approx 0.003862 \quad (-0.18\sigma \text{ against the PDG 2026 direct comparator; first-r} \tag{2}$$

Comparator, named (Rev32.2). Against the PDG 2026 direct value $|V_{ub}| = 0.00389 \pm 0.00016$, the displayed 0.003862 gives -0.18σ under the stated central-value Gaussianization (full precision -0.1752; the displayed 0.003862 sits on the -0.175 tie; Rev32.9). [LIB2-067, LIB2-215] The historical 0.003855 variant is retained only where explicitly labelled as that alternate rounding; it gives -0.22σ. The

former 0.52σ stamp is retired because its comparator was never recoverable from print. first-row unitarity gives

$$|V_{ud}| = \sqrt{1 - |V_{us}|^2 - |V_{ub}|^2} \approx 0.97429. \tag{3}$$

Comparator (supplied Rev32 — this row previously had none anywhere in the suite): PDG 2026 $|V_{ud}| = 0.97367 \pm 0.00032$, central-value Gaussianized, places the prediction at $+1.94\sigma$ — the second-largest mixing-sector comparator distance in the suite. [LIB2-065] It is printed here so the row is compared rather than left uncompared; the first-row LOADED typing (§5) applies. The first row is LOADED: Route B was selected after comparison with kaon data. [LIB2-063, LIB2-237, LIB2-239, LIB2-293, LIB2-294, LIB2-295, P-017] With the framework solar angle, the canonical current value is $|V_{us}| = 0.225256$ (the exact framework angle $\arctan(\sqrt{3}/\phi^2) = 33.4880°$, printed from the kernel line `s1162`; the rounded display angle 33.49° gives 0.225268, the figure printed through Rev32.4; the Rev32.5 "correction" to 0.225247/33.4877° was itself a hand calculation that matched no angle in the chain — the digit we got wrong twice, corrected Rev32.6 from `s1162`, A1572), compared with PDG 2026 0.22431 ± 0.00085, giving $+1.11\sigma$ under the stated central-value Gaussianization. [LIB2-063, LIB2-237, LIB2-238, LIB2-239, LIB2-293, LIB2-294, LIB2-295] Feeding the measured NuFIT solar angle through the same formula gives the separate diagnostic value 0.22687. Against the $|V_{us}|$ comparator error alone that is $+3.01\sigma$; propagating the NuFIT θ_{12} error as well gives about $+1.0\sigma$ (0.97–1.00, house recompute A1641), and only the propagated figure is a distance (Rev32.9). [LIB2-064] These are different readout objects; neither changes the derived $1/\sqrt{6}$ block weight, and neither upgrades the first-row provenance. [LIB2-064]

2.1 CKM matrix assembly and unitarity check (one angle under-specified)

Read first (tiering). "Assembly" is deliberate: this section inserts the framework's four independent entries into the standard unitary parameterisation and checks consistency — so unitarity holds *by construction*, not as an independent success. [LIB2-071] The matrix is *not* fully closed: the $(2,3)$ Peirce block has no explicit $|V_{td}|$ formula, and the $\beta = \pi/8$ conjecture (§2.2; a theorem through Rev32.8, retyped at Rev32.9) disagrees with the angle $\beta_{\text{recon}} = 23.44°$ reconstructed from these entries. [LIB2-072] The framework is under-specified here; see §2.2.

Using the standard parameterisation with the moduli-preserving insertion $s_{13} = |V_{ub}| = |V_{us}||V_{cb}|/\sqrt{6}$, $s_{12} = |V_{us}|/c_{13}$, $s_{23} = |V_{cb}|/c_{13}$, where $|V_{us}| = \sin\theta_{12}^{\text{PMNS}}/\sqrt{6}$ and $|V_{cb}| = 1/(9\sqrt{7})$ (Rev32.9: through Rev32.8 the sines were set equal to the moduli, which drops $c_{13} = 0.9999925$ and would display $|V_{us}| = 0.22525$; the matrix below is the moduli-preserving one; unitarity is unaffected), and $\delta_{\text{CKM}} = \arctan\sqrt{3(\phi + 5\phi^{-5})} = 68.1297°$ (Structural Result 2.1, ⋆⋆⋆; PDG 2026: $+0.64\sigma$ vs. the direct UT-angle γ and $+1.40\sigma$ vs. the global-fit Dirac phase in its published radian representation (R-29) — two distinct comparator objects, both named in §2.1), the magnitude matrix is

$$|V_{\text{CKM}}| \approx \begin{pmatrix} 0.97429 & 0.22526 & 0.00386 \\ 0.22512 & 0.97343 & 0.04200 \\ 0.00878 & 0.04125 & 0.99911 \end{pmatrix}. \tag{4}$$

One line, reconciling these magnitudes with $V_{\rm CKM} = I$ (*Rev29*; the reconciliation requested at S215 and open since). The framework's *structural* value is $V_{\rm CKM} = I$ — recovery-fixed module naturality forbids generation mixing outright (§8) — while the matrix printed immediately above is a *readout-layer* object delivered by the loaded Route B correspondence, and the two are compatible exactly to the extent that the loading is declared: what is tabulated above is the measured size of the controlled defect from I, read out through a correspondence this paper types LOADED, and it is not the framework structurally predicting a mixing angle.

The σ figures in this suite are under reconciliation (*Rev29*). They disagree across components and, for $|V_{ub}|$, inside this paper. The state is recorded here rather than resolved by silently picking one, because the failure mode this box guards against is silent divergence.

Comparators, named. Every σ in Sections 2–5 is *central-value Gaussianized* against a single-number comparator — historically (Rev29 print) PDG 2025 review direct determinations for the CKM magnitudes, NuFIT 6.1 (NH, w/SK) for the PMNS rows — and none is a profile-likelihood confidence level. [LIB2-069, LIB2-070] Where the comparator uncertainty is asymmetric, the error on the side facing the framework value is used (Rev32.9: through Rev32.8 this sentence said "the lower error", which is not what the printed numbers use — $\delta_{\rm CKM}$ against γ uses the upper $+2.7°$). [LIB2-070]

Rev32.2 comparator reconciliation. The current scorecard uses one named PDG 2026 comparator for each CKM object and one rounding convention for each framework value:

$$|V_{us}| = 0.225256\ (+1.11\sigma), \qquad |V_{cb}| = 0.041996\ (+1.00\sigma), \qquad |V_{ub}| = 0.003862\ (-0.18\sigma).$$

The measured-angle $|V_{us}|$ value and the 0.003855 $|V_{ub}|$ rounding remain explicitly labelled controls. The historical $0.52/0.38/1.3\sigma$ stamps are retained only in audit prose and are not current comparator results.

Rev32 comparator refresh (PDG 2026). The named CKM comparators move: $|V_{cb}|$ $0.0408 \pm 0.0014 \to 0.0407 \pm 0.0013$, taking the prediction 0.04200 from $+0.85\sigma$ to $+1.00\sigma$; $|V_{ub}|$ $0.00382 \pm 0.00024 \to 0.00389 \pm 0.00016$, and *the deviation changes sign*: 0.003862 moves from $+0.17\sigma$ to -0.18σ (the 0.003855 variant chain sits at -0.22σ). [LIB2-216, LIB2-230] For $\delta_{\rm CKM}$ the suite now names *both* PDG 2026 comparator objects explicitly rather than pairing a 2024-era γ-fit with an unnamed global fit: the global-fit Dirac phase $\delta = 1.154 \pm 0.025$ rad gives $+1.40\sigma$ in that published representation, which is primary (R-29; the degree cells $66.12° \pm 1.43°$ give $+1.41\sigma$), and the direct UT-angle $\gamma = 66.4^{+2.7}_{-2.8}$ gives $+0.64\sigma$ (a numeral comparison: it scores the inserted $\delta = 68.1297°$, while the assembled matrix's own angle is $\gamma(V) = 68.0924°$, $+0.63\sigma$; Rev32.9) — two distinct objects, each labelled where quoted. [LIB2-069, LIB2-070, LIB2-222] $|V_{ud}|$ gains its first comparator (0.97367 ± 0.00032, $+1.94\sigma$; see the first-row block). The internal $0.52\sigma/0.38\sigma/1.3\sigma$ reconciliations above remain OPEN as recorded — this refresh renames comparators, not the adjudications.

Because the matrix is built from the standard unitary parameterisation, all six unitarity relations hold identically. [LIB2-071] In particular,

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1, \tag{5}$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1, \tag{6}$$

$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1, \tag{7}$$

and likewise for the three column sums. Rev29 therefore carries CKM unitarity as a structural internal consistency result rather than as a post-fit numerical accident.

3 PMNS sector

The operative leptonic angle relations are (Rev29; θ_{23}: internal $\sin^2\theta_{\text{mismatch}} = 7/16$ theorem-grade; physical octant LOADED-CORRESPONDENCE (A1566) — pre-Rev29 stamp THEOREM $\star$100, Session 14, D575; θ_{13} STRUCTURAL $\star\star\star$, round-trip framing T^2, exponent-2; forcing pending [s502/A1275], A608/E113)

$$\tan\theta_{12}^{\text{PMNS}} = \frac{\sqrt{3}}{\phi^2} \quad \Rightarrow \quad \theta_{12}^{\text{PMNS}} \approx 33.49^\circ, \quad \text{(solar angle — THEOREM)} \tag{8}$$

$$\sin^2\theta_{23} = \frac{7}{16} \quad \Rightarrow \quad \theta_{23} \approx 41.41^\circ, \quad (\text{F}_4 \text{ mismatch geometry — internal identity theorem-grade; physic} \tag{9}$$

$$\sin^2\theta_{13} = \frac{3-2\sqrt{2}}{8} \quad \Rightarrow \quad \theta_{13} \approx 8.42^\circ. \quad \text{(double } \beta{=}\pi/8\text{; STRUCTURAL } \star\star\star, \text{ round-trip } T^2 \text{ exponent-2 [s502/A1275]; A608/E113)} \tag{10}$$

The identity

$$\phi^2 + \phi^{-2} = 3 \tag{11}$$

shows immediately that

$$\tan^2\theta_{12} = \frac{3}{\phi^4} = \phi^{-2} + \phi^{-6}, \tag{12}$$

so the solar-angle numerator is structural rather than fitted. The derivational status of these cross-generation angle relations under a recent hostile parity-grading audit is reconciled in §6.

3.1 The Jordan-adjoint mixing carrier $X_\nu = J_{\text{vac}}^{\#}$ (Rev9, D572; renamed at *Rev29*)

Name change, not content change (*Rev29*). Through Rev28 this object was called the "neutrino mass operator". Paper 5 shows that the literal mass reading fails by $\sim 9\times$ on $\Delta m^2_{21}/\Delta m^2_{31}$ (A712, verified in house), so the physical-mass predicate named an interface this suite does not have. The definition below is untouched and the object remains valid where it is constructed; X_ν is carried as a mixing/structural carrier only.

Novelty label: new specialization proved here.

Theorem 3.1 (X_ν definition and self-duality)**.** *The Jordan-adjoint mixing carrier is, at tree level,*

$$X_\nu \;=\; J_{\text{vac}} \times J_{\text{vac}} \;=\; J_{\text{vac}}^{\#} \;=\; \text{diag}(\phi^{-1}, 1, \phi),$$

the Freudenthal adjoint of J_{vac}. The pair $(J_{\text{vac}}, J_{\text{vac}}^{\#})$ is self-dual: $(J_{\text{vac}}^{\#})^{\#} = J_{\text{vac}}$. Both operators sit on the Freudenthal cubic $N = 1$ surface.

Proof sketch (Grok D572, $\star$100). Using the cross-product formula $X \times Y = X \circ Y - \frac{1}{2}\,\text{tr}(X)Y - \frac{1}{2}\,\text{tr}(Y)X + \frac{1}{2}[\text{tr}(X)\,\text{tr}(Y) - \text{tr}(X \circ Y)]I$ with $X = Y = J_{\text{vac}}$: $\text{tr}(J_{\text{vac}}) = 2\phi$ and $\text{tr}(J_{\text{vac}}^2) = 4$ (using $\phi^2 = \phi + 1$, $\phi^{-2} = 2 - \phi$). Diagonal entries: $(2\phi - \phi^2,\ 1,\ \phi^{-2} + 2\phi - 2) = (\phi^{-1}, 1, \phi)$. Off-diagonal entries vanish because J_{vac} is purely diagonal. The double-adjoint $(J_{\text{vac}}^{\#})^{\#} = J_{\text{vac}}$ follows by the same calculation with $\phi \leftrightarrow \phi^{-1}$ swapped. Both have $N(J_{\text{vac}}) = N(J_{\text{vac}}^{\#}) = \phi \cdot 1 \cdot \phi^{-1} = 1$. □

Key geometric invariants.

$$\langle J_{\rm vac},\, J^{\#}_{\rm vac}\rangle = \mathrm{tr}(J_{\rm vac}\circ J^{\#}_{\rm vac}) = \phi\cdot\phi^{-1} + 1\cdot 1 + \phi^{-1}\cdot\phi = 3, \tag{13}$$

$$\|J_{\rm vac}\|^2 = \|J^{\#}_{\rm vac}\|^2 = 4, \tag{14}$$

$$\cos\theta_{\rm mismatch} = \frac{3}{4}, \qquad \sin^2\theta_{\rm mismatch} = \frac{7}{16}, \tag{15}$$

$$J_{\rm vac} - J^{\#}_{\rm vac} = \mathrm{diag}(1, 0, -1). \tag{16}$$

Which premises the angle uses (Rev32.9; A1680, A1704). The equal norms above are those of the selected vacuum. The mismatch angle needs only their product, $\sin^2\theta_{\rm mismatch} = 1 - \langle J, J^{\#}\rangle^2/(\|J\|^2\|J^{\#}\|^2)$, and DET-7 with $N(J) = 1$ fixes $\langle J, J^{\#}\rangle = 3$ and $\|J\|^2\|J^{\#}\|^2 = 16$, which gives $7/16$ without equal norms. Equal norms $\|J\|^2 = \|J^{\#}\|^2 = 4$ need in addition (H1), inversion symmetry of the spectrum (Appendix E): the diagonal $J = \mathrm{diag}(\sqrt{x}, \sqrt{x}, 1/x)$ with $x^3 = (11 + \sqrt{105})/4$ has $N(J) = 1$ and Gram determinant 7, but $\|J\|^2 \approx 3.818$ and $\|J^{\#}\|^2 \approx 4.191$. The mismatch vector $\mathrm{diag}(1, 0, -1)$ lies entirely in the $(1, 3)$ Peirce sector (middle entry zero), so it directly seeds θ_{13}. The internal identity $\sin^2\theta_{23} = 7/16$ (physical octant LOADED-CORRESPONDENCE, A1566) is the $(1, 3)$-sector mismatch invariant (tree level, NH-like; NuFIT 6.1 NH (w/SK) global best fit is itself lower-octant, $\sin^2\theta_{23} = 0.470^{+0.017}_{-0.014}$, 2.3σ from the $7/16$ branch; the upper-octant local comparator ≈ 0.561 lies $\approx 9.5\sigma$ away); $\|J_{\rm vac}\|^2 = 4$ is now a theorem from DET-7+$N(J) = 1$+(H1) inversion symmetry (AX2 eliminated, D575; the (H1) premise was not printed here through Rev32.8), and $7/16$ follows from DET-7+$N(J) = 1$ alone through the norm product (remark above) (pre-Rev29 marking THEOREM $\star$100, superseded — per §5 no row in Sections 2–5 may be cited as zero-parameter; the row stands at DERIVED-CONDITIONAL). The internal mismatch identity $\sin^2\theta_{\rm mismatch} = 7/16$ is theorem-grade. Its attachment to the physical atmospheric angle retains an octant-registration ambiguity: the chirality and octant $\mathbb{Z}_2$ torsors admit two equally equivariant identifications, and `s501` selects negative writhe $\mapsto$ lower by convention rather than by a registered action or readout map (A1566/D1244; `s1156`). Because the current census records the retained lower branch as selected against data, the physical θ_{23} row is LOADED-CORRESPONDENCE under the override of §5. The value $7/16$ is unchanged.

$\sin^2\theta_{13}$ **from the doubled** D_4 **half-angle.** The $(1, 3)$ block hosts the D_4 outer automorphism R of Theorem A.2 (the half-angle $\pi/8$; its reading as the CKM angle β is the open conjecture of §2.2, retyped at Rev32.9, and nothing below uses that reading). Applying the half-angle rotation twice — once for $J_{\rm vac}$ (vacuum selector) and once for $X_\nu = J^{\#}_{\rm vac}$ (the Freudenthal adjoint carrier) — gives the quartic suppression

$$\sin^2\theta_{13} = \sin^4\frac{\pi}{8} = \left(\frac{2-\sqrt{2}}{4}\right)^2 = \frac{3-2\sqrt{2}}{8} \approx 0.02145. \tag{17}$$

This prediction is theorem-level *conditional on the named readout rule*; the suite tier remains STRUCTURAL (the unit-endpoint-ray readout and the round-trip T^2 squaring (exponent-2) are not yet derived from first principles): the D_4 triality map $8_v \leftrightarrow 8_s$ sends the $\pi/4$ Cartan angle of $J_{\rm vac}$ to the half-angle $\pi/8$, and the Jordan-manifold norm squares the resulting suppression to

$$\sin^2\theta_{13} = \sin^4\frac{\pi}{8} = \frac{3-2\sqrt{2}}{8}.$$

NuFIT 6.1 (NH w/SK, $\sin^2\theta_{13} = 0.02248^{+0.00055}_{-0.00059}$) places the result at 1.8σ.

The leptonic Dirac phase is derived in Rev29 from the V_3 rotation theorem:

$$\boxed{\delta_{CP} = -\frac{2\pi}{\sqrt{5}} \approx -160.997^\circ} \quad (J_{CP} \approx -0.011). \tag{18}$$

The free parameter $\beta_\delta = 11/(6\pi)$ is eliminated; three of the four PMNS parameters (θ_{12}, θ_{23}, δ_{CP}) carry theorem-grade internal constructions (physically: θ_{12} and δ_{CP} DERIVED-CONDITIONAL, the θ_{23} octant attachment LOADED-CORRESPONDENCE — §5, A1566), with θ_{13} STRUCTURAL ⋆⋆⋆ (round-trip framing T^2, exponent-2; forcing pending [s502/A1275]) (see Appendix B, §B.5 and Appendix C, §C.5 for the proofs). NuFIT 6.1 (NH): 0.36σ (central-value σ from the NuFIT 6.1 NH global best fit $212^{+26}_{-36}{}^\circ$; the full $\Delta\chi^2$ profile is non-Gaussian; see Paper 7 for falsification discussion).

Theorem 3.2 (DET-7 on the selected vacuum; the Rev29 stamp ⋆100 is superseded by §5). Let $G = \left(\begin{smallmatrix} \langle J_{\rm vac}, J_{\rm vac}\rangle & \langle J_{\rm vac}, J^{\#}_{\rm vac}\rangle \\ \langle J^{\#}_{\rm vac}, J_{\rm vac}\rangle & \langle J^{\#}_{\rm vac}, J^{\#}_{\rm vac}\rangle \end{smallmatrix}\right) = \left(\begin{smallmatrix} 4 & 3 \\ 3 & 4 \end{smallmatrix}\right)$ be the Gram matrix of $(J_{\rm vac}, J^{\#}_{\rm vac})$. Then $\det G = 4 \cdot 4 - 9 = 7 = n_{23}$. Proof uses only $\varphi^2 + \varphi^{-2} = 3$ and $\langle J_{\rm vac}, J^{\#}_{\rm vac}\rangle = 3$ on the selected vacuum. (Rev32.9: through Rev32.8 this read "both forced by AX2" and "$n_{23} = 7$ is derived". AX2 is eliminated by DET-7 (D575), and Appendix E types DET-7 as a structural *postulate* whose integer is 7, so $\det G = 7$ is that postulate evaluated on the selected vacuum, not a derivation of n_{23}.) The '7 structure': $\sin^2\theta_{23} = 7/16$ and $|V_{cb}| = 1/(9\sqrt{7})$ share the integer $n_{23} = 7$. **Note:** The Rev29 authoritative CP phase is $\delta_{CP} = -2\pi/\sqrt{5}$ (V_3 rotation); $2\pi/7$ was a deprecated PSL(2, 7) artifact and is not in the current derivation.

Candidate Construction 2.1 ($G_7 = \phi + 5\phi^{-5}$ and the retained CKM-phase numeral; Sessions 22–23, A578+A579; ⋆⋆⋆). *Status first (Rev32.1, A1538 F10): COINCIDENCE-CLASS / CANDIDATE-CLASS — step (B) below is house-proven circular as a derivation; the numeral and its σ-comparisons stand.* [LIB2-068] The CKM Dirac phase is *constructed* from the sub-leading eigenvalue correction of $J_3(\mathbb{O}_s)$:

$$G_7 = \phi + 5\phi^{-5},$$
$$\delta_{\rm CKM} = \arctan\sqrt{3(\phi + 5\phi^{-5})} = 68.1297^\circ$$

($+0.64\sigma$ from the PDG 2026 direct $\gamma = 66.4^{+2.7}_{-2.8}{}^\circ$; $+1.40\sigma$ from the PDG 2026 global-fit $\delta = 1.154 \pm 0.025$ rad)

Census note (R187-b; Appendix X, "N-1 blind formula census", `s1206` v3): in a fixed expression language of complexity ≤ 6 this form is reached only at the ceiling (complexity 6), where every in-band value is at least as simple; the bare-ϕ control 65.59° (-0.37σ, inside) is reached at complexity 3 with rank 396. The correction moves $\delta_{\rm CKM}$ away from the global fit and costs three units of complexity. The correction $5\phi^{-5}$ is obtained in four algebraic steps (step (B) circular as a derivation — see the classification note below):

(C) The minimum closed Peirce-graph loop $P_{13} \to P_{13}$ requires $n = 5$ steps through $T_{\rm half}$; Fibonacci coefficient $F_5 = 5$.

(B) The split-octonion (4, 4) signature of $\mathbb{O}_s$ forces coupling weights $w_{P_{13},k} = \frac{5}{8}\, w_{{\rm total},k}$: compact e_1, e_2, e_3 cancel; scalar + non-compact e_4–e_7 contribute (5 of 8 directions). Weighted resolvent sums equalize: $\delta M_{P_{13}} = \delta M_{P_{22}} = 5\phi^{-4}$.

(A) Equal shift $\varepsilon = 5\phi^{-4}$ on Peirce masses ϕ^2 and 1 gives $G_7 = \phi + \varepsilon/\phi = \phi + 5\phi^{-5}$.

(D) This deformation is the *unique* $O(\phi^{-4})$ perturbation of $J_{\rm vac}$ preserving the Albert-algebra characteristic-polynomial form (no free parameter *inside* the deformation; the $(c, k) = (5, 5)$ choice itself is the menu selection recorded in the control below).

CKM magnitudes under the PDG 2026 scorecard: $|V_{us}| = 0.225256$ ($+1.11\sigma$, first row LOADED), $|V_{cb}| = 0.041996$ ($+1.00\sigma$, LOADED-CORRESPONDENCE attachment, Structural formula — A1566), and $|V_{ub}| = 0.003862$ (-0.18σ, inherited first-row loading). The pre-Rev29 THEOREM/$\star$100 stamps are historical and are superseded by §5; no row in Sections 2–5 may be cited as zero-parameter. The CKM phase remains COINCIDENCE-CLASS: $+0.64\sigma$ against the direct UT angle and $+1.40\sigma$ against the global-fit phase, with the first-principles origin of G_7 open. *Zero-correction control, printed beside it (Rev32.2; house* `s1106`*, lane A1546 C4, PI ruling S294).* The *uncorrected* argument $X = \phi$ gives $\delta = \arctan\sqrt{3\phi} = 65.587°$: -0.37σ against the global-fit 1.154 ± 0.025 rad and -0.29σ (the lower error, the side facing the value) against the direct $\gamma = 66.4^{+2.7°}_{-2.8}$ — closer than G_7 on both comparators. [LIB2-075] The $5\phi^{-5}$ correction ($+2.54°$) is not supported by current data, and 28 of its 64 siblings $\phi + c\,\phi^{-k}$ ($c, k \in \{1, \ldots, 8\}$) land within 1σ of the global fit, so the numeral's agreement carries no significance beyond the density of the menu; no further (c, k) scans are made. The object type of δ_{CKM} is an *angle* ($\tan\delta = \sqrt{3}\sqrt{G_7}$ has the PMNS-angle functor shape $\tan\theta = \sqrt{3}\,P_k$); the phase-type invariant is the Jarlskog area, and the suite's $J_{\mathrm{read}} = 3.30 \times 10^{-5}$ is assembled POST-INSERTION of the loaded CKM magnitudes — not an independent prediction (PDG 2026: $J = 3.16^{+0.13}_{-0.11} \times 10^{-5}$). Against that comparator (source-ledger row SL-10) the read value sits at $d_{\mathrm{cmp}} = +1.08$ ($+4.4\%$; ruling R147). The row is DIAGNOSTIC: J is a function of $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ and δ_{CKM}, all four already scored, so it adds no evidence and is counted in no tally of independent successes. Tier stays COINCIDENCE-CLASS; reopen only by a registered operator whose eigenvalue or invariant is $\sqrt{G_7}$. [LIB2-316] *Rev14 classification note:* independent formal scrutiny (D772/D773, artifact-backed) classifies the $5\phi^{-5}$ correction as *numerology-class pending derivation*: step (B)'s equal-shift property was shown to be an algebraic identity of the construction (house-proven circular as a derivation), and an attempted kill of the integer 5 via the $\alpha(0)/m_\tau$ error budget does not land (required coefficient 4.9635 ± 0.087, containing 5 at 1σ; scheme-dependent). [LIB2-316] The numerical statement and its σ-comparisons are unchanged; the derivational status is candidate-class — neither derived nor dead.

2.2 The CKM off-diagonal phase: $\beta = \pi/8$ (Rev9, E22–E26; retyped Rev32.9 as an open conjecture)

Conjecture 3.3 (formerly Theorem 3.3; retyped Rev32.9, ruling R-31). *The CKM unitarity-triangle angle satisfies $\beta = \pi/8$ at leading order, from the D_4 $(1,3)$-block half-angle identity.* **Why this is no longer a theorem.** The argument printed below does not establish β (A1641 F02; house kernel `s1169`, A1648): (1) Step 3 projects onto $z = -V_{ud}V_{ub}^*/(V_{cd}V_{cb}^*)$, whose argument is the angle γ, not β: $\arg z = 68.0924°$ on the matrix this paper prints, under either insertion; (2) that matrix has $\beta = \arg[1/(1-z)] = 23.444°$, and none of the rephasing-invariant angles checked (eighteen, under five transforms) equals $\pi/8$; $\beta = \pi/8$ would need $\delta = 51.62°$ or $83.53°$ at the registered moduli; (3) $R(\mathcal{E}_4) = e_1 + \mathcal{E}_4$ is null in the printed plane with coefficient angle $\pi/4$, so the halving to $\pi/8$ is asserted, not computed. The numeral $\pi/8 = 22.5°$ is kept as a conjecture with this proof gap. The reactor-angle construction of §3.1 uses the D_4 half-angle as a readout rule (Appendix B) and does not rest on this conjecture.

The argument as printed through Rev32.8 (its gap is stated in the box above). **Step 1: Peirce block identification.** The $(1,3)$ off-diagonal block of $J_3(\mathbb{O}_s)$ decomposes as $J_{13} \cong \mathbf{8}_s$ under the D_4 triality action carried by the $\mathrm{Spin}(4,4)$ subalgebra of $F_{4(4)}$ (*corrected Rev29*: the compact $\mathrm{Spin}(8)$, dim 28, does *not* embed in the maximal compact $\mathfrak{sp}(3) \oplus \mathfrak{su}(2)$ of $F_{4(4)}$, dim 24; the split real form $\mathfrak{so}(4,4)$ is

the one that acts here), where $\mathbf{8}_s$ is the positive-chirality spinor representation. The canonical unit element is $\mathcal{E}_4$, the highest-weight vector of $\mathbf{8}_s$ stabilised by the J_{vac} vacuum fixed-point condition; $\mathcal{E}_4$ is a split imaginary unit with norm $N(\mathcal{E}_4) = -1$.

Step 2: The outer automorphism R and charge conjugation. The D_4 triality outer automorphism R exchanges $\mathbf{8}_s \leftrightarrow \mathbf{8}_c$ while fixing $\mathbf{8}_v$. It is the order-two *outer* automorphism of $\mathfrak{so}(8)$ — the diagram involution swapping the two spinor nodes of the D_4 Dynkin diagram — and as such lies in the *disconnected* component of $\text{Aut}(\mathfrak{so}(8)) \cong \mathfrak{so}(8) \rtimes S_3$. It therefore *cannot* be written as exp of a Lie-algebra element (an exponential lands in the identity component); R is realised concretely by the discrete triality (diagram) automorphism of Appendix A restricted to the spinor transposition $\mathbf{8}_s \leftrightarrow \mathbf{8}_c$. In Peirce-frame language R is the charge-conjugation operator C: it swaps particle and antiparticle quantum numbers within the $J_{13} \cong \mathbf{8}_s$ block. The blocks $J_{12} \cong \mathbf{8}_v$ and $J_{23} \cong \mathbf{8}_c$ remain real under R (triality covariance), so *CP violation is sourced exclusively by the $J_{13} \cong \mathbf{8}_s$ block.*

Step 3: Im/Re ratio and the $(4,4)$ signature. Acting on $\mathcal{E}_4$, the automorphism gives $R(\mathcal{E}_4) = e_1 + \mathcal{E}_4$, a sum of one compact direction ($N(e_1) = +1$) and the original split direction. Projecting onto the CKM cross-ratio plane $z = -V_{ud}V_{ub}^*/(V_{cd}V_{cb}^*)$,

$$\frac{\text{Im}(R(\mathcal{E}_4))}{\text{Re}(R(\mathcal{E}_4))} = \sqrt{2} - 1 = \tan\tfrac{\pi}{8}. \tag{19}$$

The $\sqrt{2}$ factor has a precise origin: the $(4,4)$ signature of $J_3(\mathbb{O}_s)$ forces R to mix exactly one compact ($N = +1$) and one split ($N = -1$) direction in the highest weight of $\mathbf{8}_s$. This is algebraically identical to the D_4 half-angle formula: $\tan(\theta/2)$ at $\theta = \pi/4$ (the natural root system angle) gives $\sqrt{2} - 1 = \tan(\pi/8)$.

Step 4: All-orders result. First-order CKM: $V_{ub} \propto \langle e_1|\mathcal{E}_4|e_3\rangle \cdot \Delta$ and $V_{cb} \propto \langle e_1|R(\mathcal{E}_4)|e_3\rangle \cdot \Delta$, where Δ is the eigenvalue difference factor from the Peirce spectrum $(\phi^2, 1, \phi^{-2})$. The ratio carries $\arg(R(\mathcal{E}_4)) = \pi/8$. Higher-order corrections vanish because J_{vac} is an *exact* Freudenthal triple fixed point: $\{J_{\text{vac}}, J_{\text{vac}}^{\#}, J_{\text{vac}}\} = J_{\text{vac}}$ (proved in Rev9, Session 8). The diagonal eigenvalues $\phi^2, 1, \phi^{-2}$ cancel in the ratio at every order. The printed conclusion was $\beta = \pi/8$ to all orders in the Jordan perturbation expansion; by item (1) of the box above, the quantity this argument controls is $\arg z = \gamma$. □ □

[Rev29: the figures in this paragraph are agreement of a loaded construction with data — see the two-layer notice at the head of this paper and Section 8.]

Shilov gaps. The conjectured tree-level value $\sin(\pi/8) = 0.3827$ compares to $\sin\beta_{\text{PDG}} \approx 0.377$ (gap $\approx 1.3\%$). Separately, $|z| = \phi^{-2} = 0.3820$ vs PDG $|z| \approx 0.375$ (gap $\approx 1.8\%$). Both gaps are confirmed to share the same $E_{6(6)}$ Shilov-boundary correction origin (Session 8, Gemini+Grok): the framework predicts the CKM triangle at the Shilov boundary; the PDG values are interior regularisation. The two gaps are *not* independent.

Internal consistency note (S45/D642/A642). The D_4 derivation above is algebraically independent of the other Rev29 CKM elements ($|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, δ_{CKM}). A CKM matrix constructed from those four elements under the standard parametrisation gives $\beta_{\text{recon}} = 23.44°$, not $22.5°$. [LIB2-072] The discrepancy ($0.94°$) indicates the framework is currently under-specified: the $(2,3)$ Peirce block (eigenvalue $\phi/2$) has not yet been mapped to an explicit $|V_{td}|$ formula. [LIB2-072] Empirically, the conjectured $\pi/8 = 22.5°$ is 0.8σ from the PDG direct measurement ($B \to J/\psi K$: $22.1° \pm 0.5°$), while $\beta_{\text{recon}} = 23.44°$ — the angle of the matrix this paper actually prints — is 2.7σ from it. Because the conjecture is not derived (box above), the 0.8σ scores a numeral, not a theorem; the matrix the suite prints is the object in 2.7σ tension (Rev32.9). Full-matrix unification via a complete D_4+Peirce sweep is underway (fullboat kernel Part N, S45).

3.2 PMNS unitarity and oscillation consistency (C3 closure)

The PMNS matrix is written in the standard exact form

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}, \tag{20}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. Unitarity is exact by construction: $U^\dagger U = I$. [LIB2-074] Three of the four PMNS parameters were at theorem level pre-Rev29: θ_{12} (THEOREM $\star$100), θ_{23} (THEOREM $\star$100), and δ_{CP} (THEOREM $\star$100, 0.36σ; A603/E110); after the re-tiering of §5 and the A1566 retype, θ_{12} and δ_{CP} are DERIVED-CONDITIONAL and the physical θ_{23} row is LOADED-CORRESPONDENCE (internal $\sin^2\theta_{\mathrm{mismatch}} = 7/16$ theorem-grade; physical octant LOADED-CORRESPONDENCE (A1566)), with their premises printed. θ_{13} is STRUCTURAL $\star\star\star$ (round-trip framing T^2, exponent-2; forcing pending [s502/A1275]; A608/E113). No fitted coefficients enter the PMNS sector; **no row may be cited as zero-parameter**.

4 Operational table

Table 1: Rev29 mixing-sector ledger. **The Status column below — and every tier marking in the Note column, including the GREEN ★★★★★ stamps and the phrase "zero fitted parameters" — is the Rev29 marking and is superseded en bloc by the re-tiering of §5**, which propagates each row's declared derivation chain: 10 of these 11 rows carry a re-tiered marking different from their printed Rev29 column (Rev32.6 count, generated from the Result table of §5 by comparing the two columns; the Rev29–Rev32.5 text said six, counting the CKM first row as one row and Algebraic as a non-tier), the CKM first row to LOADED because its route was selected against data, δ_{CKM} to COINCIDENCE-CLASS, and the physical θ_{23} row with $|V_{cb}|$ to LOADED-CORRESPONDENCE (A1566). Two of the eight mixing observables survive at DERIVED-CONDITIONAL or above; the internal 7/16 mismatch identity is theorem-grade and recorded separately. **No row in this table may be cited as zero-parameter.** No number in this table changes.

Quantity	Algebraic source	Status	Note
V_{us}	Peirce J_{12} block (Route B: $\|V_{us}\| = \sin\theta_{12}/\sqrt{6}$)	LOADED	Framework-angle value 0.225256 (`s1162`) against PDG 2026 0.22431 ± 0.00085: $+1.11\sigma$. The measured-angle readout 0.22687 is a separate diagnostic ($\approx +1.0\sigma$ with the NuFIT θ_{12} error propagated; $+3.01\sigma$ against the comparator error alone). Route selection used kaon data.
V_{ud}	first-row unitarity after fixing $\|V_{us}\|$ and $\|V_{ub}\|$	ALGEBRAIC	$\|V_{ud}\| = \sqrt{1 - \|V_{us}\|^2 - \|V_{ub}\|^2}$.
$\|V_{ub}\|$	hierarchy relation $\|V_{us}\|\|V_{cb}\|/\sqrt{6}$	LOADED	0.003862 against PDG 2026 0.00389 ± 0.00016: -0.18σ. The 0.003855 variant is an explicitly labelled rounding control; the former 0.52σ stamp is retired.
$\|V_{cb}\|$	$1/(9\sqrt{7})$ via DET-7 '7 unification'	LOADED (physical attachment, A1566; Structural formula)	0.041996 against PDG 2026 0.0407 ± 0.0013: $+1.00\sigma$ under central-value Gaussianization.

Quantity	Algebraic source	Status	Note
δ_{CKM}	$\arctan\sqrt{3(\phi+5\phi^{-5})}$; $G_7 = \phi + 5\phi^{-5}$ from $J_3(\mathbb{O}_s)$ sub-leading eigenvalue + $\mathbb{O}_s(4,4)$ coupling weights	Structural ($\star\star\star$)	68.1297°; PDG 2026 comparators, both named (Rev32): $+1.40\sigma$ vs. the global-fit Dirac phase 1.154 ± 0.025 rad ($66.12° \pm 1.43°$; R-29) and $+0.64\sigma$ vs. the direct UT-angle $\gamma = 66.4^{+2.7}_{-2.8}$; the first-principles algebraic motivation for G_7 remains incomplete (Coincidence-class pending G_7, §5). Bare-ϕ control printed beside it (Rev32.2): 65.59°, -0.37σ global / -0.30σ direct; J_{read} post-insertion.
CKM first-row unitarity	$\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2 + \lvert V_{ub}\rvert^2$	Algebraic	Framework predicts exact; the current PDG global fit shows $\sim 2.3\sigma$ tension from unity (the first-row/Cabibbo anomaly) — monitoring required.
$\theta_{12}^{\mathrm{PMNS}}$	$\tan\theta_{12}^{\mathrm{PMNS}} = \sqrt{3}/\phi^2$	Theorem	PMNS solar angle (proved as theorem, Rev9); normal ordering, conditional on the R_d ladder (Paper 2 §3, Structural) and on its identification with the propagation eigenstates (interface undeclared; Rev32.2, `s1108`/A1546 C6).

Quantity	Algebraic source	Status	Note
θ_{23}	F_4 mismatch geometry J_{vac} vs $J^{\#}_{\text{vac}}$	LOADED (physical; internal identity THEOREM)	$\sin^2\theta_{23} = 7/16 \approx 0.44$ (internal $\sin^2\theta_{\text{mismatch}} = 7/16$ theorem-grade; physical octant LOADED-CORRESPONDENCE (A1566); pre-Rev29 stamp THEOREM $\star$100, D575); AX2 eliminated: $\|J\|^2 = 4$ from DET-7+$N(J) = 1$+(H1); the 7/16 identity needs only DET-7+$N(J) = 1$ (norm product 16); NH tree-level; NuFIT 6.1 NH global best fit lower-octant (0.470), 2.3σ away; UO comparator ≈ 0.561 at $\approx 9.5\sigma$.
θ_{13}	D_4 triality $8_v \leftrightarrow 8_s$: $\theta \to \pi/8$; $(1,3)$ Peirce sector	STRUCTURAL ($\star\star\star$)	$\sin^2\theta_{13} = \sin^4(\pi/8) = (3-2\sqrt{2})/8 \approx 0.0214$ (STRUCTURAL $\star\ \star\ \star$, round-trip framing T^2, exponent-2; forcing pending [s502/A1275]; A608/E113); 1.8σ NuFIT 6.1; no fitted coefficients, but not zero-parameter — see the supersession note in the caption.

Quantity	Algebraic source	Status	Note
δ_{CP}	V_3 rotation: $T_{\text{half}}\|_{V_3}$ 3D rotation, $\Omega = \sqrt{5}/2$, $t^* = 2\pi/\sqrt{5}$	THEOREM (⋆100)	$\delta_{CP} = -2\pi/\sqrt{5} = -160.997°$ ($J_{CP} \approx -0.011$); 0.36σ NuFIT 6.1 (NH); β_δ eliminated; C1 Theorem: $c = \frac{1}{2}$ exact (A558); Uniqueness Theorem: T_{half} unique up to scale among G_2-invariant generators in the $(1,3)$ sector (A603/E110); 2/8 PMNS+CKM parameters at DERIVED-CONDITIONAL or above after the §5 re-tiering, with their premises printed ($\theta_{12}^{\text{PMNS}}$, δ_{CP}; the physical θ_{23} row is LOADED-CORRESPONDENCE since A1566, its internal 7/16 identity theorem-grade); the pre-Rev29 tally of 6/8 at ⋆100 is superseded.
PMNS unitarity	exact by construction	ALGEBRAIC	$U^\dagger U = I$; not an independent falsification target.

Stamps in this table (⋆100, THEOREM, Proved) are the Rev29-era marking, superseded en bloc by the Rev29 re-tier (Paper 3 §"Row-by-row re-tiering"); the current tier of each row is the re-tier table's. Footer added Rev32.5.

5 Re-tiering the Rev29 ledger against the structural layer

The table above was written before this paper carried a structural layer. It now does (§8), and the two must be reconciled row by row rather than left to a reader to reconcile for themselves. This section performs that audit. **No result below is withdrawn and no number changes.** What changes is what several rows are entitled to be *called*.

5.1 The rule, fixed before any row was scored

A claim can be no stronger than the weakest link in its own derivation chain:

$$\text{tier}(\text{row}) \;=\; \min\Big(\text{tier}_{\text{own}},\; \min_{d \,\in\, \text{deps}} \text{tier}(d)\Big),$$

with one override that dominates everything else:

The override. If any *selection* in the chain — a route, a branch, an octant, an ordering — was made by comparing candidates to measurement, the row is capped at LOADED and tagged ROUTE-SELECTED-BY-DATA, however exact the surviving formula is.

This is the entire point of the audit. A formula can be exact, its weight can be derived from first principles, and the claim can still fail to be zero-parameter, because the decision to use *that* formula rather than a sibling was made by looking at the answer. In this programme "zero free parameters" is a claim about the whole derivation chain, not about the last step of it.

The lattice is PROVED > DERIVED-CONDITIONAL > STRUCTURAL > LOADED > COINCIDENCE-CLASS. *Algebraic* is not a tier: a row that is an exact consequence of its inputs adds no independent support and inherits their tier exactly. The propagation is computed by kernel `s959`, which also verifies that no row was re-tiered *above* its own declared tier — propagation only ever weakens.

5.2 Result

Row	Printed (Rev29)	Re-tiered	Binding reason	Fixed by (Rev32.2, `s1107`)
$\theta_{12}^{\mathrm{PMNS}}$	Theorem	DERIVED-COND.	normal ordering, ladder- and interface-conditional	assumed: mass ordering (NO | IO), ladder- and interface-conditional
θ_{23}, $\sin^2\theta_{23} = 7/16$	Theorem ⋆100	LOADED (two axes: internal identity theorem-grade; physical attachment LOADED-CORRESPONDENCE)	exact internal mismatch theorem; physical octant registration unselected	current lower branch selected against data; `s501` is a non-unique candidate map, not a selector (A1566/D1244)
$\delta_{CP} = -2\pi/\sqrt{5}$	Theorem ⋆100	DERIVED-COND.	$(1,3)$ framing, ordering	chosen: framing sector $(1,3)$; assumed: Dirac
θ_{13}	Structural ⋆⋆⋆	STRUCTURAL	*unchanged*; forcing pending	chosen: exponent $n = 4$ in $\sin^n(\pi/8)$, by identity/heuristic
$\lvert V_{cb}\rvert = 1/(9\sqrt{7})$	Algebraic	LOADED (physical attachment, A1566)	Structural formula; inherits the loaded θ_{23} attachment and the integer 9 remains unforced	by data: the integer 9 (small-integer menu; no numeric range was pre-declared — `s1107` v2 states 1..9 as a lower bound)
$\lvert V_{us}\rvert$	Algebraic	LOADED	**route selected by data**	by data: first-row route (A | B), kaon override
$\lvert V_{ub}\rvert$	Algebraic	LOADED	inherits $\lvert V_{us}\rvert$	inherits the route bit
$\lvert V_{ud}\rvert$	Algebraic	LOADED	inherits $\lvert V_{us}\rvert$	inherits the route bit
first-row unitarity	Algebraic	LOADED	inherits the first row	inherits the route bit
δ_{CKM}	Structural ⋆⋆⋆	COINCIDENCE	G_7 motivation absent	by fit: $(c,k) = (5,5)$ in $\phi + c\phi^{-k}$, 64-menu, 28 within 1σ
PMNS unitarity	Algebraic	PROVED	*unchanged*; exact by construction	derived: no choice (singleton hom-set)

Stamps in this table (⋆100, THEOREM, Proved) are the Rev29-era marking, superseded en bloc by the Rev29 re-tier (Paper 3 §"Row-by-row re-tiering"); the current tier of each row is the re-tier table's. Footer added Rev32.5.

10 of 11 rows carry a re-tiered marking different from their printed column (count generated from the table above, Rev32.6; the earlier hand count of six treated the CKM first row as one row and Algebraic as a non-tier). 1 row is unchanged (θ_{13}), and PMNS unitarity — relabelled from the non-tier Algebraic to PROVED — is if anything strengthened, because it was always exactly what it said it was.

The fifth column (Rev32.2; house `s1105/s1107`, lane A1546 C5, PI ruling S294). For each non-theorem row it names the discrete choice(s) that cap the row — fixed by data, assumed, chosen, or derived — with the menu stated in words. Every listed discrete choice affects at least one row. Rows may inherit more than one choice; in particular $|V_{cb}|$ inherits both the atmospheric-octant registration and the independently unforced integer 9, and δ_{CP} inherits both the mass ordering and the framing

sector (`s1107` v2 reverse census, A1566). [LIB2-066, LIB2-285, LIB2-286, LIB2-288] The menus are therefore dependent, and no additive bit total is asserted. None is a continuous fit: the loaded content of the sector is a handful of framing and orientation data (a route, an octant registration, an ordering, a sector label, an exponent, an integer, a (c, k) pair) evaluated through theorem-grade maps. No bit total is printed here: the nominal count over declared menus is dependent, not additive information, and lives in the tracker as an attack list. One structural fact belongs in print: the naive S_3-orbit-average functor $F(\theta) = \sin\theta/\sqrt{6}$ fails on $|V_{cb}|$ by 6.6× (`s1105`: $F(\theta_{23}) = \sin\theta_{23}/\sqrt{6} = 0.270031$ at $\sin^2\theta_{23} = 7/16$, which is 6.635 times the PDG comparator 0.0407 and 6.430 times the framework's own $1/(9\sqrt{7})$), so $1/\sqrt{6}$ is a Peirce-*block*-dependent weight, not a functor; the object whose rule would free $|V_{us}|$, $|V_{ud}|$, $|V_{ub}|$ and $|V_{cb}|$ together is that block-weight rule, and it is not registered. [LIB2-073]

5.3 The two findings that matter

1 · The CKM first row is route-selected. This paper states it in its own words: *"Route B is the canonical first-row prediction because it reduces the tension with direct kaon data to* 0.62σ*."* Route A gave 0.22749, a 3.7σ tension, and was deprecated.

The $1/\sqrt{6}$ weight is separately derived and that derivation stands (`s499`: $1/\sqrt{2}$ trace-form $\times$ $1/\sqrt{3}$ three-generation $= 1/\sqrt{3!}$). But a derived weight sitting inside a route that was chosen by comparing both routes to the measurement is not a zero-parameter prediction. [LIB2-050] The free parameter is not in the formula; it is in the choice of formula.

$|V_{us}|$, and everything the first row builds on it — $|V_{ub}|$, $|V_{ud}|$, first-row unitarity — is therefore LOADED. [LIB2-287] This is the same defect the A654 supersession names in §8, found independently and one layer further down.

2 · δ_{CKM} cannot hold a structural rating. The table's own note reads: *"the first-principles algebraic motivation for G_7 remains incomplete."* An expression in powers of φ — $G_7 = \varphi + 5\varphi^{-5}$ — agreeing with measurement at 1.62σ, whose motivation is admitted to be missing, is precisely what this programme's standing rule against numerological support exists to catch. It drops to COINCIDENCE-CLASS and stays there until G_7 is derived. The number is not withdrawn; the *rating* is.

[LIB2-253, LIB2-316]

5.4 What survives, and it is not nothing

Two physical rows sit at DERIVED-CONDITIONAL, each with its premise printed beside it rather than buried: $\theta_{12}^{\mathrm{PMNS}}$ and δ_{CP}. [LIB2-060]

$\delta_{CP} = -2\pi/\sqrt{5}$ is the strongest physically attached claim in this paper and the audit leaves it there. It has the C1 theorem $c = \frac{1}{2}$ exact (A558) *and* a genuine uniqueness theorem — T_{half} is, up to scale, the unique G_2-invariant generator in the $(1, 3)$ sector (A603/E110); the scale is the one that gives it the V_3 frequency $\Omega = \sqrt{5}/2$ (Rev32.9). [LIB2-052] A uniqueness theorem is exactly what separates a derivation from a coincidence, and no other row in the ledger has one.

The separate internal identity $\sin^2\theta_{\mathrm{mismatch}} = 7/16$ remains a Jordan-geometry theorem: DET-7 $+ N(J) = 1$ fix $\langle J, J^{\#}\rangle = 3$ and the norm product $\|J\|^2\|J^{\#}\|^2 = 16$, which eliminated AX2 (D575) and gives 7/16 as a corollary (the equal norms $\|J_{\mathrm{vac}}\|^2 = 4$ need (H1) inversion symmetry in addition, Appendix E; Rev32.9) (no new free parameter enters at this step; the row itself is not citable as

zero-parameter, §5). [LIB2-356] A1566/D1244 finds that `s501` does not uniquely register that mismatch to the physical lower atmospheric octant: the chirality torsor and the octant torsor admit exactly two equivariant bijections and `s501` chooses one by convention. [LIB2-002, LIB2-356] The current physical θ_{23} row is therefore LOADED-CORRESPONDENCE; the number and the internal theorem do not move. [LIB2-002, LIB2-004, LIB2-048, LIB2-235, LIB2-240, LIB2-241, LIB2-243, LIB2-283, LIB2-284]

One tension that must print beside its tier, not beneath it. $\sin^2\theta_{23} = 7/16 = 0.4375$ sits 2.3σ from the NuFIT 6.1 NH global best fit of 0.470. That is real falsification pressure on the framework's sharp loaded lower-branch target, and a reader meeting a $\star 100$ label should meet the tension in the same breath. [LIB2-002] It is recorded here as a live falsifier, not a footnote. (The upper-octant comparator at ≈ 0.561 lies $\approx 9.5\sigma$ away, so the selected correspondence is at least on the correct side of the octant question.) Exclusion attacks the current physical correspondence; it does not by itself refute the internal mismatch identity unless the readout attachment is first derived (A1566).

5.5 The conclusion sentence does not survive

This paper's conclusion states that *"six of the eight mixing parameters are theorem-level"*. After propagation, two of eight sit at DERIVED-CONDITIONAL or above: $\theta_{12}^{\mathrm{PMNS}}$ and δ_{CP} (Rev32.5 printed three, with θ_{23}; A1566/D1244 retyped the physical θ_{23} row, Rev32.6). The theorem-grade internal mismatch identity 7/16 is recorded separately; its physical atmospheric-octant attachment is LOADED-CORRESPONDENCE. The sentence is corrected accordingly, and the corrected count is printed rather than the claim quietly dropped.

What this audit did and did not do. It did not re-derive anything, refute anything, or move any observable. Every formula, every number and every agreement figure in the Rev29 ledger stands exactly as computed. What the audit changes is the vocabulary: which rows this programme is entitled to call theorems, which are conditional and on what, which are loaded, and which are for now coincidences with the algebra still owed. The agreement figures quoted in §§2–5 — 0.62σ, 1.62σ, 1.8σ, 0.36σ — are agreement of a partly loaded construction with data. They are not, and should never have been read as, independent evidence for the structure.

6 Derivational status under Definition A: a hostile-audit reconciliation (`s270`–`s282`)

An in-house adversarial cycle (kernels `s270`–`s283`; the occupation side is folded in Appendix M) tested whether the Standard-Model generation slots, the gauge groups, and the cross-generation mixing of this paper can be *derived* from the pure bulk algebra with no phenomenological input, and concluded they cannot, recommending that the PMNS/CKM angles be reclassified as empirical boundary inputs and that the geometric V_3 derivation be downgraded to coincidence-class. We have verified that recommendation against the banked framework and *partially* accept it: the two genuinely new honest-negatives are banked, but the proposed downgrade of the geometric angle theorems *does not go through*, for the reasons recorded below. No public observable moves and no theorem rating in this paper changes.

Banked (the valid honest-negatives). (i) *Occupation is not bulk-forced* (`s270`–`s272`): the slot set $\{0, 3, 11, 17\}$ is not fusion-closed (condensation fails), is no gapped-boundary subgroup of the achiral double $D(\mathbb{Z}_{60})$ (none of the 12 Lagrangian algebras match), and is not reachable by any bulk J_{vac} symmetry-breaking potential (no per-rung tuning). (ii) *The weak sector is not native to the diagonal frame* (`s273`): the Eilenberg–Moore category of the Reynolds monad P_{code} isolates the Weyl-singlet diagonal $(+,+)$ frame but annihilates the off-diagonal $(-,-)$ Peirce spaces where the non-commuting $SU(2)_L$ generators live, so $SU(2)_L$ is not extracted from the invariant frame. (iii) *The dynamical cross-parity spurion is fatal* (`s275`–`s282`): using the odd defect $O_{\text{odd}} = \frac{1}{2}\,\text{diag}(1, 0, -1)$ as a propagating background VEV to bridge generations clears the kinematic hurdles but cannot source the $\theta_{13} \ll \theta_{12}$ hierarchy (the golden weights on the two odd rungs differ by only ≈ 1.17), and is killed outright by charge conservation in that mechanism. These sharpen the input ledger (Appendices M, N, X): the occupation slots and the colour real-form remain named inputs.

Verified to stand (why the geometric downgrade is declined). The audit's central move — that an exact parity grading forces a block-diagonal mass matrix and hence $\theta_{12} = \theta_{13} = 0$ — does not bind the derivation of §3, on three independent grounds.

(i) *Layer separation.* The parity-$\mathbb{Z}_2$ grading is a property of the *discrete* $\mathbb{Z}_{60}/\mathbb{Z}_2$ occupation clock and governs which rungs are *stable* (Appendix M, Theorem on the exact universal grading, `s267`). The angles of §3 are fixed by the *continuous* Jordan/Freudenthal geometry (J_{vac}, $J^{\#}_{\text{vac}}$, the D_4 triality R, and the V_3 rotation $\exp(t^{\star}T_{\text{half}})J_{\text{vac}} = J^{\#}_{\text{vac}}$). This paper invokes the parity grading nowhere; importing a discrete occupation-stability rule as a selection rule on the continuous mass geometry is a category error.

(ii) *Already adjudicated in-house.* The framework has separately established and recorded (`s269`, Appendix M) that parity is *falsified as a mixing rule* — about two-thirds of the empirical CKM weight, including $|V_{cs}| \approx 0.973$ and $|V_{tb}| \approx 0.999$, sits in parity-forbidden entries. "Parity forbids cross-generation mixing" is therefore a position the project already rejected.

(iii) *The hierarchy is the wrong target object.* PMNS mixing is the misalignment of the neutrino flavour basis carried by $X_\nu = J^{\#}_{\text{vac}}$ with the charged-lepton basis, not charged-lepton↔charged-lepton mass mixing; parity protection of the charged-lepton mass eigenstates does not constrain that misalignment. The mismatch $\text{diag}(1, 0, -1)$ enters §3 as a *kinematic* angle between two diagonal vacuum frames, not as an EM-charged propagating VEV, so the hypercharge/photon-mass obstruction of `s282` — which applies to the dynamical insertion, and uses a hypercharge operator not defined in this suite — does not reach it. (The solar angle is moreover large and well-measured, $\theta_{12} \approx 33.4°$; a literal $\theta_{12} = 0$ prediction would contradict both data and the framework's own result.)

Definition-A ledger (mixing sector). *Derived/structural (unchanged):* the geometric PMNS/CKM angle relations of §3 (kinematic, continuous-geometry; survive the `s270`–`s282` audit) — with the two-axis split of A1566: internal geometry, theorem-grade $\sin^2\theta_{\text{mismatch}} = 7/16$; physical PMNS attachment, LOADED-CORRESPONDENCE pending a canonical octant registration — and the parity-$\mathbb{Z}_2$ generation grading (occupation-stability bedrock; Appendix M). *Fenced inputs:* the occupation slots $\{0, 3, 11, 17\}$ (Appendix M), the colour real-form / gauge-group assignment (Appendix N), and any *dynamical* cross-parity mass-mixing mechanism (the O_{odd} spurion route is closed, `s282`). The verification adjudication is recorded for recheck in the session notes; no public observable moves.

7 Conclusion

Rev29 presents the mixing sector as a precise set of source maps rather than as a bundle of numerically isolated coincidences. The full 3×3 CKM matrix is best read as a *completion map*: two of the eight mixing parameters sit at DERIVED-CONDITIONAL or above after the §5 re-tiering ($\theta_{12}^{\mathrm{PMNS}}$, δ_{CP}, with their premises printed; the theorem-grade internal mismatch identity 7/16 is recorded separately and its physical θ_{23} attachment is LOADED-CORRESPONDENCE, A1566), θ_{13} is structural, $|V_{cb}|$ is LOADED-CORRESPONDENCE (physical attachment) with a Structural formula, the CKM first row is LOADED because its route was selected against data, δ_{CKM} is COINCIDENCE-CLASS pending G_7, and the standard-form construction completes the matrix from those inputs — it is not an independent eight-parameter prediction. The CKM matrix is structurally unitary once the Peirce-derived angle set is specified. The PMNS matrix is exactly unitary by construction. The open question is not whether these matrices can be written down, but whether upcoming oscillation and flavour data continue to track the particular algebraic map selected by the exceptional- Jordan framework.

References

[1] I. Todorov and M. Drenska, "Octonions, exceptional Jordan algebra and the role of the group F_4 in particle physics," *Adv. Appl. Clifford Algebras* **28** (2018) 82, `arXiv:1805.06739`.

8 The structural home for $V_{\mathrm{CKM}} = I$, and a supersession

This paper has carried $V_{\mathrm{CKM}} = I$ as a posture for several revisions: a statement of what the framework does *not* derive, held in place because the alternative was to assert a mixing structure the derivation did not support. At the Rev29 fold that posture acquires a structural reason, and one older claim is formally superseded.

8.1 The controlled-defect frame

The module-Dirac census of A1471 (house-verified at s954) establishes the following under one named premise.

> **Premise (recovery-fixed module naturality).** The physical finite Dirac operator is a fixed point of the recovery channel in the Heisenberg picture, and is natural for the exact Fibonacci parent module action.

> **Theorem (A1471, forced under the premise).** The recovery channel's Heisenberg fixed points form a 9-dimensional space; exact Fibonacci-module naturality cuts this to *exactly* 3, spanned by $\hat{D}(A, B, C)$; and $[D_F, \mathcal{A}_{\mathrm{gen}}] = 0$ — the finite Dirac operator commutes with the generation algebra. The visible compression commutes with the census.

The immediate consequence is the one that matters for this paper. *Strict module naturality forbids generation mixing altogether.* The mixing space is $3_{\mathbb{C}}/3_{\mathbb{R}}$ with Hilbert–Schmidt gap weights $\varphi^{-2}, \varphi^{-4}, 1$, and every nonzero mixing is a departure from the naturality that made the three-family structure forced in the first place.

Mixing is therefore not an output of this framework and not an omission from it. It is a **controlled defect**: a measured deviation from an exact structural constraint, whose cheapest instance is the $(1 \leftrightarrow \varphi^{-1})$ link with penalty $\frac{1}{2}\varphi^{-4}\varepsilon^2\|K\|^2$ (Appendix X).

What this is not. This is not a derivation of the CKM matrix, and nothing here predicts a mixing angle. The framework's printed value remains $V_{\mathrm{CKM}} = I$, and the defect statement is *naturality-conditional* — it inherits every condition of the premise above. The correct reading is: the framework now says why it gives I, and what kind of object the observed departure from I would have to be. It does not say how large that departure is. The φ powers in the gap weights come from the module structure the theorem is stated in; they are not numerological support for anything, and the programme's standing rule against such support applies here in full.

8.2 Supersession: the A654 zero-parameter CKM claim

At S46 an assessment (A654) recorded a COMPLETE, ZERO-FREE-PARAMETER CKM determination via the $J_{12} \circ J_{13} \to J_{23}$ route, quoting $|V_{td}|$ and $|V_{ts}|$ at 0.25σ and 0.05σ. That claim is **superseded**.

It is superseded rather than retired because the underlying computation was real and is still on file. What was wrong was its *type*. A zero-free-parameter claim in this programme means a quantity fixed by structure with no measured input anywhere in its derivation chain. The route A654 used inherits the loaded Route-B correspondence, whose six formulae are readout-conditioned; θ_{13} and δ_{CKM} sit behind that conditioning, and CKM-β was never resolved. A result that agrees with experiment to a quarter of a standard deviation while carrying an unaudited loaded correspondence is not a zero-parameter derivation — it is a good fit whose parameters are somewhere else.

The arrow, printed once. A654 (S46, zero-free-parameter CKM, $J_{12} \circ J_{13} \to J_{23}$) $\longrightarrow$ the Rev29 posture: $V_{\mathrm{CKM}} = I$ with mixing typed as a controlled defect under recovery-fixed module naturality. The superseded claim may not be cited as live, and no revision of this suite prints a zero-parameter CKM number. This is a statement about what "zero free parameters" is allowed to mean here: it is a claim about the whole derivation chain, not about the last step of one.

Leibniz Quantum Beats Newton

Paper 4 (Rev33.2): Higgs Sector and Electroweak Symmetry Breaking

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August 2026

Abstract

The Higgs sector of the current $J_3(\mathbb{O}_s)$ programme is no longer presented as an isolated Yukawa note. In the current reading it sits inside a larger algebraic chain whose first step is already physically restrictive: the empirical equivalence of the $(1,3)$ and $(3,1)$ Lorentz-signature conventions motivates a neutral $(4,4)$ inner product — an *adopted premise* (a stated model choice, not a forcing argument; Paper 1, Rev29), and the unique real normed composition algebra with that norm is the split-octonion algebra $\mathbb{O}_s$. Imposing a three-sector (rank-three) structure then selects the 3×3 Hermitian Jordan algebra $J_3(\mathbb{O}_s)$ over $\mathbb{O}_s$ — the generation *count* 3 is an external input (algebraic replication into 3×16 is refuted; A691/A693/A694), and the rank-three Jordan structure realizes, rather than derives, it. The Higgs sector is therefore not built on a merely convenient algebraic choice; it is attached to the same adopted signature-neutral premise (plus two genuine uniqueness steps) that underwrites the rest of the suite.

At the local level the same-block *carrier* coupling (Peirce unit-eigenvalue theorem, A674; the physical SM Higgs $H_{u,d}$ sits in the vector **10**, and the literal Yukawa is the cross-block triple of §Paper 6 §4.4) is the cubic invariant

$$\mathcal{L}_{\text{carrier}} \propto \mathrm{Tr}\big[J_{\text{vac}} \circ (\Psi \times \Phi_H)\big]. \tag{1}$$

With $J_{\text{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1})$ this projects onto the middle Peirce idempotent and fixes the tree-level boundary value $y_t(M_{\text{Pl}}) = 1$. The same Rev29 upgrade also integrates the corrected Cayley–Dickson split-octonion multiplication and SplitCD inner product, the theorem-level vacuum selector $J_{\text{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1})$, the D=5 to D=4 architecture $E_{6(6)}/F \to E_{7(7)}/SU(8)$, and the CKM+PMNS mixing ledger at its Rev29 re-tiered statuses (2/8 Derived-conditional or above since the A1566 retype, Rev32.6; CKM first row Loaded; physical θ_{23} and $|V_{cb}|$ Loaded-correspondence; δ_{CKM} Coincidence-class).

What remains open is stated plainly. The D=5 scalar manifold $E_{6(6)}/F_{4(4)}$ is pseudo-Riemannian, while the D=4 $E_{7(7)}/SU(8)$ target is Riemannian and ghost-free. The F_4-invariant tree-level potential has an exactly flat eigenvalue-ratio direction, so the golden-ratio hierarchy is not selected at tree level and must come from a beyond-tree mechanism such as Coleman–Weinberg lifting. The diagonal-vacuum weak-angle route is dead; the surviving electroweak route is the non-diagonal exact-color orbit with algebraic target $\sin^2\theta_W = \phi^{-3}$ and one remaining selector problem. Rev29 is therefore the honest EWSB statement of the suite: the Higgs geometry and the cubic Yukawa are sharp, the mixing sector carries no fitted coefficients (θ_{13} and δ_{CKM} readout-conditioned, the CKM β consistency open; Paper 3), and the weak-angle selector plus the full bosonic extraction remain the principal open tasks.

1 Introduction

Paper 4 has a narrower purpose than Papers 0, 3, and 5. It does not attempt to re-derive the entire architecture of the programme. Its job is to state the Higgs/EWSB layer in language a referee can audit quickly: what the algebra fixes exactly, what the D=5 and D=4 scalar targets are, where the ghost question is solved, what happened to the weak-angle story, and which pieces are still open.

Two changes since the old Rev7 paper are decisive. First, the upstream algebra has been corrected and sharpened: all split-octonion products are now taken in Cayley–Dickson form, the SplitCD inner product is the one compatible with the non-compact real form, and $T_{\text{half}} \in \mathfrak{f}_{4(4)}$ is kernel-verified. Second, the flavour sector is no longer an uncertain backdrop. Two of the eight CKM and PMNS mixing parameters survive the Rev29 row-by-row re-tiering (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`; A1566 retype, Rev32.6) at DERIVED-CONDITIONAL or above ($\theta_{12}^{\text{PMNS}}$, δ_{CP}), with no fitted coefficients *inside* the correspondence map; the CKM first row is LOADED because its route was selected against kaon data, δ_{CKM} is COINCIDENCE-CLASS, θ_{13} is structural, and the physical θ_{23} row and $|V_{cb}|$ are LOADED-CORRESPONDENCE (the internal 7/16 mismatch identity stays theorem-grade). The Higgs paper therefore sits on a firmer algebraic floor than any earlier revision.

2 Algebra selection and the vacuum theorem

The algebraic core is selected by a short chain of physical requirements.

(1) The metric conventions $(1,3)$ and $(3,1)$ describe the same Lorentzian physics.

(2) We therefore *adopt*, as a stated model choice, a convention-neutral algebraic framework accommodating both simultaneously — a neutral $(4,4)$ norm form. This is an adopted premise, not a forcing argument: nothing proved here excludes a convention-fixing framework (Paper 1, corrected form).

(3) Given that adoption, the composition-algebra demand then selects $\mathbb{O}_s$: it is the unique real normed composition algebra with signature $(4,4)$.

(4) Imposing a rank-three (three-sector) structure then selects the 3×3 Hermitian Jordan algebra $J_3(\mathbb{O}_s)$; the generation count 3 is an external input (replication refuted, A691/A693/A694), realized but not derived by the rank-three Jordan structure.

No Higgs-sector statement in the present paper is prior to this selection principle.

The working vacuum is now carried at theorem level in the programme state rather than treated as a freely fitted ansatz.

$$J_{\text{vac}} = \text{diag}(\phi, 1, \phi^{-1}), \qquad \phi = \frac{1+\sqrt{5}}{2}. \tag{2}$$

It is carried as the unique fixed point of the modular element $M_1 = T^3 S \in PSL(2,\mathbb{Z}) \subset F_{4(4)}$ acting on the Peirce diagonal, with the companion Freudenthal fixed-point relation

$$\{J_{\text{vac}}, J_{\text{vac}}^{\#}, J_{\text{vac}}\} = J_{\text{vac}}. \tag{3}$$

The diagonal vacuum $\text{diag}(\phi, 1, \phi^{-1})$ is proved at theorem level in session-level derivations [A641/Q5A: uniqueness for $n_{23} = 7$]; the proof is not reproduced in the present suite and is the subject of a forthcoming appendix. In the Higgs context this matters because the vacuum is no longer an unconstrained ansatz whose role is merely to make the Yukawa extraction work.

3 Scalar manifold, Higgs placement, and local cubic coupling

The correct D=5 scalar manifold is

$$E_{6(6)}/F_{4(4)}, \tag{4}$$

not $F_{4(4)}/K$. The dimension count is $78 - 52 = 26$, matching the split-magic-supergravity scalar sector built from the $N(h) = 1$ constraint on the 27 of $J_3(\mathbb{O}_s)$. $F_{4(4)}$ is the isotropy group, not the kinetic numerator.

Two readings must be kept distinct. In the *carrier* reading (the same-block construction of A674), a Higgs-like field Φ_H and a fermion doublet Ψ are placed in the same $(1,3)$ Peirce block; this is the unit-eigenvalue *carrier* theorem, not the literal SM Higgs placement. The *physical* SM Higgs doublets $H_{u,d}$ instead live in the vector $\mathbf{10} = J_{12} \oplus \mathbb{R}c_1 \oplus \mathbb{R}c_2$, and the literal one-generation Yukawas are the cross-block trilinear $\{J_{12}(H), J_{13}(\Psi_L), J_{23}(\Psi_R^c)\}$ (A686–A689; Paper 6 §4.4). In the carrier reading the full $(1,3)$ block is eight-real-dimensional, so the carrier doublet is not identified with the whole block. Under

$$\mathrm{Spin}(1,3) \times \mathrm{Spin}(3,1) \subset \mathrm{Spin}(4,4), \tag{5}$$

the 8_+ spinor decomposes into the same-chirality pairing

$$8_+ \longrightarrow \sigma_1 = (\tfrac{1}{2},0)_1 \otimes (\tfrac{1}{2},0)_2 \oplus \sigma_2 = (0,\tfrac{1}{2})_1 \otimes (0,\tfrac{1}{2})_2. \tag{6}$$

The carrier doublet is identified with one four-real-dimensional block; the other four real dimensions are the orthogonal completion in the split-signature pair, not a second doublet. The dimensional mismatch is therefore resolved by the $(1,3) + (3,1)$ signature split, not ignored. The same-block *carrier* coupling (the Peirce unit-eigenvalue theorem of A674, *not* the literal SM Yukawa) is

$$\mathcal{L}_{\text{carrier}} \propto \mathrm{Tr}[J_{\text{vac}} \circ (\Psi \times \Phi_H)], \tag{7}$$

where the Freudenthal cross product is defined using the corrected Cayley–Dickson multiplication and the SplitCD inner product. (The literal $16 \times 10 \times 16$ SM Yukawa is the cross-block triple above, with $H_{u,d} \in \mathbf{10}$.) For traceless $(1,3)$ fields one has

$$\Psi \times \Phi_H = -2\,\mathrm{Re}(\psi\bar{h})\, e_2, \tag{8}$$

so contraction with J_{vac} extracts the middle eigenvalue $\lambda_2 = 1$ and yields the exact tree-level result

$$y_t(M_{\text{Pl}}) = 1. \tag{9}$$

This remains a tree-level eigenvalue statement. Standard one-loop running carries it to the low-energy effective theory,

$$\frac{dy_t}{d\ln\mu} = \frac{1}{16\pi^2}\,\beta_t(y_t, g_1, g_2, g_3, \ldots). \tag{10}$$

The measured-side coupling at M_Z, $y_t(M_Z) = 0.970 \pm 0.002$ (SL-57, `s1337`), is a data comparator, not this boundary value's endpoint (the 0.967 printed here before Rev33.2).

The same Peirce geometry also fixes the colour-safe subgroup placement used elsewhere in the suite:

$$SU(3)_c \subset Sp(3) \subset K = [Sp(1) \times Sp(3)]/\{\pm 1\}. \tag{11}$$

This becomes relevant again in the weak-angle discussion, where the exact-color slice is the only surviving bosonic route. The anomaly-cancellation check for the identified Standard Model matter content is not repeated here; it is closed in Paper 1 (the s588 audit, formally closing hostile-board item T3-P) and in Appendix F under the explicit $B + \mathrm{SU}(5)$ map.

4 Ghost-freedom and the D=5 to D=4 architecture

The ghost question must be stated on two levels.

4.1 D=5 intermediate level

At the D=5 stage the scalar target is $E_{6(6)}/F_{4(4)}$ with tangent-signature

$$(14, 12) \tag{12}$$

on the traceless 26-dimensional sector. Equivalently, on the full 27-dimensional Jordan space the quadratic form has signature $(15, 12)$. This is the honest intermediate statement: the D=5 level is pseudo-Riemannian and contains ghost-like directions if one mistakes the intermediate tangent metric for the final physical scalar metric.

4.2 D=4 final level

After reduction on S^1, the split-magic scalar count follows the chain

$$26 \text{ (split-magic D=5 scalars)} + 27 \text{ (dualized vectors)} + 1 \text{ (KK dilaton)} = 54. \tag{13}$$

The group-theoretically exact D=4 target of *maximal* $N = 8$ supergravity is

$$E_{7(7)}/SU(8), \quad \dim = 133 - 63 = 70, \tag{14}$$

where the standard formula uses 42 D=5 scalars from the maximal D=5 manifold $E_{6(6)}/USp(8)$ ($\dim = 78 - 36 = 42$), not the split-magic manifold $E_{6(6)}/F_{4(4)}$ ($\dim = 78 - 52 = 26$). The discrepancy is

$$70 - 54 = 16 = (42 - 26) \text{ scalars outside the Jordan-algebra sector.} \tag{15}$$

These 16 additional degrees of freedom are present in maximal supergravity but are not directly produced by the $J_3(\mathbb{O}_s)$ reduction. Their identification is an open architecture task (T1-N). $SU(8)$ is the maximal compact subgroup of $E_{7(7)}$; all 70 coset generators are non-compact and the coset metric is positive-definite, so the final D=4 maximal target is Riemannian and ghost-free.

(Superseded, S73/S77 note: the identification is no longer open. On the certified $E_{7(7)}$ substrate the missing 16 is pinned — it is the identity-anchored Jordan complement $(b) = 42 \ominus 26 = \ker(p_{42} \xrightarrow{s \mapsto s \cdot I} 27)$, orthogonal to the E_6/F_4 orbit-tangent 26, and selected over the $SO(5,5)$-spinor look-alike by the kernel-identity lock (the two candidate 16-planes lie in different $USp(8)$ orbits, stabilizers 12 vs 20). The S77 potential layer then computes the dynamics of exactly this sector; see Appendix F. The T1-N identification task is retired to the historical record.)

Status of the two-16 identification (S60, A714). A natural conjecture identified this 16-scalar gap with the 16 broken generators of the vacuum-stabilizer breaking $\mathrm{Spin}(4,4) \to \mathrm{Spin}(1,3) \times \mathrm{Spin}(3,1)$ ($28 - 12 = 16$), via a putative branching $42 \downarrow F_{4(4)} = 26 \oplus 16$. This identification is now *excluded* at the group-theory level (verified independently in house by two methods). First, F_4 has no 16-dimensional irreducible representation—its dimension spectrum begins $1, 26, 52, 273, \ldots$—and the 42 is the $USp(8)$-noncompact tangent of $E_{6(6)}/USp(8)$, not an $F_{4(4)}$-module, so the proposed branching is ill-posed. Under the commuting involutions defined by $USp(8)$ and $F_{4(4)}$ (common subgroup

$USp(6) \times USp(2)$), the 42 instead grades as $42 = 28 \oplus 14$ and the 26 as $26 = 12 \oplus 14$; no canonical 16-dimensional normal complement arises. Second, the Goldstone 16 is the well-defined vector $\otimes$ vector representation $(\mathbf{2}, \mathbf{2}; \mathbf{2}, \mathbf{2})$ of $\mathrm{Spin}(1,3) \times \mathrm{Spin}(3,1)$, whereas the natural spinorial candidate $8_s \oplus 8_c$ decomposes into spinor $\otimes$ spinor blocks $(\mathbf{2}, \mathbf{1}; \mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{2}) \oplus (\mathbf{2}, \mathbf{1}; \mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{2}; \mathbf{2}, \mathbf{1})$; the two carry different quantum numbers and are not equivariantly isomorphic. The equality $42 - 26 = 28 - 12 = 16$ is therefore a dimensional coincidence, not an identification. Consequently the heavy-quark threshold corrections of the rest-mass sector cannot be attributed to "eaten Goldstone scalars" of this gap; that mechanism must be derived, if at all, from an explicit interaction sector rather than from the scalar-count equality.

Status of the heavy-quark threshold carrier (S61–S62, A715–A717). The explicit-interaction-sector search was subsequently carried to completion and closed *negative* at the derivation level. The genuine (A713) residual pattern selects a weak-isospin coupling $\delta m_f/m_f = -\kappa T_3(f)$, $\kappa \simeq 1.824\%$, uniquely among single standard-model charges (A715, two independent assessments). The gauge-invariant carrier named here is the weak-adjoint Higgs spurion $(H^\dagger \tau^a H)\, \mathcal{O}^a_{\text{Yukawa}}$ built on the verified cross-block Yukawa trilinear $\{J_{12}(H_{u,d}), J_{13}(Q), J_{23}(u^c/d^c)\}$; it is writable and charge-correct, but the sign-forcing test (A716, computed in the exact A688/A689 component map that reproduces the standard-model Yukawas) shows that $J_3(\mathbb{O}_s)$ does *not* force its relative up/down sign: the conjugation $H_d = \bar{H}_u$ yields the *same* sector sign, the vacuum grading assigns $\lambda_2 = +1$ to both channels, and the structure constants carry only component-level orientation signs. Within A716's test set the τ^3 weighting is therefore not supplied, and the operator is STRUCTURAL, not derived. (Rev33.2: the weighting is carried by the registered slot-norm involution $P = 2Q - (B - L)$, and a leg-local P deformation forces the relative sign $\delta_t = -\delta_b$, `s1218`; the leg is a choice of operator — a singlet-leg dimension-four reweighting agrees with the spurion on every neutral entry — and what stays by hand is the deformation's existence, one sign fixed by no registered symmetry, and $|\kappa|$, `s1349`; Paper 2 §3.6.) A boundary-condition revision is likewise excluded as a principled alternative (A717): it reduces no parameter. The honest Rev29 statement is that the heavy-quark sector carries bare algebraic boundary values plus a phenomenological structural T_3 threshold term with κ and the charm factor χ_c not derived; its value depends on the κ normalisation (see Paper 2 §3.6 for the full audit).

Ghost-free reading (Rev12 honest form, updated S77). The D=5 intermediate manifold $E_{6(6)}/F_{4(4)}$ is pseudo-Riemannian with signature $(14, 12)$ on the traceless sector. The split-magic KK reduction yields $26+27+1 = 54$ D=4 scalars. The full maximal-SUGRA target $E_{7(7)}/SU(8)$ with 70 scalars requires 16 additional fields beyond the Jordan-algebra sector; these are now *identified* (S73): the missing 16 is the identity-anchored Jordan complement $42 \ominus 26$ inside the maximal-SUGRA coset, pinned on the certified $E_{7(7)}$ substrate. [LIB2-007] The D=4 scalar content is ghost-free, and the full 70-scalar maximal completion is reached *operationally*: the scalar potential of the gauged theory is computed on the full 70 at the published normalization and certified against published vacua (Appendix F). The consistent-truncation question (T4-C) is now *settled, negatively and sharply* (S78b, Appendix F): the Jordan-54 is *not* a Lie-triple system in $E_{7(7)}/SU(8)$ (order-one leakage, computed), so no consistent scalar truncation retains exactly the Jordan-visible 54 — while the missing-16 carries the corresponding *local* statement, re-typed at *Rev29*: the missing-16 tangent plane is a **Lie triple system** in $\mathfrak{e}_{7(7)} \ominus \mathfrak{su}(8)$ at the basepoint, so by Cartan's theorem it exponentiates to a totally geodesic submanifold of $E_{7(7)}/SU(8)$ *in a neighbourhood of that basepoint*, locally isometric to the symmetric space $SO(4,4)/(SO(4) \times SO(4))$, the split-octonion norm-form Grassmannian. [LIB2-009] **The local computation stands as printed and no number in it is withdrawn**; what is withdrawn is the scope. The global step — geodesic completeness of the exponentiated leaf, closedness (as against a dense immersed image) of its embedding in $E_{7(7)}/SU(8)$, and the resulting determination of the *global* isometry type, i.e. which real form and which quotient by which discrete subgroup the leaf actually is — **is not supplied here or anywhere in the suite**. The equality sign in "$= SO(4,4)/(SO(4) \times SO(4))$" is therefore to be read as a local isometry, not as a global identification of manifolds. [LIB2-010] The 54-scalar model is definitively a counting/structural correspondence; the full 70 is the arena. (The missing-16 is thus *identified*; whether it couples dynamically to the golden vacuum — the linear/quadratic portal tensors B_a, H_{ab} at J_{vac} — is a separate, open computation, pre-registered as the G3–G7 drill in Appendix I.)

The same architecture should also be read with one standard supergravity caveat: the full $E_{7(7)}$ symmetry is on-shell, i.e. a symmetry of equations of motion plus Bianchi identities, not of the off-shell local action in a naive sense.

The $F_{4(4)}$-invariant potential on the 26-dimensional scalar manifold $E_{6(6)}/F_{4(4)}$ contains the SM Higgs doublet within a larger scalar spectrum. The additional scalars in the split-magic reduction $(26 + 27 + 1 = 54)$ and the 16-scalar gap to the maximal $E_{7(7)}/SU(8)$ target are not hidden Higgs degrees of freedom already matched to the SM. Nor, as it turns out, are they projected out: the S77 potential-layer computation finds *no* gauging in the scanned electric family that consistently truncates to the Jordan 54 (no rank-zero leakage row), and at the dynamically selected $SO(6,2)$ point the 16 extra scalars are instead *stabilized* — strictly positive missing-16 Hessian at the electric origin, non-negative (quartic-lifted) at the dyonic Minkowski vacuum (Appendix F).

5 The Marked Carrier Theorem (selection level)

Once the compact-positive Standard-Model boundary $\mathfrak{B}_{\text{phys}}$ is *loaded*, a sharper statement holds: the boundary carries *no continuous physical selection freedom.* In the carrier $\mathbf{27} = \mathbf{16} \oplus \mathbf{10}_H \oplus \mathbf{1}$ (with the Higgs the $SO(10)$ vector $\mathbf{10}_H \to \mathbf{5}_H \oplus \bar{\mathbf{5}}_H$, $\mathbf{5} \to (\mathbf{3},\mathbf{1})_{-1/3} \oplus (\mathbf{1},\mathbf{2})_{+1/2}$):

Novelty label: physical interpretation not established here.

Theorem 5.1 (Marked Carrier Theorem, selection level; S232)**.** *The loaded compact-positive boundary selects each of the three marked loadings with zero continuous physical moduli:*

- ***EW–σ:*** *the requirement colour-singlet ∧ weak-doublet ∧ $Y = +\frac{1}{2}$ picks the rank-2 Higgs doublet uniquely; the raw vev-line $\mathbb{CP}^1$ is exactly one $SU(2)_L \times U(1)_Y$ gauge orbit, so the physical direction moduli are 0.*
- ***Heavy quark:*** *$SU(3)_c$ is irreducible on the colour* **3** *(real commutant $\mathbb{C} \cdot I$), so by Schur the invariant line metric is unique up to scale and $Z_{\text{can}} = I$ — no form-factor family; P_b has rank 3, P_τ rank 1.*
- ***Colour:*** $\mathbf{8} = \mathbf{3} \oplus 5 \cdot \mathbf{1}$*; embeddings of the unitary* **3** *differ by the compact $U(3)$ frame = gauge, so 0 physical moduli (the gauging obstruction stands — colour is loaded, not gauged).*

All three residuals are discrete markings and gauge orbits.

The theorem is *selection level only*: it does not derive the boundary $\mathfrak{B}_{\text{phys}}$ itself (frame-free selection is closed negative by the frame-free $E_{7(7)}$ no-go of Appendix F), does not gauge colour, and touches no running, α, CKM, or scale. It is contingent on the discrete single-Higgs $\beta = 1$ ($\mathbb{Z}_2$) marking: retaining both refinement lifts would leave an unaligned two-doublet model with a continuous $\tan\beta$ residual, so the result leans on $\beta = 1$ being a genuine loaded $\mathbb{Z}_2$ selection, not a hidden tune. No public observable moves.

6 Flat Direction Theorem and Coleman–Weinberg outlook

The tree-level potential is tightly constrained by $F_{4(4)}$ invariance. On $J_3(\mathbb{O}_s)$ the invariant ring is generated by

$$T = \text{Tr}(X), \qquad Q = \frac{1}{2}\text{Tr}(X^2), \qquad \text{Det}(X) = N(X), \tag{16}$$

so the most general renormalizable F_4-invariant potential on the traceless subspace is built from Q and Det alone. The Session 24 Lagrangian analysis establishes the key result.

Novelty label: new specialization proved here.

Theorem 6.1 (Flat Direction Theorem)**.** *On the traceless eigenvalue space, the F_4-invariant tree-level potential has a rank-one Hessian in the (d_1, d_2) block. Equivalently, the eigenvalue-ratio direction is exactly flat: the ratio d_1/d_2 cannot be fixed by any polynomial F_4-invariant tree-level potential.*

At a representative point $(a, b, 0)$ with $a + b + 0 = 0$ suppressed only after projection, the relevant Hessian entries are

$$H_{11} = 8\lambda_4 a^2, \qquad H_{12} = 8\lambda_4 ab, \qquad H_{22} = 8\lambda_4 b^2, \tag{17}$$

with the upper 2×2 block proportional to the outer product $[a, b]^T[a, b]$. Its eigenvalues are therefore

$$\{8\lambda_4(a^2 + b^2),\, 0\}, \tag{18}$$

so the tangent direction to the constant-Q_0 curve is exactly unlifted.

The physical consequence is clean. The golden-ratio hierarchy is not a tree-level output of the F_4 potential. If the programme is right, $d_1/d_2 = \phi$ must come from a beyond-tree mechanism, with Coleman–Weinberg lifting from the 24 off-diagonal modes the most natural next step. The flat

direction is therefore not an embarrassment to be hidden; it is the theorem telling us precisely what kind of mechanism the vacuum selector needs.

A second tree-level result is just as important for flavour: the same stationarity conditions force the off-diagonal Peirce masses to vanish at quadratic order,

$$H_u = H_d = 0 \qquad \text{at tree level,} \tag{19}$$

so any CKM or weak-angle route needing nontrivial Peirce mass splitting must live beyond tree level.

5.1 Coleman–Weinberg lifting: current status

The one-loop Coleman–Weinberg analysis of the 24 off-diagonal Peirce modes now has a substantial chain of results [D332–D340]. The key theorems are as follows.

Bosonic sector: a graded index, not a determinant cancellation (*Rev29*). $V_{\rm CW}^{\rm bos} \equiv 0$ holds, and the number is not withdrawn — but it holds as an *algebraic* Γ*-graded (Witten-type) target-space index*, not as a physical one-loop cancellation. Each Peirce block carries four positive and four negative Hessian eigenvalues from the $(4,4)$ split signature of $\mathrm{Spin}(4,4)$, the bosonic modes sit in $8_v = (4,1) \oplus (1,4)$ under $SO(4) \times SO(4)$, and with the $\mathbb{Z}_2$ parity grading operator $\Gamma = \mathrm{diag}(+1,+1,+1,+1,-1,-1,-1,-1)$ the graded trace $\mathrm{Tr}_{8_v}(\Gamma\, \mathcal{M}^4 \log \mathcal{M}^2)$ vanishes mode-by-mode, $\sum(+4-4)\, m^4 \log m^2 = 0$ [D332]. *That is a statement about the constructed graded object and about nothing else.* The physical one-loop bosonic effective potential sums *all* bosonic modes with a single statistics sign; a ghost-free bosonic sector supplies no relative minus sign of its own. Promoting the index to a determinant cancellation therefore requires one attachment, named here explicitly and **not supplied anywhere in this suite**: **an independent path-integral measure/BRST derivation showing that the functional measure weights the 8_v modes by the $SO(4,4)$ grading operator Γ.** Absent that derivation the claim is INDEX-LEVEL only. Accordingly the earlier reading printed here — that "the determinant contributions cancel exactly and the bosonic one-loop correction vanishes to all orders," promoting D=4 ghost-freedom to GREEN — is withdrawn to the index tier; the dispatch numbers [D332, D333] and the value 0 stand unchanged, only their type is corrected. This restores agreement with Appendix A §A.7.4 ("Algebraic index and topological triviality of the bosonic CW sector," Rev26 tier caveat), which already carried the correct tier; the disagreement was an intra-suite contradiction, not a stale leftover.

Fermionic lifting and natural scale. $V_{\rm CW}^{\rm ferm} \neq 0$: fermion loops break the $\pm\eta$ degeneracy and create a local minimum. The renormalization-group stationarity (PMS) condition

$$\left.\frac{\partial \kappa^*}{\partial \log \mu}\right|_{r^*=\phi^2} = 0 \tag{20}$$

is a pure function of the vacuum ratio $r^* = \phi^2$ alone; the overall y^4 prefactor cancels from the critical-point equation. Hence the unique RG-stationary scale at the golden-ratio vacuum is

$$\kappa_{\rm PMS} = \log \frac{h^\vee(F_4)}{Q_0} = \log \frac{9}{4}, \tag{21}$$

determined by the Jordan vacuum geometry and the dual Coxeter number $h^\vee(F_4) = 9$ together with $Q_0 = 4$ (the exact Fibonacci identity $\phi^2 + 1 + \phi^{-2} = 4$), independently of the Yukawa strength y^2. [Proved, D340.]

Natural Yukawa. The Yukawa coupling $y^2 = 2/h^\vee(F_4) = 2/9$ is derived from AX1 via the canonical 't Hooft coupling $g_{F_4}^2 = 1/h^\vee$, the Dynkin index $\ell(8_v, \mathrm{Spin}(4,4)) = 1$, and the two-fold Peirce eigenvalue degeneracy. No additional axiom is required. [Proved, D337.]

Loop coefficient structure. The perturbative corrections carry fully rational coefficients:

$$\delta\kappa = \frac{57}{800}\, y^4 + \frac{3249}{1{,}280{,}000}\, y^6 + \frac{1}{1260}\, y^8 + \cdots \tag{22}$$

evaluated at $y^2 = 2/9$, these give a three-loop residual of 1.6×10^{-6} relative to $\kappa_{\mathrm{PMS}} = \log(9/4)$, confirming convergence. The full theorem statement and Peirce-sector derivation appear in Appendix A, §A.7.3.

Current status. The AX2 CW gate is at PLAUSIBLE ⋆⋆⋆⋆⋆. Steps 1, 3 and 4 of the proof (flat direction, PMS scale, natural Yukawa) are PROVED. Step 2 is re-typed at *Rev29*: the bosonic vanishing is PROVED AS A GRADED INDEX and UNATTACHED AS A DETERMINANT CANCELLATION, pending the BRST/measure derivation of the Γ weight named above. Step 5 (exact perturbative convergence to $\log(9/4)$) carries a quantified 1.6×10^{-6} residual at three-loop and is PLAUSIBLE; four-loop confirmation or Borel resummation of the rational-coefficient series is the remaining task.

Path A retraction (A342). Assessment A342 retracted the Coleman–Weinberg/PMS convergence argument as the primary derivation of $\kappa = \log(9/4)$ (Path A). The surviving and authoritative source of the $\kappa_{\mathrm{PMS}} = \log(9/4)$ value is Path B: the unique fixed point of the modular element $M_1 = T^3 S \in PSL(2,\mathbb{Z}) \subset F_{4(4)}$ acting on the Peirce diagonal (see §3 above). The CW derivation above is retained as corroborating evidence for the scale, not as the primary proof chain.

7 Weak angle: algebraic correspondence and precision status

Two earlier weak-angle routes are closed. A pure diagonal vacuum in the F_4 derivation algebra gives $\sin^2\theta_W = 1/2$ identically, permanently closing that route. The older 3/13 tree-level F_4 trace-ratio estimate is retired separately: it arises from a distinct algebraic path (trace-ratio counting of the **26** representation), not from the diagonal-vacuum route, and is no longer the operative Rev29 weak-angle statement.

The surviving electroweak statement is the non-diagonal exact-colour orbit on the 56-dimensional $E_{7(7)}$ side. The algebraic boundary value is the constant

$$\sin^2\theta_W = \phi^{-3} = \sqrt{5} - 2 \approx 0.23607. \tag{23}$$

In the corrected reading (A697, kernel-verified) $\phi^{-3} = 0.23607$ is a ∼2% structural value lying on the standard running curve of the *effective* weak mixing angle. That angle rises below M_Z from $\sin^2\hat{\theta}_W(\overline{\mathrm{MS}}, M_Z) = 0.23122 \pm 0.00006$ (PDG 2026 EW review; the on-shell value is 0.22348 ± 0.00010 and the effective leptonic value 0.23161 ± 0.00004 — three distinct scheme objects). [LIB2-225] The ∼2% structural fence below is binding: this row is not converted to a σ distance. The effective angle rises toward ≈ 0.2386 at $q \to 0$, sub-GeV/GeV-order scale. A proper multi-threshold treatment (S124; Appendix I §B4) makes this a scheme- and threshold-dominated crossing pushed sub-GeV ($\mu_{\mathrm{cross}} \sim 0.15\,\mathrm{GeV}$, band $[0.09, 0.55]$; the once-quoted 1.8 GeV lies *outside* it), so no fixed matching scale is pinned. Running *upward* from this low-scale crossing to M_Z still decreases the value to 0.23122 — the correct direction. The earlier very-large-significance "failure" framing was an artifact of two retired errors: comparing ϕ^{-3} directly to the ultra-precise M_Z anchor (the difference is ordinary running), and assigning ϕ^{-3} to a high/VEV scale and running downward. At 246 GeV

the running value is ≈ 0.231, confirming that ϕ^{-3} is a low-energy, not VEV-scale, quantity. The honest status is therefore a +2.10% structural offset from the PDG 2026 $\overline{\text{MS}}(M_Z)$ comparator, not a precision prediction at that scale. that the matching scale $\mu_\star$ is not yet derived from $J_3(\mathbb{O}_s)$ and is scheme-dependent; until a structural reason fixes it, ϕ^{-3} is a structural *target* for the low-energy weak angle rather than an absolute prediction. It is consistent with the atomic-parity-violation value (0.4σ) but in mild (2–3σ) tension with the more precise Qweak/E158 determinations and is not the $q \to 0$ Thomson limit.

Scheme identification (A698) sharpens this. Being a scale-free Peirce eigenvalue ratio, ϕ^{-3} is most naturally the tree-level Lagrangian mixing parameter $s_0^2 = g'^2/(g^2+g'^2)$ at the algebraic boundary — not the $\overline{\text{MS}}(M_Z)$, effective-leptonic, or on-shell value. [LIB2-199] Two routes to promote it to a parameter-free prediction both fail: (i) as a running value the matching scale $\mu_\star$ is scheme-dependent and undetermined; (ii) as a tree value bridged by standard electroweak radiative corrections, the required shift is not uniquely fixed — the ordinary on-shell→effective correction is +3.9% (the wrong sign for the needed −2%), and forcing $s_W^2 = \phi^{-3}$ on shell would require $\Delta r \approx 0.073$, about twice the normal size. [LIB2-289, LIB2-290, LIB2-291, LIB2-292, LIB2-295, LIB2-310, LIB2-343] Declared (Rev33.2): ϕ^{-3} is the tree-level Lagrangian s_0^2 at the algebraic boundary and its cell is UNSCALED — no SM comparator at a known scale until a boundary scale is derived; the +2.10% against $\hat{s}^2(M_Z)$ is an audit number, and the running-curve reading above (A697) with its $\mu_\star$ crossing is kept as history, retired as a declaration (`s1336`).

AX6 and the weak-angle boundary — two objects, one name (repaired Rev32.2). The S47 algebraic result separates two statements. First, $\phi^{-3} = 1/(2(\Omega_{12}+\Omega_{23}))$ is a constant identity in the Peirce eigenvalues; it is not a function of any orbit angle. Second, the orbit selector — and here two distinct objects have carried the one label "AX6" and the one symbol $\psi_\star$ in print (house `s1135`, lane-cross-verified A1549; PI ruling S295). **AX6**$_{\text{pol}}$ is the polarisation postulate $\Lambda_\star = 3P/Q(Y) = 6\phi - 1$ on the exact-colour $I_4 = \phi^3$ orbit (Appendix A: $\tan^2\psi_{\text{pol}} = (6\phi-1)/4$, $\psi_{\text{pol}} \approx 55.87°$), whose registered readout $\sin^2\theta_W = 3/(4+\Lambda_\star) = \frac{3}{4}\cos^2\psi_{\text{pol}}$ gives ϕ^{-3} *exactly* (because $3+6\phi = 3\phi^3$; $\cos^2\psi_{\text{pol}} = 4/(3\phi^3)$ is quadratic over $\mathbb{Q}$); the three CW no-gos [A617,A620,A621] were evaluated at this angle. [LIB2-200, P-006] The **Det^2 object** is the degree-six selector $\cos(6\psi) = \frac{1}{2}$ with its 50° branch; it is *not* a solution of AX6$_{\text{pol}}$ ($\tan^2 50° = 1.4203 \neq 2.1771$), gives 0.3099 under the readout, and is the sole object of A667's degree-three Galois argument. [P-026] The Det^2 object is therefore **retired as a selector of AX6**$_{\text{pol}}$; A667 remains correct for the 50° object only and does not constrain AX6$_{\text{pol}}$. [LIB2-200, LIB2-249, P-026] The value identity establishes the readout once AX6$_{\text{pol}}$ is assumed; it does not derive or select AX6$_{\text{pol}}$, which remains an independent postulate at tier.[1] The remaining task is to derive the (scheme- and threshold-dominated, sub-GeV/GeV-order) matching scale from the algebra — the once-quoted 1.8 GeV is not pinned (Appendix I §B4); until then ϕ^{-3} is a structural target for the low-energy weak angle, not an absolute prediction. [LIB2-200, LIB2-249, LIB2-289, LIB2-292, LIB2-310] The earlier high-scale, downward-running test is superseded (A697).

8 The fit-free mixing sector and the common T_{half} engine

The Higgs paper is no longer insulated from the flavour sector. The same algebra that supports the cubic Yukawa carries the mixing ledger below.

[1] Binding vocabulary from Rev32.2: "AX6$_{\text{pol}}$" and "the Det^2 object"; never a bare $\psi_\star$. Readout identity: $3/(4+(6\phi-1)) = 1/\phi^3 = \sqrt{5}-2$. Any earlier statement of "AX6" as $\cos(6\psi_\star) = \frac{1}{2}$ with $\psi_\star = 50°$ refers to the Det^2 object.

Rev29 supersession — read before the table. The Status column is the *pre-Rev29* marking and is superseded en bloc by the row-by-row re-tiering (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`; A1566 retype, Rev32.6): two of the eight stand at DERIVED-CONDITIONAL or above ($\theta_{12}^{\rm PMNS}$, δ_{CP}), the CKM first row is LOADED, $\delta_{\rm CKM}$ is COINCIDENCE-CLASS, θ_{13} is structural, and the physical θ_{23} row and $|V_{cb}|$ are LOADED-CORRESPONDENCE. **No row may be cited as zero-parameter.**

Observable	Exact expression	Status
$\theta_{12}^{\rm PMNS}$	$\tan\theta_{12} = \sqrt{3}/\phi^2$	THEOREM ⋆100
$\theta_{23}^{\rm PMNS}$	$\sin^2\theta_{23} = 7/16$	physical octant LOADED-CORRESPONDENCE; internal identity theorem-grade (A1566; pre-Rev29 stamp THEOREM ⋆100)
$\theta_{13}^{\rm PMNS}$	$\sin^2\theta_{13} = \sin^4(\pi/8) = (3-2\sqrt{2})/8$	STRUCTURAL ⋆ ⋆ ⋆ (round-trip framing T^2, exponent-2 identified [s502/A1275]; *Stamps in* forcing pending)
δ_{CP}	$\delta_{CP} = -2\pi/\sqrt{5} = -160.997^\circ$	THEOREM ⋆100
$\lvert V_{us}\rvert$	$\lvert V_{us}\rvert = \sin\theta_{12}^{\rm PMNS}/\sqrt{6}$	THEOREM ⋆100
$\lvert V_{cb}\rvert$	$\lvert V_{cb}\rvert = 1/(9\sqrt{7})$	LOADED-CORRESPONDENCE (physical attachment) / Structural formula (A1566) (pre-Rev29 stamp THEOREM ⋆100)
$\lvert V_{ub}\rvert$	$\lvert V_{ub}\rvert = \lvert V_{us}\rvert\lvert V_{cb}\rvert/\sqrt{6}$	THEOREM ⋆100
$\delta_{\rm CKM}$	$\delta_{\rm CKM} = \arctan\sqrt{3(\phi+5\phi^{-5})}$	STRUCTURAL ⋆⋆⋆ (phase origin open)

this table (⋆100, THEOREM, Proved) are the Rev29-era marking, superseded en bloc by the Rev29 re-tier (Paper 3 §"Row-by-row re-tiering"); the current tier of each row is the re-tier table's. Footer added Rev32.5.

The Higgs-sector reason to record this here is not numerical decoration. The same generator that drives the leptonic CP rotation is now proved to lie in the derivation algebra of $J_3(\mathbb{O}_s)$:

$$T_{\rm half} \in {\rm Der}(J_3(\mathbb{O}_s)) = \mathfrak{f}_{4(4)}, \qquad \text{derivation error } 3.39\times10^{-15}. \tag{24}$$

So CP violation is generated internally by an automorphism of the same algebraic structure that fixes the Higgs placement and the cubic Yukawa. The CP source is not an externally attached phase coefficient.

9 Mass pyramid and the Sprint 3 outlook

The suite now has a clear tier structure. Tier 1 is the zero-fit mixing sector (no fitted PMNS/CKM angle, phase, or normalization inside the correspondence map; two entries readout-conditioned). Tier 2 is the mass-sector construction now forming above it. The current mass operator is carried in the Jordan-automorphism form

$$M_{\text{lepton}} = U_{T_{\text{half}}}\, D_R\, U_{T_{\text{half}}}^{-1}, \qquad D_R = \text{diag}(R_1, R_2, R_3), \tag{25}$$

with

$$R_d = \frac{1}{\phi^{2d} - 1}, \qquad t^* = \frac{2\pi}{\sqrt{5}}, \qquad U_{T_{\text{half}}} = \exp(t^* T_{\text{half}}). \tag{26}$$

The same generator that gives δ_{CP} therefore also drives the charged-lepton mass ordering.

The charged-lepton mass sector supplies one compact algebraic ratio. It was printed in earlier revisions as a zero-free-parameter result; *Rev29* re-tiers it to Structural / Loaded-correspondence / Reproduced (see Paper 7 § "Lepton-family validation" for the provenance chain and the fit-gap accounting). With the single external anchor

$$m_\tau = 1776.93\,\text{MeV}, \tag{27}$$

the tree-level ratio is

$$\frac{m_\mu}{m_\tau} = \left(\frac{\phi}{\sqrt{5}}\right)^{8/3} \frac{\sqrt{2}}{10} = 0.059684, \tag{28}$$

which gives

$$m_\mu = 106.05\,\text{MeV} \qquad (+0.37\%\ \text{PDG}). \tag{29}$$

The exponent $p = 8/3$ is an exact dimension ratio *read as* a topological winding ratio, and $C = \sqrt{2}/10$ divides by $\dim J_2(\mathbb{O}_s) = 10$. *Rev29:* neither step is closed as a derivation — an exact dimension ratio is not by itself a mass exponent, and A618 records the divide-by-dimension step as "argued by analogy" and lists the formal derivation as Blocking, closed nowhere. **The +0.37% above is the fit gap**, not a loop correction: reproducing the measured ratio requires $C = 0.140894$ against the offered $\sqrt{2}/10 = 0.141421$, and C enters multiplicatively. The same Jordan automorphism reverses the resolvent ordering,

$$(R_1, R_2, R_3) \longmapsto (R_3, R_2, R_1), \tag{30}$$

so the generation ordering is forced algebraically rather than chosen by hand. The electron mass is Koide-consistent ⋆⋆⋆ under external $K = 2/3$: $m_e = 0.5076\,\text{MeV}$ (-0.66% PDG) from the QED-corrected Koide chain [A636, A637]. The charged-lepton chain is *structurally pinned with mixed provenance* (m_τ anchor; m_μ Structural / Loaded-correspondence / Reproduced at *Rev29*, *not* zero-parameter; m_e Koide-consistent under the external rule); the individual masses are not jointly derived, and the compact ratio's relation to the Appendix O two-gap operator remains a pending dual-route reconciliation. Authoritative ledger: Appendix O, *The authoritative charged-lepton ledger*.

10 Falsification criteria

The Higgs/EWSB layer now has both experimental and theoretical failure modes.

Criterion	Present target	Failure mode (**Rev32.1 reading rule: tiers name mathematical provenance; physical attachments are stated separately; no row may be cited as zero-parameter; displayed σ values are named comparator distances, not profile likelihoods.**)
Metric-signature foundation	*adopted* neutral $(4,4)$ model choice; the composition-algebra demand then selects $\mathbb{O}_s$	If the adoption is rejected — i.e. if a convention-fixing framework reproduces the same structure — the physical motivation for $J_3(\mathbb{O}_s)$ weakens upstream of numerics. The falsification target is the *adoption*, not a forcing claim
Weak-angle route	Exact-color orbit, algebraic correspondence $\phi^{-3} \approx 0.23607$	$\sim$2% **STRUCTURAL.** ϕ^{-3} lies on the effective $\sin^2\theta_W(\mu)$ running curve at a sub-GeV, scheme-dominated crossing (A697; corrected S124/Appendix I §B4); the PDG 2026 $\overline{\text{MS}}(M_Z)$ comparator is 0.23122 ± 0.00006 and the on-shell comparator is 0.22348 ± 0.00010. The matching scale is not pinned, so this is a structural target, not an absolute Z-pole prediction; no σ distance is assigned.
Leptonic CP	$\delta_{CP} = -160.997^\circ$	Stable disagreement at multi-σ in long-baseline data
Atmospheric angle	$\sin^2\theta_{23} = 7/16$ (physical octant LOADED-CORRESPONDENCE, A1566)	Stable exclusion of the 7/16 branch in global fits — falsifies the loaded lower-octant registration first
Mass bridge	$m_\mu/m_\tau = (\phi/\sqrt{5})^{8/3}\sqrt{2}/10$; $m_e = 0.5076\,\text{MeV}$ via QED-Koide chain [A636,A637]	Failure of the muon ratio at its *Rev29* tier (STRUCTURAL / LOADED-CORRESPONDENCE; *not* zero-parameter — the +0.37% is the fit gap), or of the QED-Koide chain giving m_e outside 2% PDG once the chain is fully specified

The Higgs quartic relation $\lambda_H = 8/63$ remains a low-priority conjectural item rather than a main falsification channel.

11 Conclusion

Rev29 turns Paper 4 into the correct interface between the algebraic Higgs story and the rest of the suite. The algebra follows from an adopted signature-neutral premise plus the composition-algebra demand, the vacuum is theorem-level, the local cubic Yukawa fixes $y_t(M_{\rm Pl}) = 1$, the D=4 scalar target is ghost-free, the tree-level F_4 potential has an exactly flat eigenvalue-ratio direction, the diagonal weak-angle route is dead, the surviving bosonic route is the non-diagonal exact-color orbit, and the CKM+PMNS sector stands at its Rev29 re-tiered statuses above the Higgs layer (2/8 DERIVED-CONDITIONAL or above since A1566, Rev32.6; no "closed" claim is licensed). The result is not a finished electroweak theory. It is the sharpest honest form of the present EWSB programme: algebraically rigid where the suite is ready, explicit about the selector and loop-level tasks where it is not.

12 Doublet–triplet splitting from the MRSS Higgs parity

The doublet–triplet splitting problem is the classic failure mode of Kaluza–Klein and grand-unified constructions: the Higgs doublet that must stay light arrives in a multiplet with a coloured triplet that must not, and no symmetry of the construction separates them.

At A1471 the splitting is *derived* in this framework, from the parity of the unique minimal chiral lift rather than from an imposed mechanism.

> **Result (A1471, s954).** The mouth-Real matrix-twist orbifold $\mathfrak{L}_{\rm MRSS}$ is the unique minimal admissible chiral lift (A1463, zero continuous parity moduli, house-proved). Its Higgs-10 parity assignment is unique, and that unique parity retains the weak doublet while projecting out the coloured triplet. The historic failure mode closes on the mouth twist.

The boundary price associated with the branch is 6 microscopic gauge coefficients and 3 visible combinations; the earlier endpoint shorthand from the A1463 era is retired.

The condition this rests on, stated in the same breath. At A1472 the uniqueness was corrected: it is CONDITIONAL ON THE LOADED LOW-ENERGY SCALAR SPECTRUM — one weak doublet, no coloured triplet. [LIB2-198] Read plainly, the parity is unique *given* the spectrum it is being asked to produce. This is a real result and a real circularity risk in the same sentence, and the register in Appendix X carries it as a LOADED clause with its discharge condition named: derive the low-energy scalar content and the Higgs parity from the parent boundary dynamics, rather than selecting the parity by the desired spectrum. [LIB2-198] Until that is done, this section states a consistency, not a derivation of the spectrum.

[LIB2-198]

Leibniz Quantum Beats Newton

Paper 5 (Rev33.2): Neutrino Physics and CP Violation

Tom O'Sieg

August 2026

Abstract

This paper isolates the neutrino and leptonic CP sector of the framework. It assigns exact closed-form expressions to four PMNS parameters *within a loaded correspondence*: three carry theorem-grade constructions on the algebraic structure of $J_3(\mathbb{O}_s)$ with $F_{4(4)}$ symmetry (marked THEOREM ⋆100 pre-Rev29; DERIVED-CONDITIONAL after the Rev29 re-tiering (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`)), the phase identification is conditional on the stated readout map, and the reactor-angle attachment is STRUCTURAL ⋆⋆⋆ (round-trip framing T^2, exponent-2; forcing pending [s502/A1275]). No fitted parameters remain in the PMNS sector; "no fit" is a statement about coefficients *inside* the map, not a claim that the correspondence map itself is derived, and **no row may be cited as zero-parameter**. The three mixing angles and the Dirac CP phase are:

$$\tan\theta_{12} = \frac{\sqrt{3}}{\phi^2} \implies \theta_{12} = 33.49^\circ \quad \text{(THEOREM internally; DERIVED-CONDITIONAL physically — Paper 3 re-tier),}$$

$$\sin^2\theta_{23} = \frac{7}{16} \implies \theta_{23} = 41.41^\circ \quad \text{(THEOREM internally; LOADED-CORRESPONDENCE as the physical octant —}$$

$$\sin^2\theta_{13} = \sin^4(\pi/8) = \frac{3-2\sqrt{2}}{8} \implies \theta_{13} = 8.42^\circ \quad \text{(STRUCTURAL ⋆⋆⋆),}$$

$$\delta_{CP} = -\frac{2\pi}{\sqrt{5}} = -160.997^\circ \quad \text{(THEOREM internally, A603/E110; DERIVED-CONDITIONAL physically).}$$

The θ_{13} attachment carries its qualifier in full: round-trip framing T^2, exponent-2, with forcing pending [s502/A1275]. *Rev30 notation correction:* these four lines previously chained a dimensionless function value directly to an angle — e.g. "$\sin^2\theta_{23} = 7/16 = 41.41^\circ$", which as written asserts $7/16 = 41.41^\circ$. The implication arrows above separate the two statements; no value has changed. The leptonic CP phase is derived from the V_3 rotation: $T_{\text{half}}|_{V_3}$ is a 3D rotation with angular frequency $\Omega = \sqrt{5}/2$, giving half-period $t^* = \pi/\Omega = 2\pi/\sqrt{5}$ (A603/E110, Appendix B, §B.5). The free parameter $\beta_\delta = 11/(6\pi)$ is eliminated. NuFIT 6.1 (NH, w/SK) comparison: $\delta_{CP} = 212^{+26}_{-36}{}^\circ$ ($-160.997^\circ \equiv 199.003^\circ$), deviation 0.36σ. Three of the four PMNS parameters were marked THEOREM ⋆100 pre-Rev29 (θ_{12}, θ_{23}, δ_{CP}). Two physical PMNS rows, θ_{12} and δ_{CP}, stand at DERIVED-CONDITIONAL after the Rev29 re-tiering (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`), each with its premises printed. The exact 7/16 mismatch identity remains theorem-grade internally, while its physical lower-octant attachment is LOADED-CORRESPONDENCE (A1566/D1244, Rev32.6); θ_{13} is STRUCTURAL ⋆⋆⋆ (round-trip framing T^2, exponent-2; forcing pending [s502/A1275]). No fitted coefficients enter the map; **no row may be cited as zero-parameter**. Normal ordering is assumed; the operative benchmark *assumes* Dirac neutrinos — the existing singlet permits an algebraic Majorana term absent an additional $B-L$ principle — and the representation content is exhaustive: the fermionic $16 = J_{13} \oplus J_{23}$ is exactly saturated by the SM states plus ν^c, so the framework *forbids* light sterile neutrinos — a confirmed sterile would falsify the verified assignment (A726; Paper 7).

1 Introduction

The neutrino sector is where the framework is most exposed to direct falsification. The paper therefore keeps the structure simple: the angle set, the phase formula, the current comparison to NuFIT 6.1, and the DUNE-era test scenario. [LIB2-345]

2 Three neutrino mixing angles

The operative angle relations are (normal ordering — conditional on the R_d ladder, Paper 2 §3, and on its identification with the propagation eigenstates, an interface that is undeclared (Rev32.2); pre-Rev29 markings: three of four PMNS parameters at THEOREM ⋆100, θ_{13} STRUCTURAL ⋆⋆⋆; current: θ_{12}, δ_{CP} DERIVED-CONDITIONAL, the physical θ_{23} octant LOADED-CORRESPONDENCE — A1566):

$$\tan\theta_{12}^{\mathrm{PMNS}} = \frac{\sqrt{3}}{\phi^2}, \qquad \text{(PMNS solar angle — THEOREM)} \tag{1}$$

$$\sin^2\theta_{23} = \frac{7}{16}, \qquad \text{(F_4 mismatch geometry; THEOREM internally, physical octant LOADED-C} \tag{2}$$

$$\sin^2\theta_{13} = \sin^4(\pi/8) = \frac{3-2\sqrt{2}}{8}. \quad \text{(STRUCTURAL ⋆⋆⋆)} \tag{3}$$

The θ_{23} row is the sharp loaded lower-octant target of the selected correspondence (A1566); the θ_{13} row carries round-trip framing T^2, exponent-2, with forcing pending [s502/A1275]. These give

$$\theta_{12}^{\mathrm{PMNS}} \approx 33.49^\circ, \tag{4}$$

$$\theta_{23} \approx 41.41^\circ, \tag{5}$$

$$\theta_{13} \approx 8.42^\circ. \tag{6}$$

Precision and structural status of the angle predictions (A678). The reactor angle $\sin^2\theta_{13} = \sin^4(\pi/8)$ is a STRUCTURAL ⋆⋆⋆ prediction: the formula reproduces the observed scale at $< 1.8\sigma$ — comparator NuFIT 6.1 (NH, w/SK), $\sin^2\theta_{13} = 0.02248^{+0.00055}_{-0.00059}$, central-value Gaussianized on the lower error and not a profile (*Rev29*: comparator named) — and three compact closed forms (1/45, $\sin^4(\pi/8)$, ϕ^{-8}) are numerically close to the measured value, but not all within the comparator this suite uses: against NuFIT 6.1 (Scorecard row 6) 1/45 sits at $d = -0.44$, the retained $\sin^4(\pi/8)$ at -1.75 and ϕ^{-8} at -2.02 (Rev32.9; through Rev32.8 this sentence said all three "lie within PDG uncertainty"); no current Freudenthal theorem selects one. [LIB2-061] The atmospheric angle $\sin^2\theta_{23} = 7/16$ is the sharp loaded lower-octant target of the selected correspondence (A1566): the correction required to reach the NuFIT 6.1 NH upper-octant local comparator (≈ 0.561) is ≈ 0.124, far larger than any natural Jordan subleading perturbation; the NuFIT 6.1 global best fit itself (0.470, lower octant) lies only 0.0325 above the prediction. Watch DUNE/Hyper-K for octant resolution.

Scope of the Jordan-adjoint mixing carrier X_ν (renamed at *Rev29*). The algebraic invariant $X_\nu = J_{\mathrm{vac}} \times J_{\mathrm{vac}} = J_{\mathrm{vac}}^{\#} = \mathrm{diag}(\phi^{-1}, 1, \phi)$ is the framework's **Jordan-adjoint mixing carrier**. Through Rev28 this suite called it the "neutrino mass operator". That name asserted an interface — from the constructed Jordan layer to the physical neutrino mass spectrum — which the

framework does not have and which the calculation in the next paragraph disproves in this very paper; the object was carried across an absent interface under a physical name. The definition is unchanged and the object remains valid in the layer where it is constructed. Only the predicate changes: from *mass operator* to *mixing carrier*. As a mixing carrier it organizes the PMNS mixing-sector geometry: the mismatch vector $J_{\rm vac} - J^{\#}_{\rm vac} = \mathrm{diag}(1, 0, -1)$ seeds θ_{13} and the $(J_{\rm vac}, J^{\#}_{\rm vac})$ Gram data seed the atmospheric angle. Its eigenvalues do *not* furnish the physical neutrino mass spectrum — and this is the ground of the rename, not a caveat appended to it: the literal reading $m_i \propto \lambda_i(X_\nu)$ predicts normal ordering but gives the scale-free ratio

$$\frac{\Delta m^2_{21}}{\Delta m^2_{31}} = \frac{1 - \phi^{-2}}{\phi^2 - \phi^{-2}} = \frac{\phi^{-1}}{\sqrt{5}} = 0.27639, \tag{7}$$

against the measured ratio 0.03002 (NuFIT 6.1 IC24 NO: $\Delta m^2_{21} = 7.537 \times 10^{-5}\,\mathrm{eV}^2$, $\Delta m^2_{3\ell} = 2.511 \times 10^{-3}\,\mathrm{eV}^2$ — dataset named, Rev32) — too large by a factor 9.21 (A712, verified independently in house). [LIB2-134]

Two readings, one family (Rev32.2; house `s1104`*, lane A1546 C2, PI ruling S294; the exact-asymmetric numbers below are certified by* `s1162`*, Rev32.6 —* `s1104`*'s* -1.04σ *was a symmetric calibration-grade estimate and is superseded).* [LIB2-140] The literal reading above is the $d = 1$ rung of the resolvent family, $m_2/m_3 = \phi^{-1} = R_1$; the suite's own d-assignment (Paper 2 §3, Appendix X) places the neutrino sector at $d = 2$, $m_{\nu_2}/m_{\nu_3} = R_2 = 1/(\phi^4 - 1) = 0.17082$, so the ratio channel reads $R_2^2 = 0.02918$ against the measured 0.03002 (-2.8%). [LIB2-140] Propagating the directional (asymmetric) NuFIT 6.1 marginal errors to first order (`s1162`; this is not an exact or profile-likelihood distance — Rev32.9), R_2 lies 1.44–1.79σ *below* the $m_1 = 0$ normal-ordering floor $\sqrt{\Delta m^2_{21}/\Delta m^2_{3\ell}} = 0.17325$ (without / with SK-ATM); equality is impossible for any $m_1 \geq 0$, because the ratio is minimised at $m_1 = 0$. **This row was typed, before Rev33.0, as a fixed below-floor edge target** — a 1.4–1.8σ tension with the $m_1 = 0$ NO floor, printed as a tension. **That typing is superseded.** The suite now types the row as a $\Delta m^2_{21}/\Delta m^2_{31}$ **ratio prediction, asserting** $m_1 = 0$, under which m_1 drops out of the ratio and $R_2^2 = 0.029180$ becomes a direct, JUNO-decidable statement rather than an edge target that no measurement can meet. $m_1 = 0$ **is an assertion of this framework, not a derivation**, and it is registered as one; nor is it a theorem of the ν ladder (`s1329`). Using the printed $R_2^2 = 0.029180$ and holding the anchoring central value fixed, the implied Δm^2_{31} from ledger row SL-01 stands at $+3.43\sigma$ against SL-02 (NuFIT 6.1 with SK-ATM, on its upper marginal width, since the implied value lies above the central value); anchoring instead on SL-02, the implied Δm^2_{21} stands at -1.44σ against the JUNO 59.1 d comparator SL-37. These are separately conditioned marginal distances: anchor uncertainties and correlations are not propagated, the two disagree in sign, and they are not independent tests — *no unqualified* σ *is printed for this row* (`s1330`). On the ledger widths (SL-01/SL-02, first order, directional) the measured ratio is $0.030016^{+0.000444}_{-0.000471}$, and R_2^2 lies *outside* its 1σ band, at -1.78σ (`s1330`). That band contains **three** members of the declared M1 monomial menu (density 98 per unit relative width) and none of M2, and the density is printed with the row because a hit inside a dense band is reported with its density or it is not reported (`s1359`; the ten members and density 186 printed in Rev33.1 counted `s1184`'s wider, calibration-grade band, not the ledger's). **What the typing costs, stated plainly:** at JUNO's design reach — the collaboration's own six-year projection (arXiv:2204.13249 Table 6; SL-42–SL-44, a declared external input) — a -2.8% miss on this ratio becomes $\mathbf{-7.73\sigma}$, and a JUNO-alone central ratio at or above 0.02950 excludes R_2^2 at one-sided 3σ (0.03035 at 100 d, 0.02938 at 20 yr): the kill is written before the data (`s1330`). The row is accordingly a falsifier, and it is printed as one. It stands only with normal ordering and $m_1 \lesssim 1$ meV (m_1 at -2σ: 0.86/0.51 meV without / with SK-ATM, `s1162`). [LIB2-140] The $d = 1$ reading remains the one genuinely falsified row. The assignment $d(\nu) = 2$ is SELECTED,

not derived: it is reproduced by the post-hoc feature rule $d = 2 + [Y_R \neq 0] + [\text{coloured}]$ and is the χ^2 minimum of an injective assignment ($\Delta\chi^2 = 17.8$ to the runner-up). [LIB2-141] Tier: STRUCTURAL (the ladder's own tier); local-band coverage 10.9–13.9% (`s1162`: the share of the local band lying within 2% of a member of its own twenty-member rung menu — the chance a random target is 'hit', a different statistic from the M1 count above); no blind credit.

Falsifiable corollaries — of the $m_1 = 0$ boundary, not of R_2. Under the asserted boundary $m_1 = 0$ with normal ordering, the boundary itself fixes $\Sigma m_\nu = 58.79$ meV and $m_\beta = 8.74$ meV. These are properties of the $m_1 = 0$ NO boundary, which the framework asserts rather than derives from R_2; they are not consequences of R_2. $m_{\beta\beta} = 0$ is Dirac-loaded (Appendix X), not predicted. Cosmology comparator, source-lined: DESI DR2+CMB baseline $\Sigma m_\nu < 64.2$ meV (arXiv:2503.14744; margin 5.41 meV); the 53 meV frequentist-constrained bound is model tension, not exclusion. [LIB2-142, LIB2-207, LIB2-208] A measured inverted ordering, or m_1 established above ~ 1 meV, contradicts the adopted $m_1 = 0$ normal-ordering assertion.

X_ν is therefore carried *only* as the Jordan-adjoint mixing carrier — a structural object of the constructed layer, which predicates nothing whatever about masses — and no statement in this suite may read it as a mass operator or as a mass ladder (*Rev29*; the matching rename is folded in Paper 3 §3.1). [LIB2-134] The absolute neutrino mass scale, the Δm^2_{ij} splittings, and the Dirac/Majorana or see-saw scale choice are not derived in this suite. The PMNS mixing predictions are independent of those absolute-scale choices, with normal ordering — conditional on the R_d ladder and on the undeclared ladder-to-eigenstate interface, never a ladder theorem (`s1108`; A1546 C6) — in the benchmark tables.

The B-map dictionary (A689) does contain a right-handed-neutrino slot, $\nu^c = e_7 \in J_{23}$, and the cross-block Peirce trilinear yields a nonzero Dirac Yukawa,

$$\tau(H_u, L, \nu^c) = \epsilon_{\alpha\beta}\, H_u^\beta L^\alpha \nu^c, \tag{8}$$

the lepton analogue of the up-type coupling (A709, verified independently in house). The Dirac partner is therefore present structurally. However, ν^c is an $SU(3)_c \times SU(2)_L \times U(1)_Y$ singlet ($Y = 0$), so a bare Majorana bilinear $M\,\nu^c\nu^c$ is gauge invariant and algebraically nonzero. The renormalizable Yukawa sector conserves an accidental *global* $B - L$ (the Higgs is $B - L$-neutral, so the electroweak vacuum does not break it), but no *gauged or fundamental* symmetry forbidding $M\,\nu^c\nu^c$ is derived. Neutrinos are therefore treated as Dirac by framework assumption; the seesaw/Majorana sector is structurally available but is not included in the operative PMNS benchmark.

The relevant symmetry is constructible explicitly as the fifth $SO(10)$ Cartan. On the spinor $S_+ = \Lambda^{\text{even}}\mathbb{R}^5 = \mathbf{1} \oplus \mathbf{10} \oplus \overline{\mathbf{5}}$, the grading $X(I) = 5 - 2|I|$ together with the A689 hypercharge rule $Y(I) = \sum_{i\in I} Y_i$ gives

$$B{-}L = \tfrac{1}{5}\,(X + 4Y), \tag{9}$$

which reproduces the standard one-generation charges $B{-}L(Q) = +\frac{1}{3}$, $B{-}L(u^c, d^c) = -\frac{1}{3}$, $B{-}L(L) = -1$, $B{-}L(e^c, \nu^c) = +1$, $B{-}L(H_{u,d}) = 0$ (16/16, verified independently in house, A710), commutes with $SU(3)_c \times SU(2)_L$, and is linearly independent of Y. It conserves the Dirac Yukawa and assigns the Majorana bilinear $B{-}L(\nu^c\nu^c) = +2$, so an *exact* $B{-}L$ would forbid it. However $B{-}L$ is not part of the gauged compact-SM group (rank 4; $B{-}L$ is the extra $SO(10)$ rank-5 Cartan), and no $B{-}L$-charged VEV is present to break it; a gauged $B{-}L$ would be anomaly-consistent with the existing **16** but is neither forced nor realized here. Gauging $B{-}L$ — extending the rank-4 SM to the rank-5 $SO(10)$-type group — was the open gauge-emergence question (T4-B). Its modal status

was settled by the S65 dispatch ladder (D727–D729, three independent assessments, unanimous [A729–A729c]): in the established frame-restriction category the gauge fields are *input, not emergent*, and the $B-L$ generator lives at the same structure-algebra/envelope layer as the successful one-generation charge table (the simple F_4 derivation route hosts a quark-sector candidate but no lepton slots [A678]), so no categorical asymmetry forces the non-gauging. The absence of a gauged $B-L$ is therefore *inherited* from the compact rank-four input choice — a choice that is phenomenologically required rather than algebraically forced: gauged-and-unbroken gives an excluded massless Z'_{B-L}, while gauged-and-broken needs a $B-L$-charged VEV not present here. Neutrinos accordingly remain Dirac by framework benchmark assumption: a confirmed Majorana mass ($0\nu\beta\beta$) would falsify this benchmark assumption, not the PMNS mixing algebra. The residual T4-B question is the selection theorem for the input vector content — $\mathrm{adj}(G_{\mathrm{SM}})$ versus $\mathrm{adj}(G_{\mathrm{SM}} \times U(1)_{B-L})$ — currently answered only by the Z' exclusion.

The PMNS solar angle may also be written as

$$\tan^2 \theta_{12}^{\mathrm{PMNS}} = \frac{3}{\phi^4} = \phi^{-2} + \phi^{-6}, \tag{10}$$

which makes the role of the exact identity $\phi^2 + \phi^{-2} = 3$ explicit.

2.1 Framework vs. NuFIT comparison (C3 status)

Quantity	Framework	Comparator: NuFIT 6.1 (NH, w/SK)	Tension (central-value Gaussianized)
θ_{12}	THEOREM 33.49°	$33.76^{+0.42\circ}_{-0.41}$ (NuFIT 6.1 IC24 NO — corrected Rev32: the $33.41^\circ \pm 0.81^\circ$ previously printed here was a pre-6.0-era value carrying a 6.1 label)	0.66σ (the earlier $< 0.1\sigma$ claim is struck)
θ_{23}	$\sin^2\theta_{23} = 7/16$: theorem-grade internal identity; physical θ_{23} LOADED-CORRESPONDENCE — the sharp, *falsifiable* loaded lower-branch target of the selected correspondence (A1566; pre-Rev29 stamp THEOREM $\star$100) 41.41°	43.3° ($\sin^2\theta_{23} = 0.470^{+0.017}_{-0.014}$, lower octant, w/SK)	2.3σ (octant SK-ambiguous; UO comparator 0.561 at $\sim 9.5\sigma$)
θ_{13}	STRUCTURAL $\sin^2\theta_{13} = \sin^4(\pi/8) = (3 - 2\sqrt{2})/8$ ($\star\star\star$; round-trip framing T^2, exponent-2; forcing pending [s502/A1275]; A608/E113), 8.42°	$8.60^\circ \pm 0.13^\circ$ (rounded symmetric degree quote)	1.4σ in this variable; 1.8σ in $\sin^2\theta_{13}$ — see the box below (*Rev29*)
δ_{CP}	THEOREM $\star$100, $-2\pi/\sqrt{5} = -160.997^\circ$ (A603/E110)	$212^{+26\circ}_{-36}$	0.36σ

Stamps in this table ($\star$100, THEOREM, Proved) are the Rev29-era marking, superseded en bloc by the Rev29 re-tier (Paper 3 §"Row-by-row re-tiering"); the current tier of each row is the re-tier table's. Footer added Rev32.5.

θ_{13}**: the two tensions this paper prints are the same fit** (*Rev29*). The table above reads 1.4σ and §3 below reads 1.8σ; both are arithmetically correct, and they are one NuFIT 6.1 (NH, w/SK) result read in two variables. In $\sin^2\theta_{13}$ the comparator is $0.02248^{+0.00055}_{-0.00059}$ against the framework's 0.021447, giving $1.75\sigma \to 1.8\sigma$ on the lower error. In degrees, the same comparator pushes forward to $\theta_{13} = 8.623^{+0.106\circ}_{-0.115}$, against which the framework's 8.421° sits at 1.76σ — also 1.8σ. [LIB2-228] The 1.4σ arises only from the *rounded, symmetrized* degree quote $8.60^\circ \pm 0.13^\circ$, which is not the push-forward of the NuFIT 6.1 interval. **The variable-consistent figure is** 1.8σ; 1.4σ is retained above and now carries the comparator that produces it. All σ figures in this paper are central-value Gaussianized against a single-number comparator, not profile likelihoods, and where the comparator is asymmetric the error on the side facing the framework value is used (Rev32.9; through Rev32.8 this said "the lower error", which is not what every printed figure uses). *Cross-suite:* the CKM σ figures are separately under reconciliation — see the reconciliation box in Paper 3, §2.1.

3 CP violation from $\mathrm{PSL}(2,7) \subset G_2 \subset F_{4(4)}$

Rev29 result: the leptonic Dirac phase is derived from the V_3 rotation theorem (A603/E110). The free parameter $\beta_\delta = 11/(6\pi)$ is eliminated. The Uniqueness Theorem (Session 16, A558) establishes that T_{half} is the unique G_2-invariant generator in the $(1,3)$ Peirce sector of $\mathfrak{f}_4$.

$$\delta_{CP} = -\frac{2\pi}{\sqrt{5}} \approx -160.997^\circ \qquad (J_{CP} \approx -0.011,\ \text{rephasing-invariant}). \tag{11}$$

Mechanism (V_3 rotation). The three-dimensional invariant subspace $V_3 = \mathrm{span}\{J_{\text{vac}},\ J^{\#}_{\text{vac}},\ T_{\text{half}} \cdot J_{\text{vac}}\}$ is closed under the adjoint action of T_{half}. Within V_3, the operator T_{half} acts as a 3D rotation with angular frequency $\Omega = \sqrt{5}/2$. The half-period satisfies $t^* = \pi/\Omega = 2\pi/\sqrt{5}$, giving

$$\exp(t^*\, T_{\text{half}}) \cdot J_{\text{vac}} = J^{\#}_{\text{vac}}.$$

The CP phase is the accumulated rotation angle: $\delta_{CP} = -t^* = -2\pi/\sqrt{5}$. The rotation angle is purely algebraic — no free parameter enters t^* — but its identification with the measured PMNS phase is a *conditional readout* (frame, ordering, Dirac benchmark; no row may be cited as zero-parameter). (The older PSL(2, 7) formula, $-\arccos(-1/3) - 2\pi/7 = -160.900^\circ$, remains as an independent cross-check; the V_3 formula is authoritative.)

Experimental comparison (NuFIT 6.1, NH w/SK): $\delta^{\text{exp}}_{CP} = 212^{+26\circ}_{-36}$; with $-160.997^\circ \equiv 199.003^\circ$, deviation $= (212^\circ - 199.003^\circ)/36^\circ = 0.36\sigma$ using the central-value Gaussianized (lower) error only. The full NuFIT $\Delta\chi^2$ profile is non-Gaussian; see Paper 7 for the falsification interpretation.

Status (Rev29; A1566 retype Rev32.6). 2/8 PMNS+CKM PARAMETERS AT Derived-conditional OR ABOVE ($\theta_{12}^{\rm PMNS}$, δ_{CP}); THE CKM FIRST ROW IS Loaded; THE PHYSICAL θ_{23} ROW AND $|V_{cb}|$ ARE Loaded-correspondence (THE INTERNAL 7/16 IDENTITY THEOREM-GRADE); $\delta_{\rm CKM}$ IS Coincidence-class; θ_{13} IS STRUCTURAL — NO FITTED MIXING COEFFICIENTS *INSIDE* THE MAP, AND NO ROW MAY BE CITED AS ZERO-PARAMETER. *The pre-Rev29 tally of 6/8 at* $\star$100 *is superseded* (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`).

$\delta_{CP} = -2\pi/\sqrt{5} = -160.997°$ ($J_{CP} \approx -0.011$) is DERIVED-CONDITIONAL (Rev29 re-tiering; the pre-Rev29 stamp was THEOREM $\star$100) from the V_3 rotation (A603/E110); all M-questions closed (A556, D576). NuFIT 6.1 (NH): 0.36σ — still the best-fitting parameter in the suite. DUNE Phase II will test this target. The rephasing invariant itself is *not yet a constraint*: the NuFIT 6.1 comparator, $J_{\rm PMNS} = -0.0140 \pm 0.0149$, has a 16–84% band $[-0.0286, +0.0022]$ that *includes zero* (the comparator is 0.94σ from CP conservation). The theory value $J_{CP} = -0.0107$ is therefore consistent with the data and equally with no leptonic CP violation. The row is DIAGNOSTIC (a function of four PMNS rows already scored), is counted in no tally of independent successes, and becomes a constraint only when the band excludes zero (kernel `s1185`; Monte Carlo comparator, correlations ignored; ruling R147). Normal ordering is assumed; the row is additionally conditional on the Structural R_d ladder and on the undeclared ladder-to-propagation-eigenstate interface (§2); Majorana phases are absent because the operative benchmark assumes Dirac neutrinos — an explicit framework assumption, not a consequence of a derived symmetry forbidding $M\,\nu^c\nu^c$ (A709). [LIB2-062, LIB2-242, LIB2-243, LIB2-244, LIB2-277, LIB2-278, LIB2-279, LIB2-280, P-032, P-033]

θ_{13}: STRUCTURAL $\star\star\star$ (A608/E113): $\sin^2\theta_{13} = \sin^4(\pi/8) = (3-2\sqrt{2})/8 \approx 0.02145$; D_4 triality $8_v \leftrightarrow 8_s$ maps $J_{\rm vac}$ at $\pi/4$ to $\pi/8$; numerical agreement at 1.8σ (comparator NuFIT 6.1 NH w/SK, $\sin^2\theta_{13} = 0.02248^{+0.00055}_{-0.00059}$, central-value Gaussianized on the lower error — the variable-consistent figure; cf. the 1.4σ degree-variable entry in the §2.1 table and the box beneath it, *Rev29*); round-trip T^2 squaring identified (exponent-2, s502/A1275); forcing pending.

C1 Theorem (Session 16, A558): the weight $c = \frac{1}{2}$ in $T_{\rm half} = T_{\rm scalar} + \frac{1}{2}T_{\rm oct}$ is proved exactly from the DET-7 ratio, Frobenius norm, and Peirce orthogonality (3-step algebraic chain; zero free parameters). [LIB2-051] Uniqueness Theorem (Session 16): $T_{\rm half}$ is the unique G_2-invariant generator in the $(1,3)$ Peirce sector of $\mathfrak{f}_4$. **CKM sector (Sessions 22–23):** the three CKM magnitudes ($|V_{us}|, |V_{cb}|, |V_{ub}|$) were marked Proved-tier ($\star$100) within the Route-B map pre-Rev29 — the first row is now LOADED (route selected against kaon data) and $|V_{cb}|$ is LOADED-CORRESPONDENCE (physical attachment) with a Structural formula (A1566); the CKM phase $\delta_{\rm CKM}$ is COINCIDENCE-CLASS / candidate construction — numerical value retained, G_7 selection absent; the Rev29 wording "conditional theorem under the readout axiom F-δ, STRUCTURAL" is retired here (Rev32.5, A1563; Paper 3 Candidate Construction 2.1; App B, §B.6). **All 8 PMNS+CKM parameters from $J_3(\mathbb{O}_s)$ alone, no fitted mixing coefficients inside the map: after the Rev29 re-tiering and the A1566 retype 2/8 stand at Derived-conditional or above, the CKM first row is Loaded, $\delta_{\rm CKM}$ is Coincidence-class, θ_{13} is structural, the physical θ_{23} row and $|V_{cb}|$ are Loaded-correspondence (readout-axiom claims, named inline).**

4 DUNE-era falsification scenario

Table 1: Rev29 neutrino-sector falsification targets. **Rev32.1 reading rule: tiers name mathematical provenance; physical attachments are stated separately; no row may be cited as zero-parameter; displayed σ values are named comparator distances, not profile likelihoods.**

Quantity	Framework target	Adopted precision scenario	Current reading
δ_{CP}	$-2\pi/\sqrt{5} = -160.997°$ ($J_{CP} \approx -0.011$) [THEOREM $\star$100; A603/E110; β_δ eliminated; 0.36σ]	DUNE-era $O(10°)$ at 1σ (2030s)	sharpest target; disagreement beyond the adopted band falsifies
θ_{23}	$41.41°$ ($\sin^2\theta_{23} = 7/16$; LOADED-CORRESPONDENCE physical target, internal identity theorem-grade — A1566; pre-Rev29 stamp THEOREM $\star$100)	DUNE-era few-degree precision (2030s)	exclusion falsifies the loaded lower-octant registration first; the internal F_4 mismatch theorem ($\|J_{\rm vac}\|^2 = 4$ from DET-7+$N(J) = 1$, AX2 eliminated) is attacked only if that attachment is independently derived
θ_{12}	$33.49°$	solar/reactor combined precision	consistency cross-check

Stamps in this table ($\star$100, THEOREM, Proved) are the Rev29-era marking, superseded en bloc by the Rev29 re-tier (Paper 3 §"Row-by-row re-tiering"); the current tier of each row is the re-tier table's. Footer added Rev32.5.

5 Conclusion

The Rev29 neutrino paper is intentionally direct. Three of the four PMNS parameters — θ_{12}, θ_{23}, δ_{CP} (V_3 rotation; A603/E110) — were marked THEOREM $\star$100 pre-Rev29. Two physical PMNS rows, θ_{12} and δ_{CP}, stand at DERIVED-CONDITIONAL after the Rev29 re-tiering, with their premises printed; the exact 7/16 mismatch identity remains theorem-grade internally, while its physical lower-octant attachment is LOADED-CORRESPONDENCE (A1566/D1244, Rev32.6). The reactor angle θ_{13} is STRUCTURAL $\star\star\star$: the formula $\sin^2\theta_{13} = \sin^4(\pi/8)$ is in numerical agreement at 1.8σ against NuFIT 6.1 (NH, w/SK), the release adopted throughout this suite — the 1.3–1.4σ quoted here previously was against the superseded NuFIT 6.0 and is retained only as that earlier reading (*Rev29*) — but the round-trip T^2 squaring is identified (exponent-2, s502/A1275) but not yet forced from first principles (D678). [LIB2-061, LIB2-281, LIB2-282, LIB2-315] The CKM magnitudes were marked PROVED-tier within Route B pre-Rev29; after the re-tiering the first row is LOADED and $|V_{cb}|$ is a STRUCTURAL formula with LOADED-CORRESPONDENCE attachment (A1566; Paper 3; Appendix B, §B.6); the CKM CP phase $\delta_{\rm CKM}$ is COINCIDENCE-CLASS (candidate construction; value retained, G_7 selection absent) and the full matrix is a readout-conditioned completion map (first-principles gap; correct numerically), and the $5\phi^{-5}$ chain is candidate-class pending derivation. If long-baseline experiments disagree decisively with these targets, the current exceptional-Jordan correspondence fails in the place

where it is easiest to test.

6 The neutrino posture, made explicit: Dirac-loaded

For five rounds the framework carried an unanswered question about its own neutrino typing — whether the parent-layer structures it uses commit it to Dirac or Majorana neutrinos. The question was raised as a standing rider and went unanswered through four dispatches. It was answered at A1472 and the answer is recorded here in the paper that depends on it.

6.1 The posture

The framework is **Dirac-loaded**: the Dirac branch is carried explicitly as a loaded choice, not derived. The charge arithmetic is exact and house-verified:

$$X(L) + X(H_u) + X(\nu^c) = 0, \qquad X(\nu^c\nu^c) = 10.$$

The first vanishing permits the Dirac Yukawa; the second is the obstruction that a Majorana mass term would have to overcome. X is conserved by every registered operator but is not supplied by the structure (it lies outside $\mathfrak{e}_{7(7)}$ together with $B - L$; `s1216`, `s1219`); forbidding the Majorana term is the loaded choice.

6.2 A disambiguation that mattered

An earlier round (SD16, folded at A1452) derived the mouth-chiral grading $\{S, H\} = 0$ from what it called a "branch-Real Majorana pair". That phrase invited a reading it did not support. At A1472 it is disambiguated: the pair is a **parent-layer Real condition**, $\mathcal{C} = \tau_1 K$ — an anti-linear structure on the parent layer — and *not* a Standard-Model Majorana mass term. The two live at different layers, and the earlier grading derivation is untouched by the neutrino typing.

The Dirac/Majorana collision watch opened during the S256 external reviews is resolved at this tier.

What the Majorana branch would cost. It is not closed off, and it is not free. Taking it requires a $\Delta(B-L) = 2$ source or seesaw block, and — because the loaded rows were audited as a dependency ledger at A1472, separating Dirac-essential rows from ν-independent ones — a complete re-audit of every row the Dirac loading touches. The branch is *census-enlarging*: it does not merely substitute one mass term for another, it widens the space the framework's uniqueness arguments run over. Appendix X carries this as a LOADED clause with that discharge condition named.

Leibniz Quantum Beats Newton

Paper 6 (Rev33.2): Renormalization-Group Running, Threshold Corrections, and the Electron-Mass Residual

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August 2026

Abstract

Paper 6 records the renormalization-group bridge used by the Rev8 suite for the weak angle and related threshold questions, and it adds the current electron-mass residual bookkeeping. The main algebraic input remains a tree-level exceptional-Jordan ratio; the main physical statement is that tree-level algebraic numbers and measured low-energy observables are connected only after ordinary field-theory running, threshold effects, and scale identification.

Rev8 also makes a second bridge explicit. The baseline electron formula

$$m_e^{\text{base}} = M_{\text{Pl}} \exp\Big[-\frac{\pi}{8\alpha}\Big]\, \phi^4 \sqrt{2} = 0.5036 \text{ MeV} \tag{1}$$

undershoots the physical electron mass by 1.454%. The bundle currently tracks five physically motivated correction phases, but their central values sum only to 1.304%, leaving a 0.15% remainder unexplained. **Rev12 S43 update:** The electron-mass residual sections of this paper are superseded. As of Session 43, $m_e = 0.5076\,\text{MeV}$ is KOIDE-CONSISTENT $\star\star\star\star$ under external $K = 2/3$ via the QED-corrected Koide chain [A636, A637]; the instanton-phase ansatz and all associated residual bookkeeping are retired. Paper 6 is retained as a historical record of the Rev8 electron-mass bridge programme.

Rev29 note (Sessions 16–18). The β-function and RGE content of Paper 6 are unchanged from Rev9. The $U(1)_Y$ coefficient is carried in the GUT-normalised convention $b_1 = 41/10$ throughout (KB9 fix, confirmed in both this paper and Appendix D). The Rev29 leptonic CP results (δ_{CP} at DERIVED-CONDITIONAL after the Rev29 re-tier) do not alter the RGE sector; they are reported in Paper 5. The muon-to-tau mass ratio is STRUCTURAL (re-tiered Rev29; Appendix O is authoritative), and this does not alter the RGE equations or threshold bookkeeping recorded here.

Rev32 restatement (superseding the Rev29 provenance overclaim that stood here, per the authoritative ruling in this paper's status section). The CKM Dirac phase δ_{CKM} retains its number (68.1297° from $G_7 = \phi + 5\phi^{-5}$ under the named readout axiom F-δ, Appendix B.6) but its *rating* is COINCIDENCE-CLASS, and it stays there until G_7 is derived: the number is not withdrawn; the rating is. What this paper adds is RGE-irrelevance — renormalization-group running and top-quark threshold corrections are *confirmed irrelevant* for δ_{CKM} (D594, A576) — and RGE-irrelevance is a statement about running, *not* algebraic provenance. The CKM first row is LOADED; $|V_{cb}|$ is LOADED-CORRESPONDENCE (physical attachment) / Structural formula (A1566); δ_{CKM} is COINCIDENCE-CLASS (canonical string, Rev32.1, amended Rev32.7 — the earlier "three CKM magnitudes are Structural" overclaimed against the first-row route cap, A1538 F7) — not a 100/100 closure of the sector.

1 Introduction

Historical note (Rev12 S43): The electron-mass closure problem described in this paper is now resolved. $m_e = 0.5076\,\mathrm{MeV}$ is KOIDE-CONSISTENT $\star\star\star\star$ under external $K = 2/3$ via the QED-corrected Koide chain (A636, A637). The content below is retained as the Rev8 historical record; it does not represent the current programme state.

The weak-angle bridge is a standard field-theory problem: the exceptional algebra supplies a tree-level ratio, and quantum field theory supplies running, scheme choice, and threshold corrections. Rev8 states that bridge openly. It also extends the same honesty to the electron-mass closure problem. The current baseline formula is structurally sharp, but the residual percent-level phase system is not yet derived from the Lagrangian. This paper exists to keep those two bridges conceptually separate.

2 Standard one-loop running

At one loop the gauge couplings satisfy

$$\mu \frac{dg_i}{d\mu} = \frac{b_i}{16\pi^2} g_i^3, \qquad (b_1, b_2, b_3) = \left(\frac{41}{10}, -\frac{19}{6}, -7\right) \tag{2}$$

— the Standard-Model one-loop coefficients, imported physics (tier INPUT; Rev32.9) — in the GUT-normalised convention (SU(5)-embedding normalisation of $U(1)_Y$, used throughout the D=5 framework; the alternative $b_1 = 41/6$ is the SM-normalised value and is *not* used here — Appendix D confirms $b_1 = 41/10$). Equivalently,

$$\frac{d\alpha_i^{-1}}{d\ln\mu} = -\frac{b_i}{2\pi}. \tag{3}$$

2.1 RGE preservation of Peirce structure

The resolvent family

$$R_d = \frac{1}{\phi^{2d} - 1} \tag{4}$$

remains stable under Standard Model running from the ultraviolet matching scale to M_Z. The leading anomalous dimensions are flavour-blind within a fixed representation, so the dominant corrections cancel in the ratios. Rev8 therefore continues to carry the ratio sector as structurally stable under ordinary one-loop running.

Artin bridge and four-point exposure [structural, A720]. By Artin's theorem, every two-generator subalgebra of $\mathbb{O}_s$ is associative. Hence ordinary 2-point and 3-point QFT corrections — propagators, self-energies, Yukawa vertices, gauge vertices, and one-loop beta functions — are computed by the standard associative $\overline{\mathrm{MS}}$ rules. The first place where split-octonion non-associativity can enter is a four-point/box topology. A bounded power-counting audit (S63) gives $\delta_{\mathrm{NA}}^{(4)} \sim (g^2/16\pi^2)^2\, L$ with an $O(1)$ associator coefficient. For the QED-corrected muon chain this is $6 \times 10^{-5}\,\%$, negligible against the quoted m_μ and m_e precisions; heavy-quark matching and the weak-angle structural row are likewise protected. Long-log running from M_{Pl} to M_Z gives $\sim 0.35\%$, so sub-percent RGE-stability statements are not sharpened below that level without an explicit two-loop/box audit. The one

external observable with discovery or exclusion power is the muon anomalous moment: a light-by-light-class associator insertion of $O(1)$ coefficient would contribute $\sim 125 \times 10^{-11}$, already in tension with the 2025 experiment–theory window of $38(63) \times 10^{-11}$. The normalization is FIXED to this LbL class (A725): the colored lines are confined at Λ_{G_2}, so the distortion runs at the hadronic loop scale with no high-scale decoupling — an independent dipole-class $m_\mu/\Lambda^2_{\rm UV}$ suppression is precluded by the framework's own architecture. The insertion scales as $C\,(\alpha/\pi)^3 (m_\mu/\Lambda_{G_2})^2\,\mathcal{I}_{\rm loop} \approx C \times 125 \times 10^{-11}$ (chirality-preserving; the flip is supplied by m_μ exactly as in standard hadronic light-by-light). No headline claim of this suite is shifted at its quoted precision by four-point non-associativity.

Associator $g-2$ null test [proved, A721, S63]. The $\{\mu, \gamma\}$ subsystem lies in the two-generator singlet subalgebra $\langle e_0, e_7\rangle$ of the verified field assignment (A688/A689), which is associative by Artin's theorem. Pure lepton–photon contributions to a_μ are therefore free of split-octonion associator insertions *to all orders*: the framework predicts $a_\mu = a_\mu^{\rm SM}$ in this sector. Non-associative exposure can enter only through colored internal lines, as a distortion of the hadronic light-by-light class. With the 2025 Theory Initiative values $a_\mu^{\rm exp} - a_\mu^{\rm SM} = 38(63) \times 10^{-11}$ and $a_\mu^{\rm HLbL} = 115.5(9.9) \times 10^{-11}$, current data imply $|C^{\rm HLbL}_{\rm assoc}| \lesssim 0.55$ conservatively, or $\lesssim 0.086$ within the present HLbL theory budget; with the LbL normalization fixed (A725), a factor-2 window improvement tightens this to $|C| \lesssim 0.25$, and a stabilized null $\Delta a_\mu = 0 \pm 15 \times 10^{-11}$ would give $|C| \lesssim 0.12$. A future confirmed *non-hadronic* a_μ anomaly would falsify the associative lepton–photon embedding (Paper 7).

3 Weak-angle bridge

Historical Rev8 content. The weak-angle value in this section (3/13) is superseded by the Rev29 operative value $\phi^{-3} = \sqrt{5} - 2 \approx 0.23607$. ***The current (S55, A697; corrected S124) reading is that*** ϕ^{-3} ***is a*** $\sim$2% **structural** ***low-energy target lying on the effective*** $\sin^2\theta_W(\mu)$ ***running curve at a sub-GeV/GeV-order, scheme- and threshold-dominated crossing (Appendix I §B4; the once-quoted*** $\mu_\star \approx 1.8\,\text{GeV}$ ***lies outside the band) — not a Higgs-VEV-scale boundary condition and not a*** Z***-pole prediction. Paper 4 §6 is authoritative; the Higgs-scale /*** 246 GeV ***language retained below is superseded.*** *See Paper 4 §6.*

The tree-level weak-angle output carried by the suite is

$$\sin^2\theta_W = \frac{3}{13} = 0.2308. \tag{5}$$

This is a tree-level algebraic ratio. The measured Z-pole weak angle is a renormalized quantity obtained only after running, threshold corrections, and scheme choice.

Weak-angle bridge. The algebraic weak-angle ratio is a tree-level output. The measured Z-pole quantity requires renormalization, threshold corrections, and scheme matching. Rev8 treats this as a standard QFT bridge problem rather than as a falsification of the tree-level algebraic relation.

4 Threshold sector

The working threshold picture places heavy Peirce-sector modes near

$$M_{\rm th} \sim 5\ \text{TeV} \tag{6}$$

with order-of-magnitude effects

$$\delta\alpha_1 \sim 10^{-3}, \qquad \delta \sin^2\theta_W \sim \mathcal{O}(10^{-4}). \tag{7}$$

These are placeholder estimates — threshold calculation not performed. The sign, scale, and qualitative mechanism are identified, but the detailed spectrum and loop integral are deferred.

$\sin^2\theta_W$ precision status (S55, A697 — corrected reading). The algebraic value $\sin^2\theta_W = \phi^{-3} = \sqrt{5}-2 \approx 0.23607$ is a $\sim$2% *structural* target lying on the standard running curve of the *effective* weak mixing angle, reached at a sub-GeV/GeV-order, scheme- and threshold-dominated crossing (corrected S124/Appendix I §B4 — pushed sub-GeV, $\mu_{\rm cross} \sim 0.15\,\text{GeV}$; the once-quoted 1.8 GeV lies outside the band); the difference to the Z-pole value 0.23122 ± 0.00006 (PDG 2026 EW review, $\overline{\rm MS}$ at M_Z; the $\sim$2% structural fence stays binding — no σ conversion) is then ordinary running in the right direction. The matching scale is *not* derived from the algebra and the scheme is not fixed, so ϕ^{-3} is a *structural target, not a completed Z-pole prediction* and *not* a theorem at M_Z. (The earlier high/VEV-scale downward-running test and the associated precision-failure/theorem language are superseded; Paper 4 §6 is authoritative.) AX6 is an independent orbit selector, decoupled from the value ϕ^{-3}.[1]

[1] Tracker-tier note (Rev25 fold; `s539`, S210; flagged crude): a one-loop exercise finds the $\overline{\rm MS}$ running $\hat{s}^2_W(\mu)$ crossing ϕ^{-3} near $\mu \approx 237$ GeV at the 0.03% level. It is a one-loop, scheme-naive computation and is recorded as a curiosity only — it does not alter the corrected sub-GeV effective-angle reading above, and no scale is derived.

4.1 Route C: heavy X-boson threshold mechanism

SUPERSEDED (S59–S60, A713/A714) — retained for the record, not operative. (Rev32.1 note: the σ figures quoted below are S59-era bare cross-scheme numbers, themselves retyped as comparator distances by A1531/T2 — see Appendix X.) This subsection presented the 16 eaten Goldstone bosons of $\mathrm{Spin}(4,4) \to \mathrm{Spin}(1,3) \times \mathrm{Spin}(3,1)$ as a leading-log threshold mechanism that closes the heavy-quark mass tensions ($\Delta_t = +\varepsilon$, $\Delta_b = -\varepsilon$). That operative reading is now *retired*: (i) **A713 (S59)** established that the heavy-quark comparator offsets (m_t : $+0.87\%$, $d_{\mathrm{cmp}} = +5.57$; m_b : -0.95%, $d_{\mathrm{cmp}} = -6.64$ (full precision, Rev32.9); m_c : $+1.49\%$, $d_{\mathrm{cmp}} = +4.22$; PDG 2026, all cross-scheme and non-status-setting) are genuine unresolved offsets and are *not* produced by standard RG/threshold matching — the "these look like ordinary 1-loop matching corrections" framing is *falsified* (top pole/$\overline{\mathrm{MS}}$ moves the wrong way; m_b/m_c only fit on a tuned scale). (ii) **A714 (S60)** proved that the SUGRA scalar-count gap 16 ($42-26$) and this $\mathrm{Spin}(4,4)$ Goldstone 16 ($28-12$) are *not* the same object (the equality is a dimensional coincidence; see Paper 4 §4.2), removing the proposed carrier. The group-theory facts below ($28 - 12 = 16$, the triality Casimir $C_X = 4$) remain correct *as representation theory*, but they may *not* be invoked as the operative source of the heavy-quark threshold. Any genuine threshold carrier must come from an explicit interaction sector, not from this Goldstone count. The arithmetic is preserved here for traceability only. (iii) **A715–A717 (S61–S62)** subsequently closed that explicit-sector search as well: the residual pattern carries a unique weak-isospin T_3 signature ($\delta m_f/m_f = -\kappa T_3(f)$, $\kappa \simeq 1.824\%$), but no $J_3(\mathbb{O}_s)$-intrinsic operator forces it — the weak-adjoint Higgs spurion is writable; its relative up/down sign is forced only through a leg-local deformation along the registered involution P (`s1218`), whose existence, one sign and $|\kappa|$ stay by hand (`s1349`; A716 for the objects that supply nothing) — and a boundary-condition revision reduces no parameter (A717). The heavy-quark sector is carried as bare algebraic boundary values plus a phenomenological structural T_3 threshold term (κ, χ_c not derived; its value depends on the κ normalisation); see Paper 2 §3.6 and Paper 4 §4.2.

Rev12 update (Sessions 48–49) [superseded by A713/A714, S59–S60]. The threshold sector now has a complete leading-log EFT mechanism. The Goldstone counting from the coset $\mathrm{Spin}(4,4)/[\mathrm{Spin}(4) \times \mathrm{Spin}(4)]$ yields exactly $28 - 12 = 16$ eaten Goldstone bosons, giving 16 heavy X-vectors with a universal mass matrix $M_X^2 = g_X^2 v_X^2 \mathbf{1}_{16}$ [PROVED, A669].

The $\mathrm{Spin}(4,4)$ Casimir in the spinor representation is [PROVED, A670 kernel]:

$$C_X(8_v) = C_X(8_+) = C_X(8_-) = 4, \tag{8}$$

a consequence of D_4 triality. With the quark doublets assigned to $J_{13} = 8_+$ (see Section 4.3), the Dynkin index is

$$\lambda_t^X = \lambda_b^X = C_X(t_L) + C_X(t_R) = 4 + 4 = 8. \tag{9}$$

The leading-log threshold corrections to the top and bottom pole masses are:

$$\Delta_f^X = -\frac{3\lambda_f^X}{16\pi^2}\, g_X^2\, \ln\frac{M_X}{\mu^\star} \tag{10}$$

$$\Delta_t^y = +\frac{a_y\, y_t^2}{16\pi^2}\, \ln\frac{M_y}{\mu^\star}, \qquad y_t = 1, \quad a_y = \tfrac{9}{2}. \tag{11}$$

The sign of Δ_f^X is negative by the $\overline{\text{MS}}$ matching-counterterm convention [CONVENTION, A673]; it is *not* derived from the $(2-d)$ propagator numerator. The sign of Δ_t^y is positive (Yukawa mass upshift). With $c_X = 3\lambda_X = 24$:

Route C result [proved arithmetic, A673].

$$g_X^2 \ln\frac{M_X}{\mu^\star} = 0.05836226069, \qquad \ln\frac{M_y}{\mu^\star} = 0.6225307807 \quad (a_y = \tfrac{9}{2}).$$

$$\Delta_t^X = \Delta_b^X = -\varepsilon, \qquad \Delta_t^y = +2\varepsilon, \qquad \varepsilon \approx 0.00887.$$

$$\boxed{\Delta_t = +\varepsilon, \qquad \Delta_b = -\varepsilon.}$$

The sign split arises from the large top-Yukawa threshold ($y_t = 1$); the bottom analogue is suppressed by $y_b^2/y_t^2 \approx 6 \times 10^{-4}$. The $16X$ gauge threshold is *universal* for top and bottom; it does *not* arise from different Peirce block assignments.

Status table:

Claim	Value	Status
$C_X(8_\pm) = C_X(8_v)$	4	PROVED
$\lambda_t^X = \lambda_b^X$	8	PROVED
a_y	9/2 (SM RGE)	CONVENTION/IMPORT
Threshold signs	$\overline{\text{MS}}$ counterterm	CONVENTION
$\Delta_t = +\varepsilon$, $\Delta_b = -\varepsilon$	kernel-exact	PROVED ARITHMETIC
$g_X^2 \ell_X$ algebraic origin	open	OPEN
$a_y \cdot \ell_y = 2.801$ algebraic origin	open; lead: $\ell_y \approx \phi^{-1}$ (-0.72%, A678)	OPEN

4.2 Peirce–Yukawa selection theorem

Rev29 update (Session 49). The following theorem is proved for generic split-octonion elements $\psi, h \in \mathbb{O}_s$ (all 8 components; kernel-verified, A674).

Let

$$J_{\text{vac}} = \lambda_1 c_1 + \lambda_2 c_2 + \lambda_3 c_3 = \phi\, c_1 + c_2 + \phi^{-1} c_3.$$

For $X, Y \in J_{ij}$, write their split-octonion entries as $\psi, h \in \mathbb{O}_s$, and let k be the remaining Peirce index. The Freudenthal cross product satisfies

$$X \times Y = -2\,\mathrm{Re}(\psi\bar{h})\, c_k. \tag{12}$$

Consequently

$$\mathrm{Tr}[J_{\text{vac}} \circ (X \times Y)] = -2\lambda_k\, \mathrm{Re}(\psi\bar{h}). \tag{13}$$

For distinct Peirce blocks $J_{ij} \neq J_{kl}$, the cross product lands in an off-diagonal Peirce block, hence

$$\mathrm{Tr}[J_{\text{vac}} \circ (J_{ij} \times J_{kl})] = 0. \tag{14}$$

The Peirce eigenvalues of J_{vac} give the tree-level Yukawa strengths:

$$J_{13} \times J_{13} \mapsto -2\lambda_2 = -2, \qquad J_{23} \times J_{23} \mapsto -2\lambda_1 = -2\phi, \qquad J_{12} \times J_{12} \mapsto -2\lambda_3 = -2\phi^{-1}. \tag{15}$$

All nine trace contractions verified symbolically (differences= 0); see kernel `d674_peirce_yukawa_generic_S49.py`.

Peirce–Yukawa correspondence [proved, A674]. Each same-block Peirce product $J_{ij} \times J_{ij}$ extracts the vacuum eigenvalue λ_k of the *missing* idempotent c_k. The three Peirce eigenvalues (ϕ, 1, ϕ^{-1}) are the three allowed tree-level Yukawa couplings in the direct Freudenthal channel.

4.3 Fermion placement in $J_{13} = 8_+$

Rev12 update (Sessions 48–49). The selection rule (14) constrains right-handed fermion placement.

Top-quark placement (carrier channel). In the same-block *carrier* reading (A674; the literal SM top Yukawa is the cross-block triple $\{J_{12}(H_u), J_{13}(Q), J_{23}(u^c)\}$ with $H_u \in \mathbf{10}$, A686–A688), one has $Q_L = (t_L, b_L) \in J_{13}$ and a carrier field $\Phi_H \in J_{13}$, and the carrier coupling

$$\mathcal{L}_t \propto \mathrm{Tr}[J_{\mathrm{vac}} \circ (\Psi_t \times \Phi_H)] = -2\,\mathrm{Re}(\psi_t \bar{h})$$

is nonzero if and only if $t_R \in J_{13} = 8_+$. A cross-block assignment $t_R \in J_{23}$ gives $\mathrm{Tr}[J_{\mathrm{vac}} \circ (J_{23} \times J_{13})] = 0$ (identically, for all $\psi, h \in \mathbb{O}_s$).

$$\boxed{t_R \in J_{13} = 8_+ \quad [\textsc{proved}, A674]} \tag{16}$$

Bottom-quark placement. Since the conjugate Higgs $\widetilde{\Phi}_H = i\sigma_2 \Phi_H^*$ is an anti-linear weak-doublet operation within the J_{13} subspace (octonionic conjugation preserves the Hermitian block structure; the $\mathrm{SU}(2)_L$ operation acts inside J_{13}), we have $\widetilde{\Phi}_H \in J_{13}$ [STRUCTURAL, A675]. The same selection rule then requires

$$\boxed{b_R \in J_{13} = 8_+ \quad [\textsc{proved conditional}, A675].} \tag{17}$$

This placement is *conditional* on the direct Freudenthal bottom Yukawa; non-direct insertion mechanisms are not ruled out. With $t_R, b_R \in J_{13}$, the $16X$ Casimir is universal ($C_X(t_R) = C_X(b_R) = 4$, $\lambda_t^X = \lambda_b^X = 8$) and the Route C sign split is produced by the Yukawa asymmetry $y_t = 1 \gg y_b \approx 0.024$, not by different block assignments.

The bottom Yukawa coupling in the J_{13} channel reads

$$\mathcal{L}_b \propto \mathrm{Tr}\Big[J_{\mathrm{vac}} \circ (\Psi_b \times \widetilde{\Phi}_H)\Big]_{\eta_b} = -2\eta_b\,\mathrm{Re}(\psi_b \overline{\widetilde{h}}), \qquad \eta_b = -1.$$

The phase $\eta_b = -1$ is currently a phase-convention/alignment datum [OPEN/ASSUMPTION, A675]; it is not derivable from octonionic conjugation, Jordan conjugation, or any Z_2 automorphism of J_{13} alone.

Bottom convention [convention, A678]. The $J_{13,\mathbb{C}}$ colour decomposition is

$$J_{13,\mathbb{C}} \to \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{3} \oplus \bar{\mathbf{3}}.$$

We use the convention that the **3** component represents the algebraic top singlet t_R, while the conjugate colour component represents the left-handed conjugate bottom field b_R^c. The physical

right-handed bottom field is $b_R = (b_R^c)^c$, appearing through the Hermitian-conjugate term in the four-dimensional Lagrangian. This convention resolves the 8-real-dimensional counting problem and is consistent with $C_X(8_+) = 4$ and $\lambda_b^X = 8$ [PROVED, A670/A677, A678]: the Casimir is a Spin(4, 4) representation-theory quantity, unchanged by field redefinition (kernel-verified, A678). The down-type phase $\eta_b = -1$ is *not* derived by this convention; it remains [OPEN/ASSUMPTION, A675].

Hypercharge embedding. With both t_R and b_R in $J_{13} = 8_+$, the hypercharge distinction $Y(t_R) = +2/3$ vs $Y(b_R) = -1/3$ requires $U(1)_Y$ to act as an internal generator or projector within the physical subspace of 8_+, rather than as a scalar on the whole block. The derivation of this projector is addressed in Section 4.4.

4.4 Hypercharge inside the $J_{13} = 8_+$ block

The direct Freudenthal Yukawa channel places both right-handed quarks in the same Peirce block, $t_R, b_R \in J_{13} = 8_+$. This does not imply that hypercharge acts as a scalar on J_{13}. If it did, then t_R and b_R would have identical hypercharge. Hence any viable embedding must realize $U(1)_Y$ as a non-scalar internal generator or projector acting within the physical subspace extracted from 8_+.

For the complexified D_4 weight lattice, the 8_+ weights are

$$8_+ : \quad \tfrac{1}{2}(\pm e_1 \pm e_2 \pm e_3 \pm e_4), \qquad \#\{\text{minus signs}\} \equiv 0 \pmod 2. \tag{18}$$

(Kernel-verified, `d676_hypercharge_projector_S49.py`.) The weight set is *centrally symmetric*: $w \mapsto -w$ maps 8_+ to itself. Therefore every Cartan charge spectrum on 8_+ is symmetric under $q \mapsto -q$. The SM-like charge multiset

$$(3, 1, +\tfrac{2}{3}) \oplus (3, 1, -\tfrac{1}{3}) \oplus (1, 2, -\tfrac{1}{2})$$

is *not* $q \mapsto -q$ symmetric, and therefore cannot arise as a single 8_+ Cartan branching [PROVED OBSTRUCTION, A676].

Furthermore, the physical compact group $SU(3)_c \times SU(2)_L \times U(1)_Y$ is not a compact subgroup of Spin(4, 4): the maximal compact subgroup of Spin(4, 4) is $\mathrm{Spin}(4) \times \mathrm{Spin}(4) \simeq SU(2)^4$, and no simple $\mathfrak{su}(3)$ subalgebra (dim 8) can embed in $\mathfrak{su}(2)^4$ (each factor dim 3) [PROVED, A676]. The diagonal Peirce operators J_{vac}, $L_{J_{\mathrm{vac}}}$, and Π_{13} act as scalars within J_{13} and are incapable of separating t_R from b_R by hypercharge [PROVED/STRUCTURAL, A676].

A chosen Cartan functional can distinguish individual 8_+ weight-level charges. For example, $H_Y = (\frac{1}{2}, \frac{1}{2}, \frac{1}{6}, \frac{1}{6})$ assigns $Y(w_0) = +2/3$ and $Y(w_6) = -1/3$ (kernel-verified). However this direction is *chosen*, not derived, and the full SM charge multiplicities cannot be realized by any single Cartan direction (centrally symmetric obstruction above).

We therefore introduce the SM gauge embedding as an explicit structural axiom.

Hypercharge embedding datum (earlier A676/A677 form — superseded). *This paragraph records the pre-B structural datum (formerly "Axiom HY1"); it is superseded by the $B + SU(5)$ hypercharge derivation later in this section, where all sixteen hypercharges are derived rather than assumed. It is retained only to motivate the SM embedding chain.* The Standard Model hypercharge generator was taken as a chosen compact generator $Y \in \mathfrak{f}_{4(4)}$ whose action on the physical subspace of $J_{13} = 8_+$ has eigenspaces

$$Y\Pi_{t_R} = +\tfrac{2}{3}\,\Pi_{t_R}, \qquad Y\Pi_{b_R} = -\tfrac{1}{3}\,\Pi_{b_R}. \tag{19}$$

The projectors Π_{t_R} and Π_{b_R} are internal projectors inside the physical 8_+ extraction, consistent with the SM gauge embedding

$$\iota_{\rm SM} : SU(3)_c \times SU(2)_L \times U(1)_Y \hookrightarrow F_{4(4)}. \tag{20}$$

This datum is required because the diagonal Peirce operators act as scalars on J_{13} (Section 4.3).

Octonionic colour decomposition (A677). The Peirce entry $x \in J_{13} \cong \mathbb{O}_s$ has 8 real components. Choosing a distinguished compact imaginary unit e_7, right multiplication R_{e_7} defines a compact complex structure on the six-real complement $W_6 = \mathrm{span}_{\mathbb{R}}\{e_1, \ldots, e_6\}$ (kernel-verified for compact $\mathbb{O}$; $R_{e_7}^2 = -1$). The $+i$ eigenbasis

$$u_1 = e_1 - ie_6, \quad u_2 = e_2 + ie_5, \quad u_3 = e_3 + ie_4$$

spans the colour triplet, giving the complexified decomposition

$$J_{13,\mathbb{C}} \;\to\; \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{3} \oplus \bar{\mathbf{3}} \quad \text{under } SU(3)_c.$$

The $\mathbf{3} \oplus \bar{\mathbf{3}}$ is the complexification of *one* six-real colour module, not two independent colour triplets. This resolves the dimensional puzzle: a single 8-real J_{13} block cannot contain both a full t_R and a full b_R as independent colour triplets; instead $t_R \sim \mathbf{3}$ and $b_R^c \sim \bar{\mathbf{3}}$ share the same colour sector. For the split algebra $\mathbb{O}_s$, compact $SU(3)_c$ is not a subgroup of split $G_{2(2)}$ (five of eight compact $\mathfrak{su}(3)$ generators fail as $\mathbb{O}_s$-derivations, kernel A677). Physical compact colour is therefore placed in $USp(6) \subset K_{F_4} = USp(6) \times USp(2)$.

Explicit $\iota_{\rm SM}$ chain (A677, kernel-verified). The embedding chain

$$SU(3)_c \times SU(2)_L \times U(1)_q \hookrightarrow U(3) \times USp(2) \hookrightarrow USp(6) \times USp(2) \subset F_{4(4)} \tag{21}$$

is realised at the Lie-algebra level by

$$A \in \mathfrak{su}(3) \;\mapsto\; \begin{pmatrix} A & 0 \\ 0 & -A^T \end{pmatrix} \in \mathfrak{usp}(6),$$

which satisfies $X^\dagger + X = 0$ and $X^T\Omega + \Omega X = 0$ for all eight $\mathfrak{su}(3)$ generators (kernel-verified, max error $= 0$). This gives $\mathbf{6}_{USp(6)} \to \mathbf{3}_q \oplus \bar{\mathbf{3}}_{-q}$ and the 26-dimensional branching

$$\mathbf{26}_{F_4} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{3},\mathbf{1})_{-2q} \oplus (\bar{\mathbf{3}},\mathbf{1})_{+2q} \oplus (\mathbf{3},\mathbf{2})_{+q} \oplus (\bar{\mathbf{3}},\mathbf{2})_{-q}.$$

The branching contains SM-like colour-isospin structure, but the charge normalization q and the physical projectors Π_{t_R}, Π_{b_R} are not fixed by the Jordan data. The embedding $\iota_{\rm SM}$ is therefore [STRUCTURAL/ASSUMPTION]—the chain is natural and explicit but not canonically derived. Note: Todorov–Drenska (arXiv:1805.06739) obtain the SM gauge group as $Spin(9) \cap (SU(3) \times SU(3)/\mathbb{Z}_3) \subset F_4$ using the *compact* F_4; that intersection route is distinct from the $USp(6) \times USp(2)$ maximal-compact route of $F_{4(4)}$ used here.

Hypercharge status (updated A689). The earlier datum (Eq. (19)) had status [STRUCTURAL/ASSUMPTION] as of A677, but is now *superseded*: all sixteen Standard Model hypercharges are DERIVED from the $B + SU(5)$ exterior-index assignment (A688/A689; see below). The operative statement is therefore $Y_{\rm SM}$ [DERIVED under explicit B], with the only residual freedom the canonical selection of the B-map orientation [OPEN]. "Axiom HY1" is accordingly retired as an independent axiom. The explicit embedding chain (21) has been kernel-verified, and the Route C Casimir $C_X(8_+) = 4$ (Eq. (8)) is independent of colour sublabelling and remains [PROVED] (A670, A677, A678).

Lepton embedding status [open, A678/A679]. For all values of the charge unit q ($q = \frac{1}{6}, \frac{1}{3}, \frac{1}{2}$), the $\mathbf{26}_{F_4}$ branch above contains no color-singlet weak doublet $(\mathbf{1},\mathbf{2})_Y$ and no charged-lepton singlet $(\mathbf{1},\mathbf{1})_{-1}$ (kernel-verified, A678). The SM lepton projectors are therefore not fixed by the present $\iota_{\rm SM}$ chain and remain an open structural datum. This is *independent* of the proved charged-lepton mass-ratio formulas (Paper 2, A616/A618/A636/A637).

Status of the $E_{6(6)}$ extension [structural, A679]. The reduced structure group of $J_3(\mathbb{O}_s)$ is $\mathrm{Str}_0(J_3(\mathbb{O}_s)) \simeq E_{6(6)}$, and $J_3(\mathbb{O}_s)$ is its real 27-dimensional minimal representation. At the complexified E_6 level, the standard GUT branching $27 \to 16 \oplus 10 \oplus 1$ contains one full SM generation, Q, u^c, d^c, L, e^c, ν^c, inside the $SO(10)$ spinor **16**. However, the compact $SO(10) \times U(1)$ chain does not transfer as a real subgroup chain of $E_{6(6)}$; the split-form analogue is $SO(5,5) \times SO(1,1) \subset E_{6(6)}$ (kernel-verified, A679). Under the current $F_{4(4)}$-nested branch $27 \downarrow F_{4(4)} = 26 \oplus 1$, the extra singlet carries only $(1,1)_0$ quantum numbers. The $E_{6(6)}$ route is [STRUCTURAL]: it supplies the right ambient representation, but an explicit intertwiner mapping the complex E_6 lepton slots to the suite's Peirce quark embedding is not yet derived. The $SO(5,5) \times SO(1,1)$ sub-chain, under which $27 \to 16 + 10 + 1$ with the **16** a split-real spinor, is the next candidate; see below.

The $\mathrm{SO}(5,5)$ spinor branch [structural, A681]. The split reduced-structure group $E_{6(6)} = \mathrm{Str}_0(J_3(\mathbb{O}_s))$ contains the split subgroup $\mathrm{SO}(5,5) \times \mathrm{SO}(1,1)$, under which the real 27 branches as (Hohm–Samtleben exceptional field theory)

$$27_{\mathbb{R}} \to 16_{\mathbb{R}} \oplus 10_{\mathbb{R}} \oplus 1_{\mathbb{R}}.$$

The $16_{\mathbb{R}}$ is a real Majorana–Weyl spinor of $\mathrm{Spin}(5,5)$ (Clifford-algebra kernel-verified, A681). After complexification and a chosen $\mathrm{SU}(5) \to SU(3)_c \times SU(2)_L \times U(1)_Y$ embedding, one obtains

$$16_{\mathbb{C}} \to (\mathbf{3},\mathbf{2})_{1/6} \oplus (\bar{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\mathbf{1},\mathbf{1})_{+1} \oplus (\bar{\mathbf{3}},\mathbf{1})_{+1/3} \oplus (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{1})_0.$$

Thus the missing lepton doublet $L = (\mathbf{1},\mathbf{2})_{-1/2}$ and charged-lepton singlet $e^c = (\mathbf{1},\mathbf{1})_{+1}$ are present in the complexified $\mathrm{SO}(5,5)$ spinor branch. However, compact $SU(3)_c$ is not contained in the maximal compact $\mathrm{Spin}(5) \times \mathrm{Spin}(5)$ of $\mathrm{Spin}(5,5)$ [PROVED OBSTRUCTION, A681]; the result is [STRUCTURAL] until the required complex structure and the Peirce-to-SM intertwiner are derived.

Peirce-sector compatibility via $\mathrm{Spin}(4,5)$ [structural, A681]. The compatibility with the $F_{4(4)}$ quark sector is controlled by the overlap

$$F_{4(4)} \cap SO(5,5) \simeq \mathrm{Spin}(4,5), \qquad \dim = 36.$$

Under this overlap the 26_{F_4} branches as $16 \oplus 9 \oplus 1$, and restricting to the previously used $\mathrm{Spin}(4,4) \subset \mathrm{Spin}(4,5)$ gives

$$16 \to 8_+ \oplus 8_-, \qquad 9 \to 8_v \oplus 1,$$

hence

$$26_{F_4} \to J_{13} \oplus J_{23} \oplus J_{12} \oplus 1 \oplus 1 = 8_+ \oplus 8_- \oplus 8_v \oplus 1 \oplus 1,$$

matching the Peirce blocks plus the two traceless diagonal directions. The proved placement $t_R \in J_{13} = 8_+$ is therefore compatible with the $\mathrm{SO}(5,5)$ spinor route. The outstanding gap is the explicit intertwiner

$$J_{13} \oplus J_{23} = 8_+ \oplus 8_- \;\longleftrightarrow\; 16_{\mathbb{C}}^{SU(5)} = Q \oplus u^c \oplus d^c \oplus L \oplus e^c \oplus \nu^c.$$

Constructing this map was the target of D682; the result is below.

Peirce–SM intertwiner status [open, A682]. Using the exterior-algebra model $S_+ = \Lambda^{\text{even}}\mathbb{R}^5$, the Spin(4, 4) chirality operator $\Gamma_{4,4} = \Gamma_0\Gamma_1\Gamma_2\Gamma_3\Gamma_5\Gamma_6\Gamma_7\Gamma_8$ splits $S_+ = 8_+ \oplus 8_-$ [PROVED, A682]. The computed eigenspaces are

$$8_+ : \nu^c \oplus u^c \oplus Q_{\text{weak},3} \oplus L_{\text{weak},3}, \qquad 8_- : Q_{\text{weak},4} \oplus e^c \oplus L_{\text{weak},4} \oplus d^c.$$

However, the $\mathrm{SU}(2)_L$ generators (E_{34}, E_{43}) map $8_+ \leftrightarrow 8_-$: the quark and lepton doublets Q and L are each split across the two chiralities [PROVED OBSTRUCTION, A682]. Thus the naive $\Gamma_{4,4}$ split does not preserve complete $\mathrm{SU}(2)_L$ multiplets and cannot yet be identified with $J_{13} \oplus J_{23}$. The Peirce–Yukawa theorem and the proved placement $t_R \in J_{13} = 8_+$ are unaffected by this obstruction. Three resolution paths were explored in D683A–C.

Coordinate $SU(5)$ tradeoff theorem [proved, A683C]. An exhaustive search over all ten weak-pair/color-triple index assignments in $S_+ = \Lambda^{\text{even}}\mathbb{C}^5$ establishes:

$$4 \notin W \;\Rightarrow\; [E_W, \Gamma_{4,4}] = 0 \text{ (weak doublets intact)}, \quad \text{but} \quad [E_{C4}, \Gamma_{4,4}] \neq 0 \text{ (color triplets split)}. \tag{22}$$

$$4 \in W \;\Rightarrow\; [E_C, \Gamma_{4,4}] = 0 \text{ (color triplets intact)}, \quad \text{but} \quad [E_{W4}, \Gamma_{4,4}] \neq 0 \text{ (weak doublets split)}. \tag{23}$$

No coordinate $SU(5)$ permutation preserves both $SU(3)_c$ triplets and $SU(2)_L$ doublets inside the chirality eigenspaces simultaneously. The D682 obstruction (weak doublets split) cannot be removed by a simple relabelling of the five exterior-algebra indices; it is merely traded for a color-triplet splitting. A non-coordinate intertwiner is required.

The two remaining open paths are: (A) the Peirce projector L_{c_1} (requires explicit $J_3(\mathbb{O}_s) \hookrightarrow S_+$ basis map, D683A/D684); and (B) the J_{vac}-stabilizer Spin(4, 4) chirality, whose correct explicit form has not yet been computed (D683B/D684).

The $SO(5,5)$ branch and the Peirce singlet [proved, A684]. The split $E_{6(6)}$ branching

$$27 \downarrow SO(5,5) \times SO(1,1) \;=\; 16_{-1} \oplus 10_{+2} \oplus 1_{-4}$$

admits three conjugate realizations, one per primitive idempotent. For the c_3-selected branch:

$$16 = J_{13} \oplus J_{23}, \qquad 10 = J_{12} \oplus \mathbb{R}c_1 \oplus \mathbb{R}c_2, \qquad 1 = \mathbb{R}c_3.$$

This matches the suite's established assignments ($t_R \in J_{13}$, J_{12} for CKM, J_{23} for gauge/CP). Note: the singlet is the primitive idempotent c_3, *not* the trace direction $c_1 + c_2 + c_3$. The trace singlet arises in the different decomposition $27 \downarrow F_{4(4)} = 26 \oplus 1$; these are distinct subspaces. The nilpotent blocks J_{13} and J_{23} are exactly the fermionic carriers "connected to c_3" via the Peirce eigenvalue condition $c_3 \circ X = \frac{1}{2}X$ for $X \in J_{13}$ or J_{23}.

Intertwiner status and dimension obstruction [open, A684]. Abstractly, the Peirce chirality on the 16 is

$$\Gamma^{\text{Peirce}} = \Pi_{13} - \Pi_{23}, \qquad (\Gamma^{\text{Peirce}})^2 = 1.$$

To write this operator in the exterior-algebra model $S_+ = \Lambda^{\text{even}}\mathbb{R}^5$ one needs the explicit basis intertwiner $B : J_{13} \oplus J_{23} \to \Lambda^{\text{even}}\mathbb{R}^5$. Non-coordinate involutions commuting with $SU(3)_c \times SU(2)_L$ do exist (kernel-verified, A684): for example $\Gamma_{\text{SM}} = +1$ on $\{Q_L, L\}$ and -1 on $\{u^c, d^c, e^c, \nu^c\}$. However, a dimensional obstruction limits the interpretation of J_{13} as an SM-invariant carrier:

$$\dim Q_L + \dim u^c = 6 + 3 = 9 > 8,$$

so no 8-dimensional SM-invariant subspace can contain both a full quark doublet Q_L and a full up-type singlet u^c. Consequently J_{13} is best understood as a *Peirce/Freudenthal Yukawa carrier*—the algebraic channel through which the direct Yukawa coupling is selected [PROVED, A674]—rather than as an SM-representation container.

Explicit SM-commuting Clifford chirality [proved, A685]. The first explicitly proved SM-commuting $8+8$ chirality in the exterior-algebra model is

$$\Gamma_{QL} := \Gamma_3\Gamma_4\Gamma_8\Gamma_9, \qquad \Gamma_{QL}^2 = 1, \qquad [E_{\mathrm{SM}}, \Gamma_{QL}] = 0$$

for all pure $SU(3)_c \times SU(2)_L$ generators (kernel-verified, A685). Its eigenspaces realise a left–right split:

$$S_+^{(+)} = Q_L \oplus L \quad (\text{all } SU(2)_L \text{ doublets}), \qquad S_+^{(-)} = u^c \oplus d^c \oplus e^c \oplus \nu^c \quad (\text{all } SU(2)_L \text{ singlets}). \tag{24}$$

This operator evades the coordinate $SU(5)$ tradeoff theorem (A683C) because it is non-coordinate. Identifying $\Gamma^{\mathrm{Peirce}} = \Gamma_{QL}$ is a structural choice [STRUCTURAL]; it is not yet derived from $J_3(\mathbb{O}_s)$.

The correct next target is not a one-block SM-irrep identification but a *trilinear projection theorem*: within the full spinor $J_{13} \oplus J_{23}$, one seeks the coupling

$$J_{13} \times J_{13} \;\longrightarrow\; Q_L \cdot \Phi_H \cdot u^c$$

such that $Q_L \in S_+^{(+)}$ and $u^c \in S_+^{(-)}$, consistently with the proved Peirce–Yukawa direct-channel placement.

The $16 \times 10 \times 16$ trilinear and the Peirce Yukawa channel [proved in exterior algebra, A686]. In the c_3-selected branch the SM top Yukawa is realised as the exterior trilinear

$$T(\psi, H, \chi) = \mathrm{Top}(\psi \wedge H \wedge \chi)\,, \qquad \psi \in Q \subset 8_+(\Gamma_{QL}), \quad H \in (1,2)_{+1/2} \subset 10, \quad \chi \in u^c \subset 8_-(\Gamma_{QL}),$$

giving

$$T(Q, H_u, u^c) \propto \varepsilon_{abc}\,\varepsilon_{\alpha\beta}\, Q^{a\alpha} H^{\beta} u^{c,bc} \quad [\text{PROVED}],$$

kernel-verified with the exact Standard Model index structure and six nonzero contractions (A686). The Higgs lives in the vector $10 = J_{12} \oplus \mathbb{R}c_1 \oplus \mathbb{R}c_2$ (acting on S_+ by Clifford multiplication), not in $8_\pm$. This is a *cross-block* trilinear,

$$J_{13} \times J_{12} \times J_{23} \to \mathbb{R},$$

distinct from the same-block Freudenthal bilinear $J_{13} \times J_{13}$ proved in A674. The same-block theorem remains the *Peirce unit-eigenvalue Yukawa-channel selection* (extracting $\lambda_2 = 1$); it is not the literal $QH_u u^c$ invariant. The Jordan triple product $\{J_{12}, J_{13}, J_{23}\}$ also extracts $\lambda_2 = 1$ in the real scalar Peirce subalgebra [PROVED, A686], forming a structural bridge between the two descriptions. The full generic $\mathbb{O}_s$ cross-block normalization is now resolved; see below.

Cross-block Peirce trilinear [proved, A687]. For generic split-octonion entries $x, y, z \in \mathbb{O}_s$, let $X = J_{12}(x)$, $Y = J_{13}(y)$, $Z = J_{23}(z)$. The Jordan triple product

$$\{X, Y, Z\} = (X \circ Y) \circ Z + (Z \circ Y) \circ X - (X \circ Z) \circ Y$$

gives (kernel-verified, A687)

$$\{J_{12}(x), J_{13}(y), J_{23}(z)\} = \tau(x,y,z)\, c_2,$$

where the split-octonionic triality trilinear is

$$\tau(x,y,z) = \mathrm{Re}((\bar{x}\, y)\bar{z}) = \mathrm{Re}((y\bar{z})\bar{x}) = \mathrm{Re}((xz)\bar{y})\,.$$

Consequently,

$$\mathrm{Tr}[J_{\mathrm{vac}} \circ \{J_{12}(x), J_{13}(y), J_{23}(z)\}] = \lambda_2\, \tau(x,y,z) = \tau(x,y,z),$$

with $\lambda_2 = 1$ because J_{13} occupies the *middle* slot, selecting the central idempotent c_2. The real part of the split-octonion associator contributes no independent correction: $\mathrm{Re}([x,y,z]) = 0$ in the relevant trace (A687). The correct SM unit-eigenvalue ordering is therefore

$$\{J_{12}(H),\ J_{13}(\Psi_L),\ J_{23}(\Psi_R^c)\},$$

matching the $16 \times 10 \times 16$ Yukawa structure proved in A686.

This cross-block trilinear is distinct from the same-block Freudenthal channel

$$\mathrm{Tr}[J_{\mathrm{vac}} \circ (J_{13} \times J_{13})] = -2\,\mathrm{Re}(\psi\bar{h}),$$

but both channels select c_2 and extract $\lambda_2 = 1$. The suite therefore has a two-channel Peirce Yukawa theorem: the same-block channel (A674) is the unit-eigenvalue Yukawa carrier; the cross-block channel (A687) is the literal SM $16 \times 10 \times 16$ coupling. The exact component dictionary mapping $\mathbb{O}_s$ entries to exterior-algebra SM fields is now fixed for the top-quark sector; see below.

Component normalization of the cross-block Yukawa [proved under displayed orientation, A688]. Using the SplitCD convention $e_1^2 = e_2^2 = e_3^2 = -1$, $e_4^2 = \cdots = e_7^2 = +1$, choose the orientation

$$H_u^3 = e_0, \qquad H_u^4 = e_7,$$

$$Q^{0,3} = e_1, \quad Q^{0,4} = -e_6, \quad Q^{1,3} = e_2, \quad Q^{1,4} = e_5, \quad Q^{2,3} = e_3, \quad Q^{2,4} = e_4,$$

$$u_{01}^c = e_4, \qquad u_{02}^c = -e_5, \qquad u_{12}^c = -e_6.$$

This is the split-real form of the compact color-triplet convention $u_a = (\text{real part}, \text{oriented imaginary part})$. A direct component audit (36 tests, 36/36 pass) gives

$$\mathrm{Re}\Big((\bar{H}_u Q)\overline{u^c}\Big) = \varepsilon_{cij}\, \varepsilon_{\alpha\beta}\, H_u^\beta Q^{c\alpha} u^{c,ij} \quad \text{[PROVED]}.$$

The Peirce cross-block triple therefore realizes the SM $QH_u u^c$ Yukawa invariant with unit normalization.

Full cross-block Yukawa normalization [proved under explicit B, A689]. The corrected one-generation component dictionary is

$$H_d^3 = e_0, \quad H_d^4 = -e_7 \quad (\text{split-octonion conjugate of } H_u),$$

$$d_0^c = e_1, \quad d_1^c = -e_2, \quad d_2^c = e_3, \qquad L^3 = e_0, \quad L^4 = e_7, \qquad e^c = e_0.$$

Direct component audits give zero mismatches for all three Yukawa sectors (A689):

$$QH_u u^c \text{ [PROVED]}, \qquad QH_d d^c \text{ [PROVED]}, \qquad LH_d e^c \text{ [PROVED]}.$$

The down Higgs is the split-octonion conjugate $H_d = \bar{H}_u$ in the (e_0, e_7) neutral plane; no additional $\mathbb{O}_s$ directions are required.

Hypercharge from the B-map [derived from $B + SU(5)$, A689]. With the $SU(5)$ exterior-index assignment $Y_0 = Y_1 = Y_2 = -\frac{1}{3}$, $Y_3 = Y_4 = +\frac{1}{2}$, the hypercharge of each exterior basis element is $Y(I) = \sum_{i\in I} Y_i$. All 16 elements of $S_+ = \Lambda^{\text{even}}\mathbb{R}^5$ give the correct SM value (16/16 matches, 0 mismatches):

$$Y(Q) = +\tfrac{1}{6}, \quad Y(u^c) = -\tfrac{2}{3}, \quad Y(d^c) = +\tfrac{1}{3}, \quad Y(L) = -\tfrac{1}{2}, \quad Y(e^c) = +1, \quad Y(\nu^c) = 0.$$

The previous HY1 field-by-field projector axiom is accordingly replaced by the $B + SU(5)$ index assignment. The remaining open item is not the individual hypercharge values, but the intrinsic $J_3(\mathbb{O}_s)$-based canonical selection of the B-map orientation.

Canonical status of the B-map (A690). A finite audit (kernel `d690_canonical_B_audit`, re-run in house) shows that the present $J_3(\mathbb{O}_s)$ data do *not* uniquely select B: there are 192 signed-permutation automorphisms of the SplitCD table, and applying any of them simultaneously to every B-component preserves all $QH_u u^c$, $QH_d d^c$ and $LH_d e^c$ tests (6/6, 6/6, 2/2; zero mismatches) and the full hypercharge table. This is not accidental: the Yukawa contractions are values of the $\mathrm{Aut}(\mathbb{O}_s) = G_{2(2)}$-invariant trilinear $\tau(x, y, z) = \mathrm{Re}((\bar{x}y)\bar{z})$, so the admissible frames form (at least) the global automorphism orbit of B_0. Neither J_{vac}, the cubic norm N, nor the Peirce eigenvalue ladder $(\phi, 1, \phi^{-1})$ pins the orientation, since all three are blind to the internal $\mathbb{O}_s$ basis inside a fixed Peirce block. The honest label is therefore

$$\text{one-generation SM Yukawa} + Y_{\text{SM}} \text{ [PROVED under explicit } B\text{]},$$
$$B \text{ canonical from } J_3(\mathbb{O}_s) \text{ alone [NOT PROVED]},$$

with the residual reduced to a single frame axiom A_B ("choose the oriented SplitCD/$SU(5)$ frame"), strictly weaker than HY1: A_B fixes one basis, after which every charge follows from $Y(I) = \sum_{i\in I} Y_i$.

A continuous double-check (A692, kernel-verified in house) sharpens this. Among e_0-fixing signed-permutation frames the valid set is exactly $\mathrm{Aut}(\mathbb{O}_s) \sqcup \mathrm{Aut}(\mathbb{O}_s) \circ \sigma$ with $\sigma = \mathrm{diag}(1, 1, 1, -1, 1, -1, -1, -1)$ a harmless global field rephasing. But the 76 Yukawa component equations do *not* pin the frame even to $G_{2(2)}$: inside the e_0-fixing $SO(4, 4)$ frame space their infinitesimal stabilizer has dimension $15 = \dim \mathfrak{g}_{2(2)} + 1$, the extra direction being the non-compact boost $e_3 \mapsto \cosh t\, e_3 + \sinh t\, e_4$, $e_4 \mapsto \sinh t\, e_3 + \cosh t\, e_4$, which preserves all 76 tests but is *not* a split-octonion automorphism. The residual axiom is therefore

$$A_B: \; B \text{ is the } G_{2(2)}\text{-compatible oriented SplitCD/}SU(5) \text{ frame}$$

— i.e. B must intertwine the full split-octonion product, not merely the SM Yukawa tensor (this removes the $SO(1, 1)$ boost). A_B is strictly weaker than HY1 (a frame choice, not a charge table) but remains an axiom until a $J_3(\mathbb{O}_s)$-intrinsic criterion selects the $G_{2(2)}$-compatible frame.

Compact gauge kills the boost (A707); the global frame is canonical (A708). The residual $SO(1, 1)_{34}$ boost is excluded by the compact Standard-Model gauge structure the suite already uses (A707, kernel-verified in house with the core re-derived independently): it preserves the split $(4, 4)$ metric but *not* the positive-definite color norm ($|e_3|^2_{\text{pos}} : 1 \to \cosh 2t = 3.76$), mixes the J_{23} fields $d^c\,(Y = +\frac{1}{3})$ and $u^c\,(Y = -\frac{2}{3})$, and fails to commute with $SU(2)_L$. A global finite audit (A708; kernel re-run in house and the count $\mathrm{Aut}(\mathbb{O}_s) = 192$ re-derived from a from-scratch Cayley–Dickson table) exhausts the e_0-fixing signed-permutation frames ($7!\,2^7 = 645{,}120$): the 76-valid set is exactly $\mathrm{Aut}(\mathbb{O}_s) \sqcup \mathrm{Aut}(\mathbb{O}_s) \circ \sigma$ ($384 = 192 + 192$), and once the compact-gauge frame is imposed the valid finite

frames reduce to the e_7-line-preserving subset ($96 = 48 + 48$) — a single component after quotienting by $G_{2(2)}$, compact $SU(3)_c \times U(1)_Y \times SU(2)_L$, and the σ-rephasing. The two natural discrete suspects, color conjugation $3 \leftrightarrow \bar{3}$ and the singlet swap $e_0 \leftrightarrow e_7$, both fail validity outright (e_0 is the octonion identity). No physically distinct B-frame survives, so the former axiom A_B is *discharged* relative to the compact SM gauge embedding:

$$\mathfrak{so}(1,1)_{34} \text{ compatible with compact SM gauge [PROVED NO]},$$
$$A_B \text{ [DISCHARGED rel. compact SM embedding]}.$$

The one-generation Yukawa $+ Y_{\rm SM}$ system therefore reads [PROVED relative to the compact SM gauge embedding]: the qualifier now names a physical input (the compact embedding — not derived from J_3 here; the former "gauge-emergence frontier" is now settled as a boundary: emergence-as-gauging is closed negative by the branching-wall theorem, the completion is unique up to $U(1)_{B-L}$ and global structure, and the maximal derivable shadow is dynamically selected — see Appendix F) rather than a frame ambiguity. The signed-permutation audit is now a theorem over all $O(4,4)$ frames, with the σ-rephasing named as the one additional equivalence it uses (Appendix X; `s1213`, A1750).

PART II — HISTORICAL Rev8 MATERIAL (superseded, non-operative in Rev29)

Everything from here to the end of Paper 6 is the Rev8 electron-mass bridge programme, retained for the historical record only. It is superseded by the QED-corrected Koide chain ($m_e = 0.5076\,$MeV, KOIDE-CONSISTENT $\star\star\star\star$ under external $K = 2/3$ [A636, A637]) and by the A674–A689 Peirce-Yukawa results (above). None of the sections below represent the operative Rev29 state; they are not part of the current claims of the suite.

5 Electron-mass baseline and residual (Rev8 historical)

The current baseline electron formula is

$$m_e^{\rm base} = M_{\rm Pl} \exp\left[-\frac{\pi}{8\alpha}\right] \phi^4 \sqrt{2} = 0.5036 \text{ MeV}. \tag{25}$$

Relative to the physical value

$$m_e^{\rm phys} = 0.5109989461 \text{ MeV}, \tag{26}$$

the baseline residual is

$$\Delta_e^{\rm obs} = \frac{m_e^{\rm phys} - m_e^{\rm base}}{m_e^{\rm phys}} = +1.454\%. \tag{27}$$

The algebraic form of Eq. (25) is currently taken seriously. The issue is not whether the baseline can be written compactly; the issue is whether the remaining percent-level bridge is a derived consequence of the same Lagrangian or a phenomenological closure layer built around the known electron mass.

6 Phase decomposition of the electron residual

Rev8 carries five correction phases. Their physical interpretations are motivated by structures already present in the suite, but their precise percentages are not yet derived from the Lagrangian. The correct status is therefore PHENOMENOLOGICAL ANSATZ.

Table 1: Rev8 phase decomposition of the electron residual.

Phase	Central value	Uncertainty	Current source	Rev8 reading
KK threshold matching	+0.50%	±0.10%	Paper 6 threshold sector	physically motivated, not derived
RGE bridging	−0.048%	±0.030%	standard one-loop running	kernel-fixed bridge term; not computed from Feynman diagram or matching condition
Möbius topology	+0.60%	exact in the current topological normalization	winding / half-twist picture	topologically motivated, percentage not derived
Associator correction	+0.15%	±0.06%	non-associative sector	physically motivated, coefficient not derived
Electroweak thresholds	+0.102%	±0.041%	low-energy matching cascade	physically motivated, coefficient not derived

The central values sum to

$$\Delta_e^{\text{phases}} = +1.304\%, \tag{28}$$

which is smaller than the observed residual in Eq. (27). Combining the quoted uncertainties in quadrature gives

$$\sigma_{\text{phases}} = \sqrt{0.10^2 + 0.030^2 + 0.00^2 + 0.06^2 + 0.041^2} \approx 0.127\%. \tag{29}$$

This is smaller than the observed mismatch between Eqs. (27) and (28). The suite therefore does not claim that the current five-phase table explains the electron mass completely.

7 The unresolved 0.15% remainder

The unexplained remainder is

$$\Delta_e^{\text{miss}} = \Delta_e^{\text{obs}} - \Delta_e^{\text{phases}} = +0.150\%. \tag{30}$$

At present there are three live interpretations:

(a) a missing sixth phase associated with structure not yet tracked in the bundle,

(b) an underestimated uncertainty or incomplete treatment in the RGE/threshold pieces,

(c) framework incompleteness in the present electron closure mechanism.

Rev8 does not decide among these options. It records them as the real sources of current uncertainty rather than as rhetorical afterthoughts.

8 Necessary or sufficient?

The five phases are currently best interpreted as *sufficient* phenomenological descriptors of the known electron residual, not as *necessary* outputs of the Lagrangian. This distinction is crucial. A necessary output would be fixed uniquely by the algebraic and field-theoretic structure before the electron mass is consulted. A sufficient descriptor is a physically motivated decomposition that reproduces most of the observed residual but still depends on post hoc calibration. Rev8 adopts the second description.

9 Muon/tau validation protocol

The next decisive test is whether the same cubic-invariant logic and the same phase structure extend to the heavier charged leptons without adjustment.

8.1 Operational protocol

(1) Extract $y_\mu(M_{\rm Pl})$ and $y_\tau(M_{\rm Pl})$ from the same cubic-invariant/Jordan bookkeeping used for the electron baseline.

(2) Apply the same standard one-loop running used for the electron bridge.

(3) Apply the same five phases with no lepton-specific retuning.

(4) Compare the resulting m_μ and m_τ predictions directly to the PDG benchmarks.

8.2 Interpretation of outcomes

Table 2: Interpretation of the muon/tau validation outcomes.

Outcome	Interpretation	Rev8 consequence
Both within about 1–2% with no new phases	framework behaves predictively across the charged-lepton family	strong case for structural phase system
Require only small RGE/threshold refinements	framework may be structurally right but quantitatively incomplete	retain cautiously as structural candidate
Need adjusted phases for each lepton	phase system behaves like phenomenological tuning	downgrade electron closure to framework-specific ansatz

Outcome	Interpretation	Rev8 consequence
Miss badly or require new topological data	electron success was not universal	record boundary and reclassify mechanism

10 Status

Rev29 status (Sessions 16–23), reworded Rev32. The operative tree-level weak-angle readout is $\sin^2\theta_W = \phi^{-3}$ under the polarisation postulate $\mathrm{AX6}_{\mathrm{pol}}$ (Paper 4 §6, which closes the earlier 3/13 estimate; this paper's Weak-angle bridge section already marks 3/13 superseded — Rev32.9: through Rev32.8 this box still gave 3/13 as the target); it is a structural target under that independent postulate (it is not citable as algebraically derived without that named premise — Appendix X); threshold corrections are open (no spectrum computed). The electron-mass phase system is a phenomenological ansatz, not a Lagrangian consequence. The electron baseline inherits the AX2 vacuum (now proved as a theorem); if that theorem were invalidated, the baseline formula would require revision. **CKM sector (Sessions 22–23, historical marking — superseded by the Rev29 re-tier and the A1566 retype: first row Loaded, $|V_{cb}|$ Loaded-correspondence attachment (physical) / Structural formula, δ_{CKM} Coincidence-class): three then-Proved-tier magnitudes + one conditional-theorem phase.** The three magnitudes ($|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$) follow from the $J_3(\mathbb{O}_s)$ Route-B map and agree with PDG within 1σ (Proved-tier, the six-of-eight mixing theorem; Appendix X); δ_{CKM} is Coincidence-class / candidate construction (value retained, G_7 selection absent; the Rev29 "conditional theorem under F-δ" wording retired Rev32.5; Appendix B.6). The phase origin is algebraic, not RGE/threshold (D594); the earlier "100/100, all green" framing is retired (S112 claim-hygiene). **Muon/tau no-retuning test:** extending the same five phases to muon and tau without lepton-specific adjustment currently fails at $\sim 111\sigma$. This is the decisive test for structural validity of the phase system. The decisive next step is independent muon/tau validation with no new percent-level tuning.

11 CKM Dirac phase: RGE and threshold contributions ruled out

Sessions 22–23 marked the CKM magnitudes Proved-tier within Route B — a marking *superseded by the Rev29 re-tier and the A1566 retype* (first row Loaded, $|V_{cb}|$ Loaded-correspondence attachment (physical) / Structural formula, δ_{CKM} Coincidence-class; the phase remains readout-conditioned) — and explicitly tested whether standard QFT effects could account for the CKM Dirac phase. The result is a clean structural division.

9.1 RGE and threshold running ruled out (D594, A576)

Grok dispatch D594 tested whether renormalization-group running and top-quark threshold corrections could shift the naive tree-level δ_{CKM} prediction into agreement with the PDG value. The verdict was unambiguous: neither α_s running from M_Z to M_{th}, nor top-quark decoupling thresholds, can produce a correction of the right sign and magnitude. RGE contributions to δ_{CKM} are *ruled out* as the explanation.

9.2 Algebraic origin: $G_7 = \phi + 5\phi^{-5}$

The correct explanation is purely algebraic. The sub-leading eigenvalue of $J_3(\mathbb{O}_s)$ receives a correction $\varepsilon = 5\phi^{-4}$ from the $n = 5$ Peirce-graph loop, giving

$$G_7 = \phi + 5\phi^{-5}, \qquad \delta_{\mathrm{CKM}} = \arctan\sqrt{3(\phi + 5\phi^{-5})} = 68.1297^\circ. \tag{31}$$

(Comparison: 0.8σ from the γ-fit; 1.6σ from the PDG direct value.) The factor $5\phi^{-5}$ arises from the split-octonion $(4, 4)$ signature of $\mathbb{O}_s$: 5 of 8 octonion directions contribute to the Peirce coupling (compact e_1, e_2, e_3 cancel; scalar + non-compact e_4–e_7 survive). This is not a QFT effect; it is a consequence of the Albert-algebra multiplication rules (Structural Result 2.1 in Paper 3; E96–E98, A579).

9.3 Status: current CKM provenance and PDG 2026 comparator record

Table reading rule. Rev32.1 reading rule: tiers name mathematical provenance; physical attachments are stated separately; no row may be cited as zero-parameter; displayed σ values are named comparator distances, not profile likelihoods.

Quantity	Algebraic value	PDG 2026 comparator
$\lvert V_{us}\rvert$	$\sin\theta_{12}^{\mathrm{PMNS}}/\sqrt{6} = 0.225256$ (`s1162`)	0.22431 ± 0.00085
$\lvert V_{cb}\rvert$	$1/(9\sqrt{7}) = 0.041996$	0.0407 ± 0.0013
$\lvert V_{ub}\rvert$	$\lvert V_{us}\rvert\lvert V_{cb}\rvert/\sqrt{6} = 0.003862$	0.00389 ± 0.00016
δ_{CKM}	$\arctan\sqrt{3(\phi + 5\phi^{-5})} = 68.1297^\circ$	global-fit $\delta = 1.154 \pm 0.025$ rad ($= 66.12^\circ \pm 1.43^\circ$; radians prima

The three CKM magnitudes were marked Proved-tier within the Route-B map pre-Rev29; after the row-by-row re-tiering (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`) the CKM first row is LOADED because its route was selected against kaon data, $|V_{cb}|$ is LOADED-CORRESPONDENCE (physical attachment) / Structural formula (A1566), and the Dirac phase δ_{CKM} is COINCIDENCE-CLASS while the motivation for G_7 is missing — not a 100/100 closure of the matrix. No fitted mixing coefficients enter the Route-B map, but **no row may be cited as zero-parameter**. The decisive implication for Paper 6 is that the CKM phase *does not belong to the RGE/threshold bridge*: it is fixed by the exceptional-Jordan algebraic structure before any running is applied (no QFT input beyond the tree-level algebra).

12 Conclusion

Paper 6 keeps the suite honest in two places. In the electroweak sector it states the weak-angle bridge as an ordinary QFT running-and-threshold problem. In the charged-lepton sector it states clearly that the current electron residual closure is not yet a derived Lagrangian consequence. The framework now has a clean next test: if the same logic predicts m_μ and m_τ without retuning, the present electron mechanism strengthens sharply; if not, its domain of validity is revealed.

Leibniz Quantum Beats Newton

Paper 7: Precision Tests and Failure Conditions (Rev33.2)

Tom O'Sieg

August 2026

Abstract

This paper is the public falsification note for the current (Rev29) $J_3(\mathbb{O}_s)$ programme; "Rev29" below names the revision in which a given result first entered the suite, not the revision of this document. It collects the measurements that can most directly confirm or exclude the current exceptional-Jordan correspondence and states them without protective ambiguity. The three highest-priority external tests remain the leptonic Dirac phase, the atmospheric mixing angle, and the weak-angle bridge. Rev29 S43 advances the internal electron-mass validation test: $m_e = 0.5076\,\mathrm{MeV}$ is *Koide-closed* $\star\star\star$ via the QED-corrected Koide chain [A636, A637], conditional on the external empirical rule $K = 2/3$ (not derived from $J_3(\mathbb{O}_s)$ alone). The current open internal test is the formal S-matrix derivation of the 7/3 lepton–quark amplitude bridge [A640].

1 Introduction

A theory that claims algebraic compression of particle-physics data should state how it can fail. Paper 7 exists for that purpose. The programme asks to be judged by measurements and by clean internal cross-checks rather than by rhetoric.

2 Primary falsification benchmarks

Table 1: Primary falsification benchmarks (entered at Rev29). Normal ordering is assumed throughout; the ordering-dependent rows are additionally conditional on the Structural R_d ladder and on the undeclared ladder-to-propagation-eigenstate interface (Paper 5 §2). **Rev32.1 reading rule: tiers name mathematical provenance; physical attachments are stated separately; no row may be cited as zero-parameter; displayed σ values are named comparator distances, not profile likelihoods.**

Quantity	Framework target	Measurement scenario	Failure condition
δ_{CP}	-160.997° ($J_{CP} \approx -0.011$) [DERIVED-CONDITIONAL, Rev29 re-tier — pre-Rev29 marking THEOREM $\star$100 superseded]	NuFIT 6.1 NH (w/SK): $212^{+26\circ}_{-36}$ ($-160.997^\circ \equiv 199.003^\circ$); current deviation **$0.36\sigma$ [excellent]**; DUNE-era $O(10^\circ)$ at 1σ (2030s)	exclusion at $\geq 3\sigma$ by DUNE Phase II global fit
θ_{23}	41.41° ($\sin^2\theta_{23} = 7/16 = 0.4375$)	NuFIT 6.1 NH (w/SK) global best fit: 43.3° ($\sin^2\theta_{23} = 0.470^{+0.017}_{-0.014}$, *lower octant*, 2.3σ from 7/16; upper-octant local comparator ≈ 0.561 at $\approx 9.5\sigma$; 3σ range includes 7/16); DUNE-era few-degree precision (2030s)	stable exclusion of the 7/16 branch at $\geq 3\sigma$ in global fits (this attacks the selected physical correspondence, LOADED-CORRESPONDENCE since A1566; the internal 7/16 mismatch identity is not refuted by it unless the readout attachment is first derived); note: the NuFIT 6.1 global best fit is itself lower-octant

Quantity	Framework target	Measurement scenario	Failure condition
$\sin^2\theta_W$ bridge	$\phi^{-3} = \sqrt{5} - 2 \approx 0.23607$ as a $\sim$2% *structural* low-energy target on the effective $\sin^2\theta_W(\mu)$ running curve (A697; Paper 4 §7 authoritative)	ϕ^{-3} lies on the running curve at a sub-GeV/GeV-order, scheme-dominated crossing (corrected S124/Appendix I §B4; the once-quoted $\mu_\star \approx 1.8\,\mathrm{GeV}$ lies outside the band); the M_Z difference is standard running (right direction). matching scale/scheme not derived $\Rightarrow$ structural target, *not* a Z-pole prediction; AX6 fixes the independent orbit branch	failure to supply a first-principles $\mu_\star$ (or a completed bridge) to the Z-pole observable

Quantity	Framework target	Measurement scenario	Failure condition
m_μ/m_τ	$(\phi/\sqrt{5})^{8/3} \times \sqrt{2}/10 = 0.059684$. STRUCTURAL / LOADED-CORRESPONDENCE / REPRODUCED (*Rev29* re-tier; this row previously printed PROVED, zero-parameter, "forced by algebra, no fit"). The exponent $8/3 = \dim \mathbb{O}_s/\mathrm{rank}\, J_3$ is an exact dimension ratio but is not, alone, a mass exponent (Paper 0 § "physical attachment"); the coefficient $C = \sqrt{2}/10$ divides by $\dim J_2 = 10$, a step A618 itself records as "argued by analogy," and A618 lists the formal derivation as BLOCKING — never closed. Provenance is target-first: D616 computes $0.0595/0.4220 = 0.141$ from PDG and *asks* whether it is algebraic in φ; A616 then offers $\sqrt{2}/10$ at 0.37% while stating that "the algebraic origin is not yet proved"; A618 supplies $\dim J_2 = 10$ afterward. [A616, A618]	$m_\mu = 106.05\,\mathrm{MeV}$ (+0.37% PDG). **The +0.37% residual *is* the fit gap**, not a loop correction: the coefficient required to reproduce the measured ratio is $C = 0.140894$ against the offered $C = \sqrt{2}/10 = 0.141421$, and C enters the formula multiplicatively, so the discrepancy passes straight through to m_μ/m_τ at the same +0.37%	Miss > 2% without retuning would invalidate the loaded-correspondence mass formula; the row may *not* be cited as zero-parameter

Quantity	Framework target	Measurement scenario	Failure condition
$\mathrm{AX6_{pol}}$: $\Lambda_\star = 3P/Q(Y) = 6\phi - 1$ (readout $3/(4+\Lambda_\star) = \phi^{-3}$ exactly); the Det^2 object $\cos(6\psi) = \frac{1}{2}$ is a distinct, retired selector (Rev32.2)	Independent orbit-selector postulate; the readout is an identity given the postulate, and does not select it	A667 degree argument (Det^2 object only) plus earlier negative CW verdicts [A617,A620,A621]	Derivation of AX6 from first principles, or failure to anchor the structural target ϕ^{-3} (via a first-principles $\mu_\star$) to the Z-pole observable
Pure μ–γ associator null	$a_\mu = a_\mu^{\mathrm{SM}}$ to all orders in the lepton–photon sector (Artin theorem on $\langle e_0, e_7\rangle$; A721)	2025 Theory Initiative: $a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 38(63) \times 10^{-11}$; HLbL $= 115.5(9.9) \times 10^{-11}$; implied $\lvert C_{\mathrm{assoc}}^{\mathrm{HLbL}}\rvert \lesssim 0.55$ (conservative) or $\lesssim 0.086$ (HLbL budget); normalization FIXED to the LbL class (A725) — no dipole-class escape	confirmed *non-hadronic* a_μ anomaly after HVP/HLbL resolution, or a required associator coefficient $\lvert C\rvert \gg O(1)$
$v_{\mathrm{EW}}/\sqrt{\sigma}$ (Rev26)	$\varphi^{105/8} = 553.30$ [CONDITIONAL / loaded-depth; Appendix O] — framed-depth operator $D = 13 + \frac{1}{8}$; $m_\tau/\sqrt{\sigma} = 4$ exact companion identity	registration relative to the marked compact-positive Higgs carrier (Paper 4); the frame-free route is closed negative (Appendix F permanence); $\sqrt{\sigma}$ registration carries $\pm 1.51\%$	a determination of $v_{\mathrm{EW}}/\sqrt{\sigma}$ inconsistent with $\varphi^{105/8}$ beyond the $\sqrt{\sigma}$ registration uncertainty, or exhibition of a *second* genuinely underivable dimensionful ratio in scope (which falsifies the one-input theorem chain, Appendix X/O)

Stamps in this table (⋆100, THEOREM, Proved) are the Rev29-era marking, superseded en bloc by the Rev29 re-tier (Paper 3 §"Row-by-row re-tiering"); the current tier of each row is the re-tier table's. Footer added Rev32.5.

3 Secondary tests

RGE scale convention for mixing targets. The PMNS and CKM angle targets are quoted at a single canonical comparison scale, which the suite takes to be the electroweak scale M_Z unless explicitly stated otherwise. In the Standard Model, running of CKM angles between M_Z and a GUT scale is numerically small because the dominant anomalous-dimension terms are flavour-universal within a fixed representation; for hierarchical Dirac-neutrino masses the PMNS angle running is likewise sub-percent, typically at the $\mathcal{O}(10^{-3})$ level. The algebraic angle values are therefore compared to low-energy global-fit values without adding a new fitted RGE offset. A future high-precision treatment should add a dedicated uncertainty line for scheme, threshold, and normal-ordering assumptions; such corrections are subleading relative to the present DUNE-era falsification bands.

The relation $y_t = 1$ is a falsifiable boundary statement, not a fitted threshold: imposed as a UV boundary condition it defines a running curve $y_t(\mu)$ that is compared with the extracted coupling at a stated scale and scheme (the $v/\sqrt{2}$ tree value sits at +0.87% vs. PDG 2026, naive cross-scheme comparator distance $d_{\rm cmp} = +5.57$ — not a tension, Appendix X; the authoritative negative running/matching audit is Paper 2 §3.6). Rev32.1 (A1538 F4): the "$\geq 19\sigma$ consistency at every scale" clause previously printed here is withdrawn as ill-formed — like the transposed "$+5.3\sigma$" admitted alongside it, a pipeline-string defect; the s450/S193 run record and its 0.17 GeV pipeline residual are carried unaltered in the kernel ledger [s450/S193].

Several additional quantities provide cross-checks:

(1) charged-lepton and down-type mass ratios from the resolvent family,

(2) CKM hierarchy relations,

(3) CKM first-row unitarity check,

(4) internal consistency of the cubic-invariant Yukawa construction,

(5) formal S-matrix derivation of the 7/3 lepton–quark amplitude bridge,

(6) the shadow charge-pattern match (S77 designated test — *executed S78, fence fired*): the residual $\mathfrak{so}(6) \oplus \mathfrak{so}(2)$ of the dynamically selected $SO(6,2)$ point hosts $\mathfrak{su}(3) \oplus \mathfrak{u}(1) \oplus \mathfrak{u}(1)$ (Appendix F); the framework-internal check was whether the two $\mathfrak{u}(1)$ charge spectra on the **56** reproduce the derived $Y_{\rm SM}$ and $3(B-L)$ patterns. *Outcome:* mismatch at the representation level at the origin point (octet present, one singlet vs nine; exact Casimir clusters), so the pre-registered demotion applies there — "stable $\mathfrak{su}(3) \oplus \mathfrak{u}(1)^2$ point selected" — with the obstruction and uniqueness theorems untouched; and the apparent moduli-scan escape was then closed by *counting* (S78b): the entire **56** carries only four $\mathfrak{su}(3)$-singlets under the residual class, versus the nine the matter pattern requires in a 27-plane, so the demotion is final for the whole $SO(6,2)/SO^*(8)$ family. The *charge wall* question — can any gauging's gauge algebra (hence any vacuum residual) contain a unitary-class $\mathfrak{su}(3)$ ($\mathbf{8} = \mathbf{3} \oplus 5 \cdot \mathbf{1}$)? — was subsequently executed (S80): the global negative holds, the second wall theorem (Appendix F), house-proved and blind-replicated. Only vector- and adjoint-class $\mathfrak{su}(3)$'s are ever gauging-realizable; the labeled shadow is excluded for every gauging, and the demotion above is global and final.

The 7/3 bridge as a finite-layer matching boundary: a falsifiable 4321/twin-Pati–Salam prediction

The 7/3 lepton–quark ratio is exact at the *finite (bare-vertex) layer*: the unit-weight condition $K_B = I_7$ is forced there by the cubic Jordan norm, and the $I_8 \to I_7$ reduction is exact under Standard-Model covariance (S146, A1016–A1020) — a conditional trace statement in the sense of Appendix G, whose physical (LSZ/S-matrix) layer is open. Read as an $\overline{\rm MS}$ mass relation it does not hold at $\mu = m_b$ or at any pre-registered fixed scale tested on charm and bottom (`s1336`, `s1339`). This is a relation imposed at the matching scale, not a closed low-energy mass theorem; whether it *survives* the running to laboratory energies is a separate, testable dynamical question. The minimal ultraviolet completion that realizes the 7/3 channel structure is a 4321 / twin-Pati–Salam model ($SU(4) \times SU(3)' \times SU(2)_L \times U(1)$), which predicts, as a concrete and *falsifiable* low-energy signature, a TeV-scale vector leptoquark $U_1 \sim (\mathbf{3}, \mathbf{1}, 2/3)$ together with a Z' and vector-like fermions. This

completion is *not* parameter-free, and the competing realization is a tuned Standard-Model Yukawa pattern (21/29, 85/29); the framework does not at present select between them.

Falsification statement. A confirmed absence of any such TeV-scale U_1 leptoquark / Z' / vector-like sector up to the energies where the 7/3 high-scale relation would have to be matched by tuned Yukawas, combined with no first-principles derivation of the matching scale, would show that the 7/3 ratio survives only as a high-scale relation with no laboratory imprint. The 7/3 ratio retains STRUCTURAL ⋆⋆⋆⋆ status: the in-house advance toward deriving the channel-count axiom $\mathrm{Tr}_3(W) = 7$ (s191) is *not yet hostile-reviewed* and is therefore *not* treated here as a closure of the bridge.

Failure dependency tree

Failure	Layer attacked
δ_{CP} miss at $\geq 3\sigma$	V_3 rotation mechanism ($T_{\mathrm{half}}\|_{V_3}$, $\Omega = \sqrt{5}/2$); does *not* by itself attack AX1/AX3 (AX2 is eliminated by DET-7, Paper 3) or the angle theorems
θ_{23} miss at $\geq 3\sigma$	first falsifies the loaded lower-octant registration (A1566); attacks the internal J_{vac} versus $J^{\#}_{\mathrm{vac}}$ mismatch geometry only if that physical attachment is independently derived
weak-angle bridge miss	threshold-sector calculation and RG bridge; does not by itself falsify the Jordan algebra core
7/3 amplitude failure	If S-matrix vertex construction shows closure amplitude $\neq n_{23}/\mathrm{rank}(J_3) = 7/3$; would falsify STRUCTURAL ⋆⋆⋆⋆ status of m_b and m_c; does not affect proved lepton masses or mixing sector — the independence is structural, not convenient: the bridge failure attacks the *S-matrix realization* of the Gram ratio as a physical amplitude, while the angle theorems use $n_{23} = 7$ only through the kinematic normalization $\sin^2\theta_{23} = n_{23}/\|J_{\mathrm{vac}}\|^2$ (the internal identity; the physical octant attachment is LOADED-CORRESPONDENCE, A1566), which a failed amplitude construction leaves untouched. **Rev27 sharpening (`s902`):** the *metric-weighted-residue* route is now a proven no-go — a unitary residue over $\mathrm{Im}\,\mathbb{O}_s$ lies in $[-4, +3]$, so 7 cannot be a signed amplitude; the 7/3 stands as an *index* ratio (finite-carrier theorem, Appendix K), and only a non-residue index-theoretic realization remains open (Appendix I, Lemma)

heavy-quark T_3 pattern failure	If improved PDG quark masses break the equal-and-opposite residual pattern $\lvert\delta m_t/m_t\rvert = \lvert\delta m_b/m_b\rvert$ (currently equal to 0.67σ of the PDG 2026 m_b uncertainty: $\lvert\kappa_t - \kappa_b\rvert = 0.194$ pp vs $\sigma_{\kappa_b} = 0.289$ pp; a PDG central value moving $\lvert\kappa_t - \kappa_b\rvert$ past 1σ breaks it), the structural weak-isospin T_3 threshold reading (A715–A717) is falsified; attacks only the structural threshold term, not the bare boundary values, the proved lepton masses, or the mixing sector
QED-Koide chain failure	If one-loop QED coefficient at Λ_{G_2} differs from $3\alpha/4\pi$ (e.g. non-associative leading-loop correction), or Koide relation breaks at required precision; would reopen m_e derivation
charged-lepton mass-chain miss	QED-Koide or muon-resolvent chain only; does not attack the mixing sector unless the shared algebraic inputs fail
non-hadronic a_μ anomaly confirmed	If a confirmed a_μ deviation survives HVP/HLbL resolution and is shown to be non-hadronic, the associative lepton–photon embedding $\{\mu, \gamma\} \subset \langle e_0, e_7\rangle$ (A721) is falsified; attacks the verified field-assignment map (A688/A689), not the mass formulas or the mixing theorems
light sterile neutrino confirmed	The fermionic content of the 27 is exactly $J_{13} \oplus J_{23} = 16$, saturated by the 15 SM Weyl states $+\,\nu^c$; the diagonal idempotent slots carry no spinor content and J_{12} is fully bosonic (A726). A confirmed extra color-singlet $Q = 0$ Weyl fermion (e.g. an eV-scale sterile resolving a short-baseline anomaly) falsifies this 16-exhaustion; attacks the representation content of the verified assignment, not the mixing theorems
confirmed Majorana neutrino mass ($0\nu\beta\beta$)	Falsifies the Dirac-neutrino *benchmark assumption* (T4-D), not the PMNS mixing algebra: the bare singlet bilinear $M\,\nu^c\nu^c$ is gauge-invariant and algebraically available within the existing 16, excluded here by choice rather than by theorem [A729–A729c]. The non-gauging of $B{-}L$ is inherited from the compact rank-four input (S65 ladder); a $B{-}L$ gauge boson or seesaw branch would require a new vector plus breaking data outside the current input
extra long-range $U(1)_{B-L}$ gauge boson	A confirmed *massless* long-range force coupling to $B{-}L$ falsifies the *empirical* half of the R_v-selection (no fifth force) and would show $R_v = \mathrm{adj}(G_{\mathrm{SM}} \times U(1)_{B-L})$ was physical; a confirmed *massive* Z'_{B-L} instead falsifies the *structural* no-breaking theorem (a $B{-}L$-charged scalar VEV / **126**-analog is absent from the on-record **27**). Attacks the R_v-selection residue (D730 [HYBRID], S66), not the $J_3(\mathbb{O}_s)$ charge construction

4 Lepton-family validation as an internal benchmark

Rev29 upgraded the lepton-family validation picture substantially; *Rev29* corrects the tier at which that upgrade may be cited. The muon-to-tau ratio

$$m_\mu/m_\tau = (\phi/\sqrt{5})^{8/3} \times \sqrt{2}/10 = 0.059684 \qquad (+0.37\%\ \text{PDG})$$

is STRUCTURAL / LOADED-CORRESPONDENCE / REPRODUCED, *not* proved and *not* zero-parameter. Both $p = 8/3 = \dim \mathbb{O}_s/\operatorname{rank} J_3$ and $C = \sqrt{2}/10$ are algebraic quantities built from Jordan-algebra dimensions, but an exact dimension ratio is not by itself a mass exponent, and the divide-by-dim J_2 step producing C is recorded in A618 as "argued by analogy," with the formal derivation listed there as BLOCKING and closed nowhere in the corpus. The provenance is target-first rather than derivation-first: D616 first computes $0.0595/0.4220 = 0.141$ from PDG values and asks whether that number is algebraic in φ; A616 then supplies $\sqrt{2}/10$ as a 0.37% match while explicitly stating that "the algebraic origin is not yet proved"; only afterwards does A618 identify the divisor as $\dim J_2 = 10$. Consequently the +0.37% is not a residual awaiting a small correction — **it is the fit gap itself**: reproducing the measured ratio requires $C = 0.140894$ where the framework offers $C = \sqrt{2}/10 = 0.141421$, and because C multiplies the whole expression the 0.37% propagates unattenuated into m_μ/m_τ and into m_μ. The muon mass is $m_\mu = 106.05\,\text{MeV}$ (+0.37% PDG) at the same loading. Paper 0 already prints the honest attachment row for 8/3; this paragraph brings Paper 7 into agreement with it.

The electron mass chain is now closed. One-loop QED matching gives $\delta_{\text{QED}} = +0.3125\%$ and $m_{\mu,\text{phys}} = 105.72\,\text{MeV}$; with $m_\tau = 1776.93\,\text{MeV}$ the Koide constraint gives $m_e = 0.5076\,\text{MeV}$ (-0.66% PDG). The remaining internal mass-sector test is not the electron, but the formal S-matrix derivation of the heavy-quark 7/3 amplitude bridge.

5 Rev25 additions: the nuclear ledger, the selector falsifier, the anomaly closure, and the island program

The nuclear cluster ledger (Paper 9). Rev25 adds a second world-facing precision theatre: seventeen sharp binding-energy passes, deuterium through calcium, each against a pre-frozen bar, plus one typed obstruction (^{28}Si: passed its pre-answer 1% bar and entered through the frozen disjunctive refinement gate — neither a sharp nor a miss at a pre-answer bar), plus the Hoyle state, eleven correct sign calls, and two glueball bands (Paper 9, with its protocol sub-appendix carrying the complete receipt chain and honesty ledger). Its failure conditions are stated in Paper 9 directly: any scored miss at a *pre-answer* frozen bar, any *untyped* miss at any frozen gate, a broken sign call, or a vault-match failure under seal attacks the cluster kernel at paper tier. (Rev28 clarification, dated: the Rev27 text scored ^{28}Si as a miss. That was an over-correction — the 0.1% figure was a post-answer refinement gate whose frozen text was explicitly disjunctive and whose obstruction branch was taken on the record. The pre-answer bar was 1% and was passed at 0.334%. The correction is made here with receipts, not silently; the irreducibility claim itself remains falsifiable — a first-circulation fix refutes it.)

The occupation falsifier $\{31, 39\}$ (Appendix M). The rest-mass occupation selector, promoted to DERIVED-CONDITIONAL at Rev25, carries a live, hash-stamped public kill condition: the selector's period-two continuation $\{31, 39\}$ (sha `7fd5775e...`, stamped before verification). An established occupied rung outside the selector's support kills the derivation and reverts the occupation to Input.

Anomaly audit: closed exactly (fold of `s572`/`s588`). All Standard-Model gauge and gravitational anomalies cancel *exactly* from the framework's own derived hypercharges — the complete exact audit (`s588`, S214) upgraded the structural check (`s572`) and formally closed the hostile-board item T3-P. Cross-reference: Appendix F (gauge boundary). As a falsification row: any demonstration that a banked hypercharge assignment produces a non-cancelling SM anomaly would reopen T3-P and attack the charge construction directly; the audit says no such demonstration exists against the current bank.

The island-of-stability forecast (named program — not a result). The cluster kernel of Paper 9 supplies closed-form ladder weights, which Paper 9's declared cluster-ledger interface carries to binding energies (*Rev29*: predication corrected; the kernel prices a weight, not an energy), and two magic numbers ($\{2, 8\}$) are native to the framework; the remaining shell structure is open. A forecast of superheavy stability — the island — would require (i) the two-circulation sector (the typed ^{28}Si boundary), (ii) leg scaling beyond ten cells, and (iii) the missing magic numbers. It is registered here as future falsification theatre at *program* tier: no number is offered, and the program's first scored forecast, whenever it exists, must arrive with a pre-frozen bar like everything else in Paper 9.

6 Rev28 additions: the selection frontier and its falsifiers

The twenty-round fold recorded in Appendices S, T and X adds no world-scored number. It adds *named, checkable* failure conditions, which are listed here so that they are falsifiable in the same sense as the rest of this paper.

1. **AX-MG.** Exhibit a derivation of either clause — coefficient-one or degree-one — from the banked boundary rows without inserting an edge-functor property (unitality, trace-preservation, idempotence). Both clauses are currently proved *independent* by exact countermodel; a derivation refutes the pricing and improves the ledger.
2. **AX-COT.** [P-003] Exhibit an isometric universal property tying chamber, Cartan spin factor and occupancy simultaneously. Ordinary equivariance is proved insufficient; such a property would discharge the clause off shell.
3. **The observation functor.** Exhibit an admissible observable that distinguishes two loaded choices sharing a scalar character $w = c\varphi^{-m}$, or that sees the signature transition through a 15-dimensional even readout. Either computes a piece of the faithfulness kernel and unblocks the freedom total.
4. **The onset.** Derive $n^* = 7$ from level structure. The current statement is that the level structure does *not* select seven; a principle that does retires a priced integer marking.
5. **Einstein universality.** Compute the AdS linearized spectrum of the sourced heat-kernel action. A spectrum inconsistent with linearized general relativity falsifies the gravity-measure programme at the tier where its existence is already closed. [LIB2-301]
6. **The mass attachment.** Exhibit an interacting pole differing from the quasi-free gap $M_N = \Lambda\sqrt{\varphi^{2N} + \frac{1}{4}}$. AX-MASS-LABEL is a one-particle attachment and claims nothing beyond it.
7. **The record.** No falsifier is possible for the realized value: it is proved to be sample data. This is stated as a limit of the theory's claims, not as a result about the world.

What would *not* count. An improved χ-ladder expression fitted against the banked ratio vector. The suite has adopted a standing rule against such rounds until a physical generator and response map exist; the two cautionary rows are the off-support 0.0331562% kill and the 0.2744490308% legal miss, both recorded in Appendix X.

[LIB2-302]

7 Interpreting success and failure

Agreement with current data does not prove the framework. Disagreement in one of the primary falsification channels would count heavily against the present correspondence map. The electron-mass chain is KOIDE-CONSISTENT under the external rule $K = 2/3$ (S43) — an algebraic mass correspondence with a dual-route reconciliation owed against the Appendix O two-gap operator, not a joint derivation of the three charged-lepton masses (authoritative ledger: Appendix O, *The authoritative charged-lepton ledger*). The current open mass-sector test is the S-matrix derivation of the 7/3 lepton–quark bridge.

D4 closed

Rev29 closes the PMNS phase problem at the level of the operative framework target. The old ansatz value $-133.36°$ is superseded. [LIB2-277] The new target

$$\delta_{CP} = -\frac{2\pi}{\sqrt{5}} = -160.997°$$

is carried as a derived result with no free phase parameter inside the selected V_3 construction; the readout premises and the reactor forcing remain as stated elsewhere (Rev32.9). The corresponding physical invariant is $J_{CP} \approx -0.011$. The failure mode is therefore now sharply localized: a future miss attacks the V_3 rotation mechanism ($T_{\text{half}}|_{V_3}$, $\Omega = \sqrt{5}/2$), not the entire Jordan-algebra programme.

Atmospheric-angle precedent

The atmospheric angle is a useful case study in real-time falsification. The current framework value is the mismatch-geometry branch $\sin^2\theta_{23} = 7/16$, rather than the older higher-octant phenomenological placeholder — printed since Rev32.6 as the sharp loaded lower-branch target of the selected correspondence (A1566: the octant registration is a candidate map, not a selector; the internal identity stays theorem-grade). [LIB2-240, LIB2-284] Recent benchmark movement sharpened the need to state the branch explicitly rather than speak loosely of a "near-maximal" target. This is how falsifiable phenomenology should behave: predictions are stated, old placeholders are retired, and the target remains exposed to future data.

8 Conclusion

Paper 7 is the suite's shortest paper and one of its most important. The programme can fail. Long-baseline oscillation data, atmospheric-angle precision, electroweak weak-angle refinement, and charged-lepton-family validation have the power to decide whether the exceptional-Jordan correspondence is physically right, merely suggestive, or wrong.

Leibniz Quantum Beats Newton

Paper 8: Junkyard — Deleted Content, Dead Ends, and Junked Ideas (Rev33.2)

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August 2026

Abstract

Paper 8 is the rev7 archive of deleted content, dead ends, and candid programme commentary. It is not a graveyard in the sense of a list of proved impossibility theorems; that role belongs elsewhere in the suite. It is the record of routes that were explored seriously enough to matter, then abandoned because a simpler, clearer, or more algebraically closed alternative replaced them.

The central rev7 example is the Yukawa sector. A linear Freudenthal projection appeared attractive in earlier drafts but could not preserve the off-diagonal Higgs geometry without additional assumptions. The cubic invariant

$$\mathrm{Tr}\big[J_{\mathrm{vac}} \circ (\Psi \times \Phi_H)\big] \tag{1}$$

replaced it cleanly, so the linear route was deleted. Similar logic governs the other entries: modular attempts to derive the Higgs vacuum expectation value, Niemeier-lattice analogies, attractor-inspired weak-scale ideas, monopole and E8 digressions, and several speculative bridges were explored, recorded, and then parked.

The point of publishing this archive is methodological. A mature theoretical programme should publish its junk as well as its gold. The deleted routes in this paper are not embarrassing leftovers. They are part of the reason the surviving rev7 bundle is cleaner than its predecessors.

1 Deleted content from rev5 to rev7

Linear Freudenthal Yukawa (deleted)

Rev5 attempted to package the Yukawa sector through a linear Freudenthal projection. The route failed because it effectively forced the Higgs into a diagonal placement if the middle eigenvalue was to be isolated cleanly. That broke the $SU(2)_L$ doublet geometry used elsewhere in the framework. Rev7 therefore deletes the linear route and replaces it everywhere with the cubic invariant

$$\mathrm{Tr}\big[J_{\mathrm{vac}} \circ (\Psi \times \Phi_H)\big]. \tag{2}$$

H-embedding ambiguity (deleted)

Earlier drafts flirted with the idea that specifying the Higgs block might require a new axiom. The cubic-invariant route resolves that issue. No extra axiom is needed. The off-diagonal Higgs placement is preserved directly by the local cubic invariant.

2 Curios (moved, not deleted)

Boolean hypercube / I Ching trigram indexing (moved from Paper 0 Box C.1, S64)

Moved here during the S64 honest-language pass: hostile reviewers consistently read the box as numerology despite its disclaimer. The content is unchanged; it remains an observation on shared combinatorial structure [A626], not a formal isomorphism and not load-bearing anywhere in the suite.

While the formal algebraic breaking of the $(4, 4)$ vacuum is governed by $J_3(\mathbb{O}_s)$, its combinatorics can be illustrated by a classical system. The basis of $\mathbb{O}_s$ is indexed by the Boolean hypercube $\mathbb{Z}_2^3$ — the same underlying combinatorial structure that generates the I Ching's eight trigrams (first identified by Leibniz in 1703 as binary arithmetic). Under this identification, $\{e_0, \ldots, e_7\}$ correspond to the eight trigrams, with $e_0 \leftrightarrow 000$ (pure yin, Earth) and $e_7 \leftrightarrow 111$ (pure yang, Heaven).

The 64 entries of the $\mathbb{O}_s$ Cayley table map naturally to the 64 hexagrams. The 8 *pure* hexagrams (squared basis elements, $e_i^2 = \pm 1$) correspond to diagonal Peirce sectors contributing zero to the fermionic Coleman–Weinberg potential ($V_{\mathrm{CW}}^{\mathrm{bos}} \equiv 0$). The 56 *mixed* hexagrams correspond to off-diagonal Peirce sectors P_{12}, P_{13}, P_{23} that drive $V_{\mathrm{CW}}^{\mathrm{ferm}} \neq 0$ via the Möbius anti-periodic boundary condition.

The solved vacuum $J_{\mathrm{vac}} = \mathrm{diag}(\varphi, 1, \varphi^{-1})$ assigns a natural reading to the three generation face-centres: φ (yang-weighted, τ), 1 (balanced, μ), φ^{-1} (yin-weighted, e). *This is an observation on shared combinatorial structure [A626], not a formal isomorphism; the Fano-plane sign structure of $\mathbb{O}_s$ has no I Ching counterpart.*

PSL$(2, 7)$ class-algebra structure constants (moved from the library, S271)

Two structure constants of $\mathrm{PSL}(2, 7) = \mathrm{GL}(3, \mathbb{F}_2)$, computed from the character table via the class-algebra formula $c_{i,j}^k = (|C_i|\,|C_j|/|G|) \sum_\rho \chi_\rho(C_i)\,\chi_\rho(C_j)\,\overline{\chi_\rho(C_k)}/\dim\rho$, are PROVED [D161, S19]:

$$c_{4A,7A}^{7A} = 14, \qquad c_{4A,7A}^{7B} = 0.$$

The second vanishing is exact, and it was the algebraic content of the old chirality argument: the subtractive combination $-\arccos(-1/3) + 2\pi/7$ would require $c_{4A,7A}^{7B} \neq 0$, so it is forbidden, leaving $-\arccos(-1/3) - 2\pi/7 = -160.900°$ as the only allowed sign.

Why it sits here. That application is DEPRECATED. The authoritative CP phase is $\delta_{CP} = -2\pi/\sqrt{5} = -160.997°$ from the V_3 rotation (DERIVED-CONDITIONAL after the Rev29 re-tiering; pre-Rev29 stamp $\star 100$; Paper 5, Appendix C), which does not involve $\mathrm{PSL}(2, 7)$ at all; the $2\pi/7$ expression survives elsewhere in the suite only as an explicitly labelled cross-check and must not be re-introduced as a derivation. [LIB2-245] The constants above therefore have no load-bearing role left anywhere. They are recorded because the algebra is correct, not because anything rests on it. [LIB2-057, LIB2-245]

On $14 = \dim G_2$. The coincidence is real, and it is why the result was noticed at all. It is **not** a link to any G_2 object in this suite. The suite's G_2 occurs in two guises — the dynamical anchor $\Lambda_{G_2} \approx 260\,\mathrm{MeV}$ ($260 \pm 0.5\,\mathrm{MeV}$, Appendix X; Appendix A, a dimensional-transmutation scale) and the split real form $G_{2(2)}$ in the Jordan chain — and neither of those is the dimension of the compact automorphism group turning up as a class-algebra integer. No mechanism relating a $\mathrm{PSL}(2, 7)$ structure constant to $\dim G_2$ has been proposed or tested anywhere in the corpus. [LIB2-057] The

equality is therefore a numerical coincidence with no established mechanism: an observation, in the same sense as the trigram indexing above, and not a formal correspondence.

Library entry: `L-StructConst_PSL27_structure_constants.md`.

3 Dead ends and why they failed

The S197/S208 Tier-C clean negatives (recorded S270)

The following results were verified, grep-anchored against this suite, and then left unbanked for sixty-five sessions. They move no observable; that is precisely why they went unrecorded, and precisely why they belong here. Each is reproduced from the S208 reconciliation ledger with its provenance.

	Provenance	**Result**
C1	T1-a	σ-mass-gap is **not** an $F_4 = \mathrm{Der}(J_3)$ Cartan object: it is the $L_{J_{\mathrm{vac}}}$ multiplication in the $\mathfrak{e}_6/\mathfrak{f}_4$ complement, $\|\mathrm{proj}_{\mathrm{Der}}\| = 10^{-16}$. A one-line no-go.
C2	T1-c, `s364`+`s391`	Charged action-level Dirac mass is **forbidden** (no-Q theorem, triple-confirmed); the neutral 2×2 Majorana channel is allowed.
C3	T1-d	912 $\Theta(A)$ gauging is **blind** to the hypercharge residue A — the compact $\mathfrak{g}_{\mathrm{SM}}$ is excluded for *every* A, including the Standard-Model $A = 2$. A decisive negative.
C4	T1-e, A1208	Extraction retired: defect ranks are congruence / frame-orbit **invariants**, so no legal frame repair exists (house verification of A1208, S197). The original kernel `s449` is not recoverable and is no longer cited as the anchor.
C5	T1-f	A1111 four-corrections errata: $T^2 = -I \not\Rightarrow$ antiperiodic/Möbius; one warp $\not\Rightarrow$ both EM and mass; the half-twist commutes with mass. The corrected facts are already on disk; the *errata block* is what was missing.
C6	T2-e	$\mathfrak{so}(6,6) = D_6$ is an $\mathfrak{e}_{7(7)}$ rung, and the 28:28 versus 30:30 distinction matters: **"30:30 = T-bridge" is false**. The $SU(2)$ = weak identification is fenced, not assumed.
C8	T2-f	Selector Σ parity-split end-state: EVEN $= U_{\mathrm{rec}} + K_*$+ gaps, ODD $= \kappa_\perp + \varepsilon$ (frozen at A1208). The honest statement is that rest-mass + mixing is *a named section, not a derivation.*
C9	T3-b, 3/3	The Σ_{wb} winding-monotone rest-mass driver is **falsified by the leptons**: the residuals alternate in sign.

Two honest notes on this table. First, **C7 is deliberately absent**: the Peirce-arrival ordered-vertex theorem ($Y_f = q \cdot J_{\text{vac}} \cdot C$ forced by chiral middle-slot routing, $\Delta = 3 - \sqrt{5}$ unique) is a *positive* result — the selection principle behind the A3/B-mixing route — and does not belong in a junkyard. Its conclusion is already folded into the mixing paper; its premise is not, and that remains owed. Second, **C10 is deliberately absent**: the charge-clock tower $(q_0^{\text{KK}})^2 = 1/24 = 1/(4 \cdot 6)$ with the Ω_Q Plücker line is marked HOLD at source — deep but conditional, gated on the Ω_Q provenance that Appendix N, *The carrier truncation is also chosen* shows is still a choice. It is library-resident by decision, not by neglect. [LIB2-355]

Still owed. The S208 ledger homes C2, C8 and C9 substantively in Appendix O (mechanism section, honest end-state, and the falsification record respectively). Recording them here makes them findable; it does not discharge those folds.

Modular-forms route to the weak scale

A modular-forms route was explored as a way of deriving the electroweak scale from the same golden-ratio data that organise the algebraic vacuum. The route never acquired an algorithmic bridge from $J_3(\mathbb{O}_s)$ to modular weights. It is parked rather than promoted.

Niemeier-lattice route

The programme explored whether the Peirce eigenvalues and the 24-dimensional Niemeier-lattice story could constrain the Higgs vacuum expectation value. Structural parallels existed; a derivation did not. The idea is deferred.

Extremal attractor mechanism for the electroweak scale

Because cubic invariants play a prominent role in supergravity attractor equations, the possibility of deriving the weak scale from an attractor construction was examined. The formal analogy was real; the scale derivation was not achieved. The route is deferred to a later revision.

Seesaw mechanism as a required neutrino ingredient

A conventional seesaw route was tested as a mandatory extra layer for neutrino masses. In the present scheme it turned out not to be structurally necessary for the basic neutrino mass ratios, so it was omitted from the operational rev7 bundle.

Proton-decay rate estimate

A genuine proton-decay calculation would require a concrete high-scale unification completion and a definite heavy gauge-boson sector. The current algebraic suite does not yet contain that completion. Proton decay is therefore deferred rather than advertised prematurely.

Monopole condensation route

A monopole-condensation story was explored as a symmetry-breaking mechanism. It proved more complicated and less directly constrained than the cubic-invariant route. It was abandoned.

E8 embedding

The programme repeatedly checked whether an E_8-based unification picture could replace or complete the $J_3(\mathbb{O}_s)$ story. Interesting parallels were found, but no concrete derivation resulted. The rev7 suite remains explicitly based on $J_3(\mathbb{O}_s)$, not on E_8.

4 Fenced: out-of-scope work (recorded, not promoted)

This section is the explicit pen for results that reach past the framework's remit — the dimensionless flavour observables of $J_3(\mathbb{O}_s)$ and the single hadronic calibration scale Λ_{G_2} — into territory the suite does not claim to model. They are not dead ends (they may be revisited) and not errors (they were computed correctly in their day); they are simply *out of scope*. We record them here, fenced, rather than fold them into the suite proper, so the falsifiable core stays clean and the strays stay visible.

Hadronic over-reach: the baryon/meson ratio $M_B/M_M \approx 6.7$ (S23–S24)

A self-consistent Faddeev-pole gap equation produced a baryon/meson mass ratio $M_B/M_M \approx 6.7$, matching the PDG proton/pion value 938/140 at a single free coupling $\alpha \approx 0.0025$ (the ratio stays in 5–8 across all perturbative α). [LIB2-246] Three reasons it is fenced rather than promoted: it reproduces *one* hadron ratio (a single number is not a spectroscopy programme); it carries a free parameter; and it was computed in the legacy ambient-27 Hessian parametrization (spectral trace $A = 144$, Morse index 17/27) that the current $\{\varphi, 1, \varphi^{-1}\}/\mathbb{O}P^2$ description has since replaced. An in-frame recompute (kernel `s284`, S175) confirms it does *not* survive: the current $N = 1$ cone yields only φ-power eigenvalue ratios, and $938/140 = 6.72$ is not a clean φ-power (nearest $\varphi^4 = 6.854$, +2%). The number is therefore **retired** — a legacy null-cone artifact superseded by the current frame, not a parked candidate. [LIB2-246, P-022] The *structural* content underneath it — that the baryonic (cubic) channel survives the quadratic one-loop cancellation that kills the mesonic channel — is kept in the suite as a theorem (Appendix I, *Confinement as the spectral geometry of the cubic norm*). [LIB2-246] Only the number is parked here. Library: `L-BaryonMesonRatio`, `L-FaddeevEvasion`, `L-A144`, `L-TMatrixResidue`, `L-HybridMassGap`. [LIB2-352]

Golden flux-tube tension $\sqrt{\sigma} = \Lambda_{G_2}\,\varphi$ (S164 era)

A flux-tube relation tying the string tension to the confinement scale by a single golden factor, $\sqrt{\sigma} = \Lambda_{G_2}\,\varphi$. It is thinly banked (no canonical library entry; it survives only as a notebook pointer), and it reproduces no datum independent of Λ_{G_2} itself — it restates the one calibration scale dressed by φ. The S175 in-frame recompute (kernel `s284`) does *upgrade* its provenance: on the current $N = 1$ cone it is exactly the off-diagonal Hessian eigenvalue ratio $2\xi_1/2\xi_2 = \varphi$ ($\xi = (\varphi, 1, \varphi^{-1})$), hence frame-current and α-free, and it is now recorded as a structural remark in Appendix I (the in-frame flux-tube remark, *Dynamics of the Golden Vacuum*). But because it still merely restates

Λ_{G_2} (no Λ-independent content), it remains out of the falsifiable core — recorded here as a suggestive coincidence pending a derivation that earns its keep. (The companion S164-era observation that the golden ray reads as a relativistic boost, $\cosh(2\ln\varphi) = 3/2$, i.e. $\varphi^2 + \varphi^{-2} = 3$ (the DET-7 condition of Appendix E; equivalent to AX1 for $\rho > 1$), is *in* scope as kinematics and belongs with the time/clock bridge, not here.)

The S103 moment ledger: parked candidate (60-tick clock, time-silence, Gogberashvili splice)

The S103 "moment ledger" (filed 2026-06-13, status CANDIDATE, never star-gated) is parked here by PI decision rather than promoted. [LIB2-201] Its three results: (i) one observed *moment* as a single traversal of the $S^1/\mathbb{Z}_2$ base rotation circle, with the 56 substrate states carrying $\Sigma|\text{winding}| = 60$ ν-ticks; (ii) *time-silence* — a clean *derivable negative*: no monomial in the banked constants $\{q_0^{\text{clk}}, \kappa, \Gamma\}$ carries a time dimension, so a duration in seconds is not derivable from the framework's mass/length-complete constant set (any seconds figure is illustrative, never banked); (iii) the *Gogberashvili splice* reconciling the house constants with an external $\{c, \hbar\}$ time axis on the shared $SO(4,4)$ signature. [LIB2-201, LIB2-202, LIB2-203] Result (ii) is a genuine scope statement — the framework is time-incomplete by construction — and is the part most worth keeping; (i) and (iii) are suggestive but unpromoted. [LIB2-202, LIB2-203] The papered Moment Clock + T-operator (Appendix H) is unaffected; only the candidate headline numbers are parked. [LIB2-201] Library: `L-S103-MomentLedger`.

> **Why these are fenced, not deleted.** The lab notebook keeps the good, the bad, and the ugly. Out-of-scope results are real work that simply sits outside what the present framework claims to predict; deleting them loses provenance, promoting them enlarges the attack surface for no predictive gain. The fence is the honest middle: visible, dated, and explicitly outside the falsifiable core (Appendices B/C). [LIB2-203] Future strays that overreach the flavour-plus-one-scale remit belong in this section.

5 PI's junked ideas

Octonion-based supersymmetry

The idea that non-associativity might itself encode supersymmetry was explored as a brainstorming direction. It would have required new particles and new axioms. The rev7 bundle is intentionally non-supersymmetric and does not pursue this path.

Gravitational coupling from the Jordan determinant

Another brainstorm attempted to read Newton's constant directly from the Jordan determinant. No natural scaling law emerged, so the idea was shelved.

Cosmological constant from a trivial Peirce block

A dark-energy interpretation of a trivial Peirce block was considered. No mechanism was found that would naturally suppress the cosmological constant to the observed scale. The idea was abandoned.

Electroweak precision tests as the best CP-falsification channel

A kaon- or electroweak-precision-based falsification route for the geometric δ_{CP} prediction was explored. The conclusion was negative: long-baseline neutrino measurements remain the more direct test.

Spinor-octonion duality

The idea that fermions might be literally octonion-valued spinors rather than ordinary spinors acted on by an octonion algebra was examined and dropped as an unnecessary over-complication.

The $\mathbb{Z}_{12}$ fused clock (Rev14, S86)

The proposal that the $\mathbb{Z}_4$ tick and the $\mathbb{Z}_3$ generation cycle fuse into a single $\mathbb{Z}_{12}$ clock generator was killed by computation: the candidate generator C is not in the normalizer of $\mathfrak{e}_{7(7)}$ ($C \notin N(\mathfrak{e}_{7(7)})$, S86 probe). Only the trivial product reading $\mathbb{Z}_3 \times \mathbb{Z}_4$ survives, which adds nothing. Junked; the tick and the generation structure remain independent.

The anti-symplectic $T^2 = +1$ moment model (superseded, Rev15, S106l→S107)

An earlier abstract model of the past↔future "moment" (S106l) built the operation T as a hand-made real swap on $\mathbf{27} \oplus \overline{\mathbf{27}}$ and read it as *anti-symplectic* with $T^2 = +1$. The live computation on the certified **56** (S107a) overturns both readings: T is a genuine element of $E_{7(7)} \subset Sp(56, \mathbb{R})$ — hence *symplectic*, not anti-symplectic — and squares to -1 (the Kramers/spinor half-turn), so antiunitary time reversal appears as $T^2 = -1$. The abstract model is therefore superseded; the correct live statement is recorded in Appendix H (Theorem H.1). Retained here only as the cautionary note that a real-swap caricature of a substrate operation can flip both its symplectic character and its square.

6 Candid commentary

The rev7 suite was improved mainly by deletion. The cubic invariant proved simpler than the linear route. The weak-angle gap proved to be an ordinary threshold problem rather than a fatal contradiction. The electroweak scale proved more stubborn than hoped. The archive therefore records a simple lesson: the most elegant route is often the one that removes assumptions rather than the one that adds them.

7 Archive index

Category	Count
Deleted rev5 content	2
Dead ends and deferred routes	7
PI brainstorm ideas	5
Total	14

Paper 8 is published with the bundle because scientific maturity includes an archive of discarded routes, not only a catalogue of surviving claims.

Leibniz Quantum Beats Newton

Paper 9 — The Nuclear Cluster Tier: Binding Energies from the Golden Fusion Category, Deuterium through Calcium (Rev33.2)

Seventeen sharp binding energies (worst untyped residual 0.77%, ^{3}H*) plus one typed obstruction (*^{28}Si*,* 0.334%*; pre-answer bar* 1% *passed, refinement-gate obstruction branch taken), the Hoyle state, eleven correct sign calls, and two glueball bands — every number scored against a bar frozen before the answer existed. The composition ladder is a closed-form categorical trace invariant. All values are ratios to* $\sqrt{\sigma}$*; the absolute scale is the framework's one registered input (Appendices O/X). Sessions S225–S226; assessments A1349–A1357; kernels* `s705`–`s747`*.*

Tom O'Sieg

August 2026

Posture (binding). This paper promotes the S225–S226 nuclear/cluster arc from the lab-notebook layer to paper tier. The promotion case is protocol-based: seventeen sharp passes at pre-frozen accuracy bars (plus one typed ^{28}Si obstruction, §9) across ten scored dispatch rounds with *zero* threshold moves, three blind vault matches (§10), and a freedom ledger audited round-by-round. Everything here is *dimensionless* (binding energies as ratios to the string tension $\sqrt{\sigma}$); the single dimensionful input of the framework is registered separately (Appendices O/X). The engine workbooks are unchanged; no public observable of the Rev24 suite moves. The continuum (Yang–Mills) functor remains the named missing object: nothing in this paper is a Clay-problem claim. [LIB2-300] **Known soft flank, stated up front:** charge *signs* and *counts* are derived, but charge *magnitudes* are open — two exposed defects are filed in §9 ($S_p(^{9}\mathrm{B}) \times 1.29$, $S_p(^{8}\mathrm{B}) \times 2.60$). Read the seventeen-sharp ledger (one typed ^{28}Si obstruction, scored as neither a sharp nor a miss at a pre-answer bar — §9) with that flank in view.

Contents

1 Scope and the one-line result **1**

2 The functor and the cluster menu **2**

2.1 The exhibited functor F_φ 2

2.2 The morphism menu 2

3 The $A \leq 4$ binding kernel **3**

4 α-closure and the gap theorems **4**

5 Bridged and Borromean systems, $5 < A < 12$ **4**

6 The ladder: a closed-form trace invariant **4**

7 The Hoyle state and the breathing kernel **5**

8 The ledger **6**

9 Typed features and open items **8**

10 Protocol sub-appendix: the receipt chain **10**

11 Status ledger **10**

1 Scope and the one-line result

The golden fusion category machinery of this suite (the Fibonacci quotient with $f_4 = 0$, the exhibited functor F_φ, and a discrete menu of φ-monomial morphism weights) fixes the ledger weights $\mathcal{T}_n$, which the declared cluster-ledger interface (2) then carries to nuclear binding energies (*Rev29*: predication corrected; the categorical machinery does not itself price an energy, the interface does). [LIB2-143] The one-line result:

> From deuterium to calcium, every scored binding energy lands inside a pre-frozen sharp bar with one typed exception — **seventeen sharp passes**, worst *untyped* residual 0.77% (^{3}H; 0.37%, ^{24}Mg, is the worst of the $A \geq 5$ non-silicon continuation), plus one *typed obstruction* (^{28}Si, 0.334%: pre-answer bar 1% passed; refinement-gate obstruction branch taken) — the Hoyle resonance passes at its frozen bar, eleven sign/stability calls are all correct, and the $n\alpha$ composition ladder closes into a one-line trace formula whose two blind forecasts matched numbers sealed in advance (a φ^2 match at 26 ppm and a $\mathcal{T}_9$ match at 0.10%).

Throughout, world values are AME binding energies; $\sqrt{\sigma} = 445(3)(6)$ MeV is imported solely to express MeV; sharps below about 1.5% are therefore central-value agreements: the scale alone moves each absolute prediction by ±1.51% (`s1209`); the inputs of every formula are A, Z, the selector depth $N_\star = 11$, and φ — nothing else. No fine-structure constant enters: the charge penalties below do Coulomb-like work with φ-monomials alone (declared, audited, and the charge *magnitude* sector is honestly the weakest — see §9).

2 The functor and the cluster menu

2.1 The exhibited functor F_φ

F_φ sends the winding category — registered at Rev33.2 as $\mathrm{Wind}_{\mathrm{ev}} := T_\delta$, framed tangles on one self-dual strand with loop value δ (`s1334`, `s1335`) — to the Fibonacci category, $A \mapsto e^{3\pi i/5}$: objects

$F_\varphi([N]) = \tau^{\otimes N}$; the mirror is the dagger, a structure and not a relation (imposing $R = R^{-1}$ would kill F_φ: $\|R - R^{-1}\| = \sqrt{5}$), and order-reversing (the covariant crossing mirror $\sigma_i \mapsto \sigma_i^{-1}$ agrees with it on generators and differs on $\sigma_1\sigma_2$, `s1280`); the four-strand Jones–Wenzl projector is killed, $F_\varphi(f_4) = 0$ (the $[5]_\delta = 0$ specialization at $\delta = \varphi$), and through six strands the kernel on TL_N is exactly the ideal it generates (`s1334`). It was exhibited with its full generator set and kernel-verified (A1349, kernel `s706`: F-involution, braid relation, both mirror-inversion laws, all $\leq 3 \times 10^{-16}$; $\mathrm{Spec}(\varphi^{-1}N_\tau) = \{1, -\varphi^{-2}\}$ exact). This is the object referred to below as the **cluster kernel**: the binding rules of this paper are morphism-weight statements in the image of F_φ. [LIB2-143]

A standing fact about the recoupling matrix, recorded as a fence (Rev30, A1480). The Fibonacci recoupling matrix is an *involution*: $F^2 = I$. House check: $(F^2)_{00} = \varphi^{-1}(\varphi^{-1} + 1) = \varphi^{-1} \cdot \varphi = 1$ with the off-diagonal identically zero. **Consequently $F^5 = F$ and $(F^5)_{00} = \varphi^{-1}$ — not φ^{-5}.** Any argument that accumulates a golden power by composing an odd number of F-moves is therefore *false as stated*, not merely unproven: the moves do not accumulate, they alternate. This is printed here as a fence rather than as a correction — no such five-move monodromy argument appears anywhere in this suite, and this note exists so that none is introduced. It bears on no banked binding formula in the menu below.

2.2 The morphism menu

Every banked binding formula is assembled from a short, closed menu; no entry was added after S226. Generators (with their derivations where banked):

- **M1, the two-body bridge** (S=1, T=0): the Fibonacci singlet channel $\tau \otimes \tau \to \mathbf{1}$ exists iff spin-symmetric/isospin-antisymmetric — the topological Pauli statement. Weight $\varphi^{-N_\star} = \varphi^{-11}$ (the deuteron). Consequence, scored 3/3: *nn* unbound, *pp* unbound, *pn* bound (A1352).
- **M2, the closed α-cell**: $f_4 = 0$ makes $A = 4$ a closed Jones–Wenzl cell; $\alpha \equiv \mathbf{1}$. The α is *saturated*: a fifth nucleon sees $\mathbf{1}_\alpha \otimes \tau = \tau$, a boundary defect with no scalar channel. [LIB2-146]
- **M3, the triple-α associator $\Theta : \mathbf{1}^{\otimes 3} \to \mathbf{1}$**: three-strand associator data survives $f_4 = 0$ while the four-strand channel dies — the same quotient that forbids ^{8}Be builds carbon (A1353).
- **M5, the Borromean Y-junction $Y : \mathbf{1} \otimes \mathbf{1} \otimes \tau \to \mathbf{1}$** (^{9}Be: cut any leg and closure dies — nature's $\alpha\alpha/\alpha n/\alpha\alpha n$ pattern). *Fence (Rev32.9, A1671 F03):* in the ordinary Fibonacci category $\mathrm{Hom}(\mathbf{1} \otimes \mathbf{1} \otimes \tau, \mathbf{1}) = \mathrm{Hom}(\tau, \mathbf{1}) = 0$, by the same fusion rule that leaves M2's saturated α with no scalar channel, so Y as printed is the zero morphism. A nonzero Y needs an enrichment this suite does not exhibit (a boundary charge or a retained τ-line capped outside the three legs, and a selector for the α-cell inside $\tau^{\otimes 4} = 2 \cdot \mathbf{1} \oplus 3 \cdot \tau$). M5 is carried as a leg-counting pattern: the ^{9}Be rule below uses it to count (cancel one annulus defect, add one bridge step), not as a matrix element, so no banked number changes; the ^{9}Be derivation inherits this gap.
- **M6, the halo dual $Y' : \mathbf{1} \otimes \tau \otimes \tau \to \mathbf{1}$** (^{6}He: the dineutron's singlet channel is typed wrong in vacuum; the closed cell supplies the annular cap — Borromean, not pairwise; A1355, with the same mechanism pre-registered blind in-house by the Skyrme-lens kernel `s735`).
- **Leg rules**: composite-**1** and τ-cluster leg typing; leg-local attachment of odd lines (the endpoint rule: two *unequal* images, φ^{-18} junction-near + φ^{-19} free-endpoint, Möbius-delayed; A1356).

- **Charge generators** (retro-derived, A1354): the half-tick $\frac{1}{2}\varphi^{-12}$ ($\mathbb{Z}_4$ half-tick averaged over the Möbius sheets); the fused pair $\varphi^{-11} = 2 \cdot \frac{1}{2}\varphi^{-12} \cdot \varphi$ (two half-ticks fused in the τ-channel pick up the Perron–Frobenius eigenvalue: *coherence multiplies by* φ — exact); the count $3 =$ the Fano line $\{0, 1, 3\}$ (the selector's own line reappearing as an image-link count).
- **The universal second-order defect** φ^{-18}: the closed-cell annulus subtraction, appearing in ^{8}Be, nn, the ^{7}Li/^{7}Be splitting, the ^{9}Be/^{9}B corrections, the endpoint rule, and the breathing kernel — one constant, seven appearances.
- **The breathing generator** $\Delta_{\mathrm{br}} = \varphi^{-15} - \varphi^{-18}$ with $15 = 8 + 7$ (triangle scale + heptad circulation), the *same* decomposition banked for the Hoyle exponent (§7).

3 The $A \leq 4$ binding kernel

The rule, filed before any number (A1349):

$$\frac{B(A,Z)}{\sqrt{\sigma}} = \varphi^{-11}\Big[\binom{A}{2} + 2\varphi^{-2}\binom{A}{3} + (\varphi^3 - \tfrac{1}{2\varphi})\binom{A}{4} - \frac{1}{2\varphi}\binom{Z}{2}\Big]. \tag{1}$$

The deuteron is the $\binom{2}{2}$ term alone (machine-exact); the scored sharps are ^{3}H 0.77%, ^{3}He 0.10%, ^{4}He 0.16% at the frozen 1% bar. *Freedom audit (carried verbatim):* the rule introduced three new φ-coefficients and was scored on three new nuclei with a triangular dependency — zero net overdetermination within $A \leq 4$, so *no blind credit* for the coefficient choice. [LIB2-145] The discriminating test was structural: $A \geq 5$ has no new binomial available, and everything from §4 on is scored against that zero-freedom continuation. [LIB2-145]

4 α-closure and the gap theorems

Novelty label: physical interpretation not established here.

Theorem 4.1 ($A = 5$ gap)**.** *$f_4 = 0$ closes the α cell, so a fifth nucleon is an external τ-line with no scalar channel: no single nucleon binds to α. [LIB2-146] Signs (choice-free): $S_n(^5\mathrm{He}) = -\varphi^{-12} < 0$, $S_p(^5\mathrm{Li}) = -\frac{3}{2}\varphi^{-12} < 0$ — both correct (A1350).*

Novelty label: physical interpretation not established here.

Theorem 4.2 ($A = 8$ gap)**.** *$\alpha \otimes \alpha = \mathbf{1} \otimes \mathbf{1}$ exposes no τ-line: no bridge is typeable, $B(^8\mathrm{Be}) = 2B_\alpha$ at leading order, and the first correction is the level-repulsive virtual $\tau\bar{\tau}$ annulus, $S_{\alpha\alpha} = -\varphi^{-18}\sqrt{\sigma} < 0$. [LIB2-147] Sign correct; the pre-frozen marginality tier was hit ($|S|/\sqrt{\sigma} = 1.7 \times 10^{-4} < 10^{-3}$; report magnitude 0.84× world). Stars make carbon through the triple-α channel because $\mathbf{1} \otimes \mathbf{1}$ offers no bridge while the three-strand associator survives (A1351, A1353). [LIB2-147]*

These two are *consequences*, not inputs: the same projector that closes the α forbids its pairwise growth and forces the associator architecture of §6. [LIB2-146]

5 Bridged and Borromean systems, $5 < A < 12$

$^{6}\mathbf{Li} = \alpha + d$ + one Fibonacci bridge one step past the selector depth (φ^{-12}); closed form $\varphi^{-11}(7 + 8\varphi^{-2} + \varphi^{3})$; sharp 0.11% (A1350). $^{7}\mathbf{Li} = \alpha + t$ with the neutral $N_{\star}$-bridge (φ^{-11}); $^{7}\mathbf{Be} = \alpha + {}^{3}$He with the charged mirror bridge ($\varphi^{-11} - \frac{1}{2}\varphi^{-12}$, the half-step penalty reused), refined by the three proton-image links ($-3\varphi^{-18}$) and completed by the breathing kernel (§7): final sharps -0.0074% and $+0.076\%$ at the tightened 0.25% bars, with the mirror differential preserved *by type* (a mirror-even object cancels identically in the difference; verified at 2.5×10^{-17}). $^{9}\mathbf{Be}$: the Y-junction cancels the $\alpha\alpha$ annulus defect and adds one bridge step, $S_n = \varphi^{-12} + \varphi^{-18} > 0$, sharp 0.18%; the mirror ^{9}B dies correctly, $S_p = -\varphi^{-13} + \varphi^{-18} < 0$, with the shadow-tier magnitude $S_p = -\varphi^{-13} + 8\varphi^{-18}$ ($8 = 2 \times 3$ Fano image links $+2$ endpoint images) at $\times 1.29$ of world (A1353, A1355). $^{6}\mathbf{He}$: the halo, $B = B_{\alpha} + \varphi^{-13} + \varphi^{-18}$, sharp 0.0005% (AME 29.2711 MeV; the 0.0039% printed through Rev33.1 was scored against a mis-transcribed 29.2701, house defect S372.2), with both A=6 signs correct (^{6}He bound, ^{6}Be killed by the already-derived two-face φ^{-11}). $^{8}\mathbf{Li}/^{8}\mathbf{B}$: the endpoint rule (two unequal images $\varphi^{-18} + \varphi^{-19}$; the neutral junction-near image is absorbed into the triton bridge under vacuum normalization for ^{8}Li, while charged ^{8}B keeps both) — sharps 0.0027% and 0.0049% (A1356, pass 2 refiling under clean receipt, mechanism-carried).

6 The ladder: a closed-form trace invariant

For n-fold α composites the banked law is

$$\frac{B(n\alpha)}{\sqrt{\sigma}} = n\,B_{\alpha} + \mathcal{T}_n\,T_3, \qquad T_3 \equiv \varphi^{-8} - \varphi^{-11} \tag{2}$$

(T_3 = the net triple-α associator weight, so $\mathcal{T}_3 = 1$ by construction), and the ladder coefficients close into one formula (A1356, A1357):

$$\boxed{\mathcal{T}_n = (n-2) + \varepsilon_n, \qquad \varepsilon_n = \mathrm{tr}_{\phi}(R_n)} \tag{3}$$

where $(n-2)$ counts the triangle slots in any pants decomposition of the n-gon and ε_n is the Markov trace of the residual boundary-circulation class of the n-gon flip graph in the Temperley–Lieb quotient $\mathrm{TL}_{\varphi}/\langle f_4 \rangle$. [LIB2-148] The derived selection rule (A1357):

$\varepsilon_5 = -\varphi^{-2}$ (the pentagon F-move coherence loop: the killed channel appears once, negative trace),

(*Rev32.9, A1671 F04:* the parenthetical is a mnemonic, not a computation. In the quotient $\mathrm{tr}_{\varphi}(\bar{f}_4) = 0$, so the sign has to come from a defined signed residual of the class R_5, which this paper does not write down; likewise the representatives R_n, the oriented hexagon pair and the $n \geq 7$ argument are banked from A1357 and not exhibited here. The values ε_n are not disputed; their derivation is owed.)

$\varepsilon_6 = 0$ (the hexagon's two boundary orientations are mirror-paired in the annular quotient and cancel),

$\varepsilon_n = \varphi^{5-n}$ $(n \geq 7)$ (exactly one noncontractible class survives, decaying φ^{-1} per added cell). $[LIB2 - 149]$

Note $\mathcal{T}_5 = 3 - \varphi^{-2} \equiv \varphi^{2}$ is exact algebra: *the pentagon identity fixes the ledger weight $\mathcal{T}_5$, and the interface* (2) *carries that weight to a binding energy* (*Rev29*: predication corrected; the identity prices a weight, not an energy). [LIB2-148] Scored consequences, all against frozen bars: ^{20}Ne 0.00031%

($\mathcal{T}_5 = \varphi^2$ — the best number on the ledger; a central-$\sqrt{\sigma}$ match, §8, look-elsewhere census) · ^{24}Mg 0.37% ($\mathcal{T}_6 = 4$) · ^{28}Si 0.33% ($\mathcal{T}_7$; typed, see §9) · ^{32}S 0.024% ($\mathcal{T}_8$) · ^{40}Ca −0.032% ($\mathcal{T}_{10}$; doubly-magic calcium at ten cells, matching the in-house blind application `s745` exactly) · ^{36}Ar report 0.018% ($\mathcal{T}_9 = 7 + \varphi^{-4}$).

Remark 6.1 (the two blind vault matches)**.** Twice, the closed form produced a number that had been extracted from world data and *sealed* in a hash-stamped pre-commit before the question was asked. (i) The s732 vault held $k_{\text{implied}}(^{20}\text{Ne}) = 2.617965$, never disclosed; the lane derived $\mathcal{T}_5 = \varphi^2 = 2.618034$ from the pentagon identity alone — 26 ppm. (ii) The s737 answer sheet held the implied $\mathcal{T}_9 = 7.1385$ behind the sole hint "at $n = 9$ the ladder does not do the obvious thing"; the rule declined the linear 7 and produced $7 + \varphi^{-4} = 7.1459$ — 0.10%, against a vault the lane never saw. Receipts, hashes, and the unseal schedule are in §10.

7 The Hoyle state and the breathing kernel

Hoyle (scored PASS at the frozen 10% bar, A1354):

$$E_{\text{Hoyle}} - E_{3\alpha} = (\varphi^{-15} + \varphi^{-18})\,\sqrt{\sigma} = 0.4033 \text{ MeV} \quad \text{vs world } 0.3796 \text{ MeV } (6.2\%).$$

The resonance that makes carbon in stars is the trace-free breathing mode of the same associator whose ledger weight enters the ^{12}C binding energy through interface (2) (*Rev29*: the associator is a constructed object and binds nothing; it supplies a weight, and the interface supplies the energy), priced by two φ-powers, with the exponent decomposition $15 = 8 + 7$ (triangle scale + heptad circulation). [LIB2-152]

The breathing kernel (A1357): the same object, typed — the trace-free breathing excitation of the $\alpha + 3$-body bridge, mirror-even and charge-blind, priced by

$$\Delta_{\text{br}} = \varphi^{-15} - \varphi^{-18}$$

with *zero new constants* ($15 = 8 + 7$ reused; $-\varphi^{-18}$ = the annulus subtraction, seventh appearance). [LIB2-162] It repaired the ledger's two worst entries (^{7}Li 0.64% → −0.0074%, ^{7}Be 0.59% → +0.076%) at *tightened* bars, and the mirror differential was preserved by type, not numerically. The refusal-then-derivation arc — the lane declined to use this object untyped in A1355, then derived its typing kernel in A1357 — is the protocol working as designed.

The baseline ruling (banked, A1357): separation energies are computed in the ledger category — $S_x(P \to D + x) = B(P) - B(D_{\text{ledger}}) - B(x)$ with the *refined* daughter always, where $B(x) = 0$ for a single emitted nucleon (the case the ruling was banked for) and the bound-fragment term is required otherwise (^{8}Be→ $\alpha\alpha$ gives 0 at leading order only with it; Rev32.9, A1674 F01; Rev32.10, A1714-F01 — this does not license the open $A = 8$ alternative baseline); neutral excitations cancel only if present in both typings. [LIB2-155, LIB2-163, LIB2-304, LIB2-306] Applied honestly it made one number worse: $S_p(^8\text{B}) = 4\varphi^{-18} + \varphi^{-19} = 0.356$ MeV = ×2.60 of world, filed as an *exposed charge-sector magnitude defect* rather than baseline-shopped (§9). [LIB2-163]

8 The ledger

Table 1: The Rev25 freeze-state nuclear ledger. All sharps are $|B_{\text{pred}} - B_{\text{world}}|/B_{\text{world}}$ against bars frozen before the answer; ten scored rounds, zero threshold moves. † = first-circulation residual, 0.334%: the value and the frozen-bar history are unchanged; the direct connected-Markov-second-moment energy attachment (A1543/D1224) is closed negative (A1544) and a reopen needs a different localized operator or a derived negative energy map (§9); the Rev30 readout fork is separate and untouched.

System	Formula ($B/\sqrt{\sigma}$ or as stated)	Sharp	Bar
d	φ^{-11}	0.52%	1%
^{3}H	eq. (1)	0.77%	1%
^{3}He	eq. (1)	0.10%	1%
^{4}He	eq. (1)	0.16%	1%
^{6}He	$B_\alpha + \varphi^{-13} + \varphi^{-18}$	0.0005%	1%
^{6}Li	$\varphi^{-11}(7 + 8\varphi^{-2} + \varphi^{3})$	0.11%	1%
^{7}Li	$\alpha + t$ bridge + Δ_{br}	-0.0074%	0.25%
^{7}Be	mirror bridge $-3\varphi^{-18}$ + Δ_{br}	$+0.076\%$	0.25%
^{8}Li	endpoint rule (pass-2 refiling)	0.0027%	1%
^{8}B	endpoint rule (φ^{-18} + φ^{-19})	0.0049%	1%
^{9}Be	$2B_\alpha + \varphi^{-12}$	0.18%	1%
^{12}C	$3B_\alpha + T_3$	0.10%	1%
^{16}O	$4B_\alpha + 2T_3$	0.17%	1%
^{20}Ne	$5B_\alpha + \varphi^{2}T_3$	0.00031%	1%
^{24}Mg	$6B_\alpha + 4T_3$	0.37%	1%
^{28}Si	$7B_\alpha + (5 + \varphi^{-2})T_3$	0.334%† *(typed obstr.)*	1% pre-answer (D1060): PASS; 0.1% refinement gate (D1061): obstruction branch (A1357)
^{32}S	$8B_\alpha + (6 + \varphi^{-3})T_3$	0.024%	1%
^{40}Ca	$10B_\alpha + (8 + \varphi^{-5})T_3$	-0.032%	1%
Hoyle	$(\varphi^{-15} + \varphi^{-18})\sqrt{\sigma}$	6.2%	10% (PASS)
$m(0^{++})/\sqrt{\sigma}$	$\sqrt{8\pi \log \varphi} = 3.4777$	in band	$[3.3, 3.9]$
$m(2^{++})/m(0^{++})$	$\sqrt{2}$	in band	$[1.3, 1.6]$

That is **seventeen sharp passes, deuterium through calcium** (worst untyped 0.77%, ^{3}H; 0.37% is the worst of the $A \geq 5$ non-silicon continuation — Rev32.9), plus one *typed obstruction* (^{28}Si, 0.334%; pre-answer bar 1% passed, §9), plus the Hoyle pass, plus **eleven sign/stability calls all correct** ($pn/nn/pp$; the $A = 5$ pair; ^{8}Be; ^{6}He/^{6}Be; ^{9}Be/^{9}B), plus two glueball numbers in their frozen bands, plus report-tier entries (^{36}Ar 0.018% · ^{10}Be 0.20% · ^{6}Be S_{2p} within 4.9% · the exposed $S_p(^{8}\text{B})$, §9). [LIB2-153] The glueball chain — the annulus functional $E(R)$ as the $-\log$ of a vacuum-normalized closed Markov correlator, saddle $m^2/\sigma = 4\pi\Delta$ with $\Delta = 2\log\varphi$, the

Lüscher vacuum term correctly cancelling in the correlator ratio — is banked at draft tier with its normalization derivation recorded (A1350–A1352).

Remark 8.1 (magic numbers: what is native and what is not)**.** Two magic numbers are native to the framework: 2 ($\mathbb{Z}_2$ doubling) and 8 (the octonion octet, the $11-3$ gap of the occupation $O=\{0,3,11,17\}$; kernels `s574`, `s702`). The remaining magic numbers are *open* — the shell-structure question, and with it any island-of-stability forecast, is a named *program* (Paper 7), not a result. This scope line is deliberate.

Remark 8.2 (the upper wall is measured: a blind negative past ^{60}Zn)**.** The upper edge of this tier is a measured failure, not an assumption. The continuation of the $n\alpha$ ladder (2) along the $N=Z$ line from ^{64}Ge to ^{100}Sn was pre-committed blind (kernel `s1101`, frozen before any AME2020 value with $A>60$ was read), with two frozen correction classes. **Both fail.** The 1% bar dies at the first blind row, ^{64}Ge (+1.64%). From there the residual grows monotonically to +6.03% at ^{100}Sn: the ladder over-binds every measured row, and there is no recovery at the ^{100}Sn closure. Kernel `s1102` identifies the missing term only qualitatively, as the net curvature of a surface term and a Coulomb term. That term needs a nuclear radius the framework does not yet have, and the fitted Coulomb scale is 43% of the physical one. This is a failed pre-registered prediction and is printed as one: the ledger above claims no pass beyond calcium, and past ^{60}Zn the ladder is falsified.

Look-elsewhere census (`s1364`). A pre-registered census (specification frozen before the run) asks how often the ledger's formula language reaches a random target as well as the printed rows do. Under Fisher's combination the aggregate is not significant at the headline (ratio) scale, $p=0.85$; at $\sqrt{\sigma}=445$ MeV it is $p=0.024$ (`s1364`, with the AME2020 ^{6}He binding 29.2711 MeV; 0.060 with the mis-transcribed 29.2701, `s1209`) — inside the pre-registered *mixed* band ($0.01<p\le 0.05$), and the move is the ^{6}He row alone, a central-$\sqrt{\sigma}$ sharp. (The product of the per-group p-values, 5×10^{-4}, is not itself a p-value.) The evidence is concentrated: ^{20}Ne's ε is the best of 51 menu values, and the $\mathcal{T}_5=\varphi^2$ vault hits 3.4×10^{-4} of random targets — at the central $\sqrt{\sigma}$ only (the implied $\mathcal{T}_5$ is 2.92, 2.618 and 2.32 at $\sqrt{\sigma}=439$, 445 and 451 MeV); for the $n\ge7$ ε family the printed rule is the unique best of 45; the deuteron and ^{12}C carry ACC 0.03 and 0.32. The composite rows ($A=3$ and 6–8) carry none.

9 Typed features and open items

^{28}Si — first-circulation residual; direct annular second-moment attachment closed negative (Gate 1, re-adjudicated Rev28; retyped Rev32.2 from A1542–A1545, PI ruling S294). For ^{28}Si the first-circulation ladder leaves a 0.334% residual. The pre-answer 1% bar was passed; the refinement gate nevertheless took its obstruction branch, so the point remains neither a sharp nor a miss at that bar. A second-order annular object was *defined* in $\mathcal{A}_\varphi=\mathrm{ATL}_\varphi/\langle f_4\rangle$ — the mirror-odd circulation $R_n=P_\tau(\rho_n-\rho_n^{-1})P_\tau/(2i)$ with connected normalized Markov second moment $\Omega_n^{(2)}$ (A1543) — but its direct physical attachment fails: the canonical connected Markov second moment is non-negative, whereas the required $n=6,7$ binding correction is negative, so the annular order does not derive the negative energy coefficient that would reverse the sign; the ribbon full-twist identity also excludes exact switch-off ($\Omega_n^{(2)}>0$ at $n=8,9,10$). [LIB2-156, LIB2-157] *Support certificate (Rev32.9, A1674 F03; kernel `s1177`).* A full-twist relation alone does not give strictness: $\rho_6=\pm1$ on P_τ is allowed and gives $\Omega_6^{(2)}=0$. Strictness follows from three stated premises: (P1)

$\rho_n^n = \theta_\tau^{1-n} P_\tau$ with $\theta_\tau = e^{4\pi i/5}$ (A1544); (P2) P_τ commutes with ρ_n; (P3) the normalized Markov trace is a faithful state on $P_\tau \mathcal{A}_\varphi P_\tau$. Then R_n has eigenvalues $\sin\big((\chi_n + 2\pi k)/n\big)$ with $\chi_n = \arg \theta_\tau^{1-n}$, and $\Omega_n^{(2)} \geq \min_k \sin^2\big((\chi_n + 2\pi k)/n\big)$ for any spectral weights. With $\chi_n/\pi = 0, -\frac{4}{5}, \frac{2}{5}, -\frac{2}{5}, \frac{4}{5}$ for $n = 6, \ldots, 10$ the bound is 0 at $n = 6$ and strictly positive at $n = 7, 8, 9, 10$ (e.g. $\sin^2(\pi/35)$ at $n = 7$). Premises P1–P3 are cited, not re-derived here. Because the coefficient in T_3 units remains open, the finite $< 0.02\, T_3$ support threshold is not scored. [LIB2-157] Thus the direct Markov-second-moment repair class is closed negative (A1544, lane `c1112`), and the perturbative annular attachment is closed negative as well (A1545); the surviving reopen class is *packing/coordination of 6–7 cells* (`s1112/s1115`, below). [LIB2-156, LIB2-158] The earlier phrase "irreducible at first circulation order" is retained only in that scoped sense; the A1542 seven-item minimum reopen package is the build specification for any reopen. [LIB2-303] No public observable or tier moves. The scoring history, with receipts, is this. The *pre-answer* frozen bar for ^{28}Si was 1% (D1060); the entry passed it at 0.334%. [LIB2-154] The 0.1% figure was a *post-answer* refinement gate, and its frozen text (D1061, Demand 3(b); pre-commit sha `s746 85c69f1f`…, stamped 2026-07-04 pre-send) was explicitly disjunctive: *"0.1% OR an honorable 'irreducible at this order' with the obstruction named, which earns FULL CREDIT."* A1357 ¶7 took that branch under the gate's own frozen terms. The kill condition therefore never fired at a pre-answer bar. Rev27's "counted as a miss" was an over-correction driven by the compressed Rev25 ledger row; it is corrected here with receipts rather than silently, and the correction is dated.

Stated plainly, as the honest caveat: the 0.1% gate post-dates the residual value; the obstruction branch was in that gate's frozen text before the gate was applied; the pre-answer bar was passed. [LIB2-154] The entry is scored as *neither a sharp nor a miss* — a typed obstruction. [LIB2-154] The scored tally is seventeen sharp passes plus one typed obstruction. The irreducibility claim itself remains falsifiable: a first-circulation fix refutes it. [LIB2-154, LIB2-303]

Fence (Rev30): the ^{28}Si *readout* is forked, and it is not the residual above. A separate campaign (N1–N5; adjudicated A1480, superseded by A1481) produced a readout for ^{28}Si that is **not** the 0.334% first-circulation binding residual discussed in this section. [LIB2-159] That readout currently admits *three inequivalent readings*, and they differ by roughly a factor of 170:

$$\underbrace{+1.457939 \times 10^{-6}}_{\text{amplitude, } \varphi^{-5}} \qquad \underbrace{+2.524367 \times 10^{-4}}_{\text{probability, } \varphi^{-10}} \qquad \underbrace{\text{nonselective channel}}_{\text{depth } 15}$$

The control for all three is the **untouched** ^{40}Ca rung ($n = 10$), which sits at -3.172350×10^{-4} — the same number this paper prints as -0.032% in the ledger above. [LIB2-159]

Consequence, binding. While that fork is open, **^{28}Si must not be quoted at 10^{-6}**: doing so silently selects the amplitude reading over the other two. Any citation of the ^{28}Si readout must print all three readings and the ^{40}Ca control, or name which reading it uses and why. [LIB2-159] This fence does not touch the 0.334% typed obstruction above, which is a different quantity with its own frozen-bar history; the two must not be conflated, and the 170× spread is exactly what conflation would hide.

The $\mathcal{T}_6$ thread (implied 3.899 vs 4, $-0.10\, T_3$ at six cells) plausibly belongs to the same sector.

Reopen class of the six-to-seven-cell deficit, retyped (Rev32.2; `s1112/s1115`, PI ruling S294). The deficit is *not* a framing or twist object of the τ annulus: the ribbon phase $\theta_\tau^{(1-n)}$ is trivial at $n = 6$, exactly where the deficit is largest, and no residue class of any clock factor produces the

window $\{6,7\}$ (`s1112`); the Fibonacci fusion-path braid representation, rebuilt independently and decomposed by total charge, shows no sector quantity that thresholds at $n = 8$ — the profile is flat at 0.46–0.56 while the deficit drops tenfold (`s1115`; independent lane build agrees to 10^{-7}). [LIB2-158] The reopen class is therefore *packing/coordination of 6–7 cells (writhe sector)*, not "annular second order"; the negative-annular closure (A1545 T4) stands. [LIB2-158, LIB2-303] Nothing here changes the ^{28}Si fence above.

The charge-magnitude sector (exposed, honestly). Charge *signs* and *counts* are derived; charge *magnitudes* wander: $S_p(^9\mathrm{B}) \times 1.29$, $S_p(^8\mathrm{B}) \times 2.60$ under the baseline ruling. [LIB2-155] The banked candidate mechanism — coherence factors are configuration-dependent ("coherence multiplies by φ") — names the problem without solving it. [LIB2-155, LIB2-304] This is the cluster kernel's known soft flank; it is stated here so that no reader mistakes the sharp ledger for a solved charge sector. ^{64}Ge, where the $N = Z$ line's deficit is steepest, is two walls at once — radius *and* charge magnitude, with an unregistered screening factor 0.47 (`s1114`, Rev32.2). [LIB2-304]

The isospin sector (typed loaded, Rev32.2; `s1122`–`s1124`, `s1129`; PI ruling S294). The isospin Casimir coefficient $\chi = \varphi^{-6} T_3$ is *selected* on the fixed-Z chains, not derived. [LIB2-160] The only registered $u \leftrightarrow d$ object is the $\mathbb{O}_s$ sign map $e_7 \to -e_7$ ($H_u \leftrightarrow H_d$; A689 dictionary; read literally, the flip of e_7 alone — `s1170` shows the phrase also fits full conjugation, and neither map is a Jordan automorphism) — a $\mathbb{Z}_2$ parity, not an $SU(2)$ — and the Peirce $\frac{1}{2}$ is scalar on blocks and induces $\chi = 0$. [LIB2-161, LIB2-305] The sector is therefore LOADED. [LIB2-160] The neutron–proton mass difference $m_n - m_p = 1.2933$ MeV is UNPRICED: the framework carries the QED half (Coulomb, with α external) and no QCD half ($m_u = m_d$ at constituent level; the e_7 flip carries no energy), so without a flip energy its sign comes out wrong (`s1129`). [LIB2-305] The row is entered in the Appendix X ledger as unpriced. Pricing it requires a new registered isospin-breaking datum: no registered object gives H_u and H_d different energy (`s1170`), and the flip is visible only in the Yukawa trilinear with $J_{23} \otimes J_{31}$ states, none of which is registered (Rev32.9).

$A = 8$ breathing accounting (open audit item). Whether Δ_{br} is present in the $A = 8$ parents is an unchecked naturality square: the alternative reading gives $S_p(^8\mathrm{B}) = 5\varphi^{-18} + \varphi^{-19} - \varphi^{-15} = +0.106$ MeV ($\times 0.78$ — better than both filings). [LIB2-162, LIB2-306] Designated next audit item (OPEN_ITEMS; post-freeze dispatch candidate). [LIB2-306]

10 Protocol sub-appendix: the receipt chain

The evidentiary weight of this paper rests on the scoring protocol, so the protocol is part of the paper.

Frozen bars and pre-commits. Every scored number was demanded against an accuracy bar frozen in a hash-stamped pre-commit kernel *before* the dispatch was sent (the S226 chain: `s732 93cd5133...`, `s738 28a1b5d1...`, `s746 85c69f1f...`; each asserted on-disk at scoring time by the citing kernel). Across the entire arc — **ten consecutive scored rounds** — there were *zero* threshold moves: no bar was loosened, moved, or reinterpreted after an answer arrived. New demands got new bars; existing bars never moved.

The three vault matches. (i) $\mathcal{T}_5$: sealed implied value 2.617965 (s732), blind-derived φ^2 — 26 ppm. (ii) $\mathcal{T}_9$: sealed answer sheet (`s737`, sha `537f244c...`) behind a deliberately unhelpful hint; blind-derived $7 + \varphi^{-4}$ — 0.10%. The sheet was unsealed on schedule and the lane re-verified the unsealed content against the sha published *before* it answered. [LIB2-151] (iii) The ^{40}Ca value matched

the in-house blind application of the banked rule (`s745`) exactly. The covenant has paid in both directions.

The honesty ledger, complete. Window A (the standing invitation to fit a number outside the typed menu) was refused with reasons **seventeen** consecutive times — the refusal count is itself lane-integrity evidence. One blind window was **VOIDED by our own disclosure error** (window E, kernel `s687`): the blinding was compromised on our side, the window was scored VOID rather than counted, and a blindness-certification gate (`s691`) was instituted before any further blind window. One process deviation (the `s738` pre-commit *contents* shipped where the sha-only line was intended) was audited in-kernel (`s739`): nothing non-public leaked; the trap stayed blind; pass 2 of the affected round was designated official. One in-house prose slip ($S_n(^8\mathrm{Li})$) was caught by re-verification and corrected on the record. These are recorded here because a ledger of hits is only as credible as its ledger of errors.

Verification. Every quoted decimal in A1349–A1357 was reproduced by deterministic in-house kernels (`run1==run2`, hash-manifested `s705`–`s747`) at deviations between 0.0 and 2.2×10^{-11}.

11 Status ledger

Result	Status	Note
F_φ functor (generator set)	DERIVED	kernel-verified, A1349
$A \leq 4$ kernel eq. (1)	SHARP-PASS-WITH-TONGS	triangular saturation declared
$A = 5$, $A = 8$ gap theorems	DERIVED	choice-free consequences
Two-body typing (S=1, T=0)	DERIVED	3/3 signs
Ladder closed form eq. (3)	DERIVED	two blind vault matches
ε-selection rule	DERIVED	A1357
Breathing kernel Δ_{br}	DERIVED	zero new constants
Baseline ruling	BANKED	drew blood honestly
Hoyle state	SCORED PASS	frozen 10% bar
Glueball pair	DRAFT TIER	in frozen bands
Charge magnitudes	OPEN	exposed defects filed
Isospin sector ($\chi = \varphi^{-6} T_3$)	LOADED	selected on fixed-Z chains; $m_n - m$
^{28}Si residual	FIRST-CIRCULATION RESIDUAL	0.334%; pre-answer 1% bar passed
Annular R_n, $\Omega_n^{(2)}$	DEFINED; ATTACHMENT CLOSED NEGATIVE	$\Omega \geq 0$ vs required negative correct
$A = 8$ breathing accounting	OPEN	next audit item
N=Z ladder past ^{60}Zn (blind)	FALSIFIED	`s1101`: 1% bar dies at ^{64}Ge; +6.03
Magic numbers beyond $\{2, 8\}$	OPEN PROGRAM	Paper 7

Bottom line. The periodic table ran deuterium to calcium on a discrete φ-monomial menu and a closed-form trace invariant, against bars frozen in advance, with the freedom ledger audited at every step and the failures typed and filed beside the hits. The associativity axiom does not *have* an energy; it **enters** one — *through the declared cluster-ledger interface* (2), $B(n\alpha)/\sqrt{\sigma} = n\,B_\alpha + \mathcal{T}_n\,T_3$, which is the only join between the constructed layer and the physical layer in this paper. [LIB2-144] At $n = 5$ the constructed weight $\mathcal{T}_5 = \varphi^2$ is what the axiom fixes; the binding energy is what the interface returns. The blind match at 26 ppm is the *coefficient* ($\mathcal{T}_5 = \varphi^2$ against the sealed 2.617965); the returned ^{20}Ne binding energy sits at 0.00031% (Rev32.9: through Rev32.8 the 26 ppm was attributed to the returned number) (*Rev29*: predication corrected — the interface, not the verb, carries the type change). What is *not* claimed: no continuum functor, no Clay claim, no derived charge magnitudes, no shell model beyond $\{2, 8\}$, and no absolute scale — the unit lives in the registered input (Appendices O/X).

[LIB2-153, LIB2-164]

Leibniz Quantum Beats Newton

Appendix A — Lagrangian Skeleton, Jordan Eigenproblem, and Cubic-Invariant Yukawa Derivation (Rev33.2)

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August 2026

Abstract

Appendix A collects the local field-theory skeleton used by the Rev29 suite and upgrades the old Rev8 appendix to the current architectural state. The purpose remains narrow and explicit. This appendix does not claim a complete ultraviolet completion of particle physics; it records the minimal local structures that are already fixed by the present $J_3(\mathbb{O}_s)$ programme and labels the remaining bridges honestly. Three upgrades distinguish Rev29 from the earlier appendix: the corrected Cayley–Dickson split-octonion algebra, the theorem-level vacuum selector $J_{\rm vac} = \mathrm{diag}(\phi, 1, \phi^{-1})$, and the full D=5 to D=4 architecture in which the pseudo-Riemannian $E_{6(6)}/F_{4(4)}$ stage is lifted by KK reduction and dualization to the ghost-free Riemannian target $E_{7(7)}/SU(8)$.

The appendix also records the formal Jordan eigenproblem needed by the emerging mass sector. The native spectral problem is $A \circ V = \lambda V$ for primitive idempotents $V \in J_3(\mathbb{O}_s)$, but the numerically correct implementation is the 27×27 real lift $L_A : Y \mapsto A \circ Y$. In that lifted form the Session 30 mass construction becomes explicit:

$$M_{\rm lepton} = U_{T_{\rm half}}\, D_R\, U_{T_{\rm half}}^{-1}, \qquad D_R = \mathrm{diag}(R_1, R_2, R_3), \qquad U_{T_{\rm half}} = \exp\!\big(\tfrac{2\pi}{\sqrt{5}} T_{\rm half}\big). \tag{1}$$

The same generator $T_{\rm half}$ that produces $\delta_{CP} = -2\pi/\sqrt{5}$ is therefore the algebraic engine of the charged-lepton hierarchy programme.

Rev29 scope note. The cubic invariant fixes a tree-level ultraviolet Yukawa eigenvalue and the suite now carries theorem-grade *internal* constructions for the mixing and charged-lepton sectors; their physical attachments are tiered separately (two of eight mixing rows at DERIVED-CONDITIONAL or above after the Rev29 re-tiering, Paper 3; no row may be cited as zero-parameter, Appendix X). It does *not* yet derive the electroweak vacuum expectation value, the exact selector on the weak-angle exact-color orbit, the formal heavy-quark S-matrix bridge, or the detailed quantum matching of the D=5 theory to the low-energy effective action. Those remain measured anchors, open bridges, or beyond-tree tasks.

A.1 Foundational algebra and notation

All split-octonion products in the current suite are taken in Cayley–Dickson form. Writing elements as pairs (p, q) over the quaternions (Rev32.9: through Rev32.8 this said "over the compact octonions",

whose doubling is sixteen-dimensional; the product below is the standard split doubling of the quaternions, $4 + 4 = 8$),

$$(p, q)(r, s) = (pr + s^* q,\ sp + qr^*), \tag{2}$$

with split inner product

$$\langle x, y\rangle_{\mathrm{CD}} = \sum_{k<4} x_k y_k - \sum_{k\geq 4} x_k y_k. \tag{3}$$

This SplitCD form is not cosmetic. With a Euclidean inner product the derivation algebra collapses to the wrong dimension, whereas the SplitCD choice restores

$$\dim \mathrm{Der}(J_3(\mathbb{O}_s)) = 52 = \dim \mathfrak{f}_{4(4)} \tag{4}$$

and makes the corrected Jordan product satisfy the Jordan identity at residual level below 6×10^{-14}.

The Jordan product and Freudenthal cross product are

$$X \circ Y = \tfrac{1}{2}(XY + YX), \tag{5}$$

$$X \times Y = X \circ Y - \tfrac{1}{2}\mathrm{Tr}(X)Y - \tfrac{1}{2}\mathrm{Tr}(Y)X + \tfrac{1}{2}\big[\mathrm{Tr}(X)\mathrm{Tr}(Y) - \mathrm{Tr}(X \circ Y)\big]I. \tag{6}$$

The local split-octonion multiplication in these formulas is always Cayley–Dickson, not the older Fano mnemonic/sign-flip surrogate.

A.2 Theorem A.1: vacuum selector

The working vacuum is

$$J_{\mathrm{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1}), \qquad \phi = \frac{1 + \sqrt{5}}{2}. \tag{7}$$

Its present status is theorem-level.

Novelty label: new specialization proved here.

Theorem 0.1 (Vacuum selector)**.** *J_{vac} is the unique fixed point of the modular element $M_1 = T^3 S \in PSL(2, \mathbb{Z}) \subset F_{4(4)}$ acting on the Peirce diagonal of $J_3(\mathbb{O}_s)$. Equivalently, the working vacuum is the unique algebraic attractor of the present modular/Jordan-boundary construction. [LIB2-337] The companion Freudenthal fixed-point relation*

$$\{J_{\mathrm{vac}}, J^{\#}_{\mathrm{vac}}, J_{\mathrm{vac}}\} = J_{\mathrm{vac}} \tag{8}$$

holds exactly.

This upgrade matters for the Lagrangian skeleton because all local cubic terms are evaluated at the vacuum. The appendix therefore no longer treats the golden-ratio vacuum as an unexplained numerical choice inside the local theory, even though the deeper physics interpretation of the selector still belongs to the architecture papers rather than this appendix alone.

A.3 Minimal gauge, fermion, and Higgs skeleton

The local gauge-field skeleton is

$$\mathcal{L}_{\mathrm{gauge}} = -\frac{1}{4}\kappa_{AB}\, \mathcal{F}^A_{\mu\nu}\mathcal{F}^{B\mu\nu}, \qquad \mathcal{A}_\mu \in \mathrm{Str}_0(J_3(\mathbb{O}_s)) \cong \mathfrak{e}_{6(6)}. \tag{9}$$

The fermion kinetic term is written schematically as

$$\mathcal{L}_\Psi = i\bar{\Psi}\gamma^\mu D_\mu \Psi, \qquad D_\mu = \partial_\mu + \mathcal{A}_\mu. \tag{10}$$

The Higgs placeholder is

$$\mathcal{L}_H = |D_\mu H|^2 - V(H), \tag{11}$$

with the explicit suite-level statement that

$$\langle H \rangle = 246\,\mathrm{GeV} \tag{12}$$

is measured rather than derived.

The minimal local assembly is therefore

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_\Psi + \mathcal{L}_H + \mathcal{L}_Y, \tag{13}$$

where $\mathcal{L}_Y$ is the cubic-invariant Yukawa term derived below.

A.4 D=5 to D=4 architecture pipeline

The local skeleton above sits inside a larger architecture whose status is now sharply differentiated.

A.4.1 D=5 Chern–Simons bulk

At the D=5 level the natural exceptional starting point is the split-magic Chern–Simons system with gauge group $E_{6(6)}$ and cubic coupling tensor C_{IJK}, the same tensor that underlies the cubic norm of $J_3(\mathbb{O}_s)$. In schematic form,

$$S_{D=5} = \int d^5x\,\sqrt{-g}\Big[-\frac{1}{4}a_{IJ}(h)F^I_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}g_{xy}(h)\partial_\mu h^x \partial^\mu h^y\Big] + \frac{1}{6\sqrt{6}}\int C_{IJK}A^I \wedge F^J \wedge F^K. \tag{14}$$

The current programme state carries this level as PLAUSIBLE: the D=5 Chern–Simons bulk with $E_{6(6)}$ and the cubic tensor is consistent and is the natural origin of the downstream Jordan and Freudenthal structures, but the full uniqueness-of-gauging proof is still open.

A.4.2 D=5 intermediate scalar manifold

The D=5 scalar target is

$$E_{6(6)}/F_{4(4)}, \tag{15}$$

with dimension $78 - 52 = 26$. This is the correct split-magic-supergravity scalar manifold. Its tangent signature on the traceless Jordan sector is $(14, 12)$, so the D=5 stage is pseudo-Riemannian.

A.4.3 KK reduction and D=4 target

After reduction on S^1, dualization of the 27 D=5 vectors, and inclusion of the KK dilaton, the scalar count from the split-magic Jordan sector is

$$\underbrace{26}_{\text{split-magic D=5 scalars}} + \underbrace{27}_{\text{dualized vectors}} + \underbrace{1}_{\text{dilaton}} = 54, \tag{16}$$

using the D=5 split-magic manifold $E_{6(6)}/F_{4(4)}$ of dimension $78 - 52 = 26$ established above (*not* the maximal-supergravity count $E_{6(6)}/USp(8)$ of dimension 42, which would give $42 + 27 + 1 = 70$). The maximal-supergravity D=4 target with full coset

$$E_{7(7)}/SU(8) \quad (\dim 70) \tag{17}$$

is therefore *not* reached by the Jordan-algebra sector alone: it would require $70 - 54 = 16$ additional fields beyond the split-magic content. These are now *identified* (S73; Paper 4 §4.2 is authoritative) as the identity-anchored Jordan complement $42 \ominus 26$ on the certified $E_{7(7)}$ substrate, though their explicit field-level extraction — and the consistent-truncation question — remain open. The 54 D=4 scalars from the Jordan sector are ghost-free (the D=5 intermediate stage is pseudo-Riemannian of signature $(14, 12)$ on the traceless sector, but the realized D=4 scalar content is Riemannian). This matches Paper 4 §4.2 (authoritative); the prior "$42 + 27 + 1 = 70$, established" wording is corrected here.

PR-C2 status (corrected, S61). The split-magic KK reduction yields $26+27+1 = 54$ ghost-free D=4 scalars from the Jordan-algebra sector. The full maximal-supergravity $E_{7(7)}/SU(8)$ target (70 scalars) is *not* established: it requires 16 additional non-Jordan fields, now identified as the $42 \ominus 26$ Jordan complement (S73) but not yet explicitly extracted as fields. Two things remain open: (i) the explicit field-level realization of the identified 16 completing $54 \to 70$, and (ii) the proof that the chosen D=4 sigma model is an exact consistent truncation of the D=5 theory rather than a counting match plus structural correspondence. Whether this identified missing-16 couples dynamically to the golden vacuum is the pre-registered portal drill of Appendix I (G3–G7).

A.5 Jordan eigenproblem and the 27-dimensional real lift

The native spectral problem in $J_3(\mathbb{O}_s)$ is not the ordinary matrix-eigenvalue problem of a 3×3 real symmetric matrix. It is the Jordan eigenproblem

$$A \circ V = \lambda V, \tag{18}$$

with V a primitive idempotent of $J_3(\mathbb{O}_s)$. The numerically correct route is the linear operator lift

$$L_A : J_3(\mathbb{O}_s) \to J_3(\mathbb{O}_s), \qquad Y \mapsto A \circ Y. \tag{19}$$

Because $J_3(\mathbb{O}_s)$ is 27-dimensional over the reals, L_A is represented by a 27×27 real matrix.

Novelty label: standard theorem used.

Proposition 0.2 (Validity of the real lift)**.** *For the Jordan eigenproblem of $J_3(\mathbb{O}_s)$, diagonalizing the 27×27 real matrix representing L_A is a legitimate computation of its spectrum: the lift is a real linear operator. Realness alone does not make it Euclidean-symmetric, so a symmetric solver such as* `eigvalsh` *is licensed only where L_A is symmetric in the stated coordinates or has a demonstrated positive symmetrization — as it is for the diagonal vacuum, but not in general (the split-Hermitian element in the l-direction of J_{12} has a lift with nonreal eigenvalues,* `s1164`*). The primitive-idempotent eigenvalues must also be identified separately from the full spectrum, which contains Peirce means (Rev32.9, A1587).*

This is the form used in Sprint 3 and in the current mass-sector roadmap. The same lifted language also reproduces the Albert–Freudenthal spectrum of $L_{J_{\rm vac}}$,

$$\{\phi,\ 1,\ \phi^{-1},\ (\phi+1)/2\times 8,\ (\phi+\phi^{-1})/2\times 8,\ (1+\phi^{-1})/2\times 8\}, \tag{20}$$

so the $(1,3)$ off-diagonal eigenvalue is

$$\Omega = \frac{\phi+\phi^{-1}}{2} = \frac{\sqrt{5}}{2}. \tag{21}$$

This same algebraic number governs the V_3 rotation frequency used in the formula $\delta_{CP} = -2\pi/\sqrt{5}$ (a theorem-grade internal construction; the physical δ_{CP} row is DERIVED-CONDITIONAL, Paper 3).

A.5.1 Mass operator and resolvent data

The current mass-sector construction introduces

$$R_d = \frac{1}{\phi^{2d}-1}, \qquad D_R = \mathrm{diag}(R_1, R_2, R_3), \tag{22}$$

with exact identities

$$R_1 = \phi^{-1}, \qquad R_2 = \frac{1}{\phi(\phi+2)}, \qquad R_3 = \frac{1}{4\phi^3}. \tag{23}$$

The lepton mass operator is then carried in automorphic-conjugation form:

$$M_{\rm lepton} = U_{T_{\rm half}}\, D_R\, U_{T_{\rm half}}^{-1}, \qquad U_{T_{\rm half}} = \exp\!\big(t^* T_{\rm half}\big), \qquad t^* = \frac{2\pi}{\sqrt{5}}. \tag{24}$$

Because $T_{\rm half} \in \mathfrak{f}_{4(4)} = \mathrm{Der}(J_3(\mathbb{O}_s))$, this is a genuine Jordan-algebra automorphism. The physical significance is immediate: the same $T_{\rm half}$ that generates the leptonic CP phase also generates the charged-lepton hierarchy map.

A.5.2 G_2 constraint and spectral identification

The automorphism group of the split octonions fixes only the real subspace, so a G_2-invariant $M_{\rm lepton}$ has real off-diagonal octonionic entries. In effect the physical mass operator is a real symmetric 3×3 object embedded in $J_3(\mathbb{O}_s)$. On the lifted side, the spectrum of $L_{M_{\rm lepton}}$ separates into three anchor eigenvalues plus 24 pairwise means,

$$\{\lambda_1, \lambda_2, \lambda_3, \tfrac{1}{2}(\lambda_i+\lambda_j)\times 8\}, \tag{25}$$

which is the correct Albert–Freudenthal bookkeeping used to identify the true Jordan eigenvalues.

A further structural result from Sprint 3 is the reversal theorem

$$(R_1, R_2, R_3) \longmapsto (R_3, R_2, R_1) \tag{26}$$

under the $U_{T_{\rm half}}$ automorphism, so the charged-lepton mass ordering is forced to be

$$d=1 \to \tau, \qquad d=2 \to \mu, \qquad d=3 \to e. \tag{27}$$

This is why the blind-drop relation $m_\mu = m_\tau/(4\phi^3)$ belongs to the same algebraic tower as the CP phase.

A.6 The cubic-invariant Yukawa theorem

The Yukawa term is

$$\mathcal{L}_Y = \mathrm{Tr}\big[J_{\mathrm{vac}} \circ (\Psi \times \Phi_H)\big]. \tag{28}$$

It is the unique cubic local object used by the present suite.

Novelty label: physical interpretation not established here.

Theorem 0.3 (Cubic-invariant Yukawa theorem)**.** *The tree-level top Yukawa eigenvalue is fixed exactly by Eq.* (28)*,*

$$y_t(M_{\mathrm{Pl}}) = 1, \tag{29}$$

and this value is uniquely forced by the simultaneous requirements

(a) F_4-equivariance of the Freudenthal cross product,

(b) Jordan homogeneity (degree two in the matter fields), and

(c) cubic closure with the vacuum element J_{vac}.

> **Interface marker (W01, *Rev29*) — where the carrier eigenvalue becomes a physical statement.** What the theorem establishes is internal: the cubic invariant, evaluated on the *carrier* field Φ_H in the $(1,3)$ Peirce block, extracts the middle vacuum weight $\lambda_2 = 1$. [LIB2-119] That is a statement about a constructed eigenvalue of a constructed carrier, and it fixes a *tree-level boundary value inside the loaded correspondence.* Reading that eigenvalue as the physical top Yukawa at the Planck scale, $y_t(M_{\mathrm{Pl}}) = 1$, is a *declared attachment*, not a derivation: it requires identifying the carrier with the physical Higgs direction, the vacuum weight with a renormalization-scheme-specific coupling, and the algebraic scale with M_{Pl}. [LIB2-120, P-027] None of those three identifications is proved here, and the notation $y_t(M_{\mathrm{Pl}})$ should not be read as though they were. [LIB2-120] Downstream consequences inherit the attachment, not the theorem: in particular the ledger row $m_t = v_{\mathrm{EW}}/\sqrt{2}$ is typed STRUCTURAL; its PDG offset is a cross-scheme comparator distance, no status (Rev32.1 retype, A1531/T2; Appendix X; `Trackers/HONESTY_LEDGER.md`).

[P-027]

Proof sketch. Place the fermion doublet Ψ and a carrier field Φ_H in the $(1,3)$ Peirce block (the same-block *carrier* reading of A674; the literal SM Higgs $H_{u,d}$ sits in the vector **10** and the physical Yukawa is the cross-block triple, A686–A689). [LIB2-119] Writing their active split-octonion components as ψ and h, one finds

$$\Psi \circ \Phi_H = \mathrm{diag}(2\,\mathrm{Re}(\psi\bar{h}),\, 0,\, 2\,\mathrm{Re}(\bar{\psi}h)), \qquad \mathrm{Tr}(\Psi \circ \Phi_H) = 4\,\mathrm{Re}(\psi\bar{h}). \tag{30}$$

Using the Freudenthal cross product and $\mathrm{Tr}(\Psi) = \mathrm{Tr}(\Phi_H) = 0$ gives

$$\Psi \times \Phi_H = (\Psi \circ \Phi_H) - \tfrac{1}{2}\mathrm{Tr}(\Psi \circ \Phi_H) I = -2\,\mathrm{Re}(\psi\bar{h})\, e_2. \tag{31}$$

Contraction with $J_{\mathrm{vac}} = \phi e_1 + e_2 + \phi^{-1} e_3$ then extracts only the middle weight,

$$\mathrm{Tr}\big[J_{\mathrm{vac}} \circ (\Psi \times \Phi_H)\big] = -2\,\mathrm{Re}(\psi\bar{h}), \tag{32}$$

so the multiplicative Yukawa factor is exactly the middle eigenvalue $\lambda_2 = 1$. [LIB2-119] □

The same theorem can also be expressed in E_6 language. The cubic Jordan invariant is the unique E_6-type cubic coupling of the local matter/Higgs system, and the present vacuum insertion is the F_4-breaking specialization of that same cubic object. In this sense the fullboat and Jordan/Freudenthal formulations are two presentations of the same local Yukawa structure.

Remark 0.4 (Peirce idempotent derivation of $y_t = 1$)**.** *The same result follows directly from the Peirce decomposition rules. For orthogonal idempotents f_i in $J_3(\mathbb{O}_s)$, the exact identity $f_k \circ x = \frac{1}{2}x$ for $x \in P_{ik}$ or P_{jk}, combined with the trace-norm normalisation $C = \sqrt{2}/10$ of the $J_2(\mathbb{O}_s)$ subalgebra* [A618]*, absorbs the SM factors $m = y \cdot v/\sqrt{2}$ and fixes the carrier eigenvalue $y_t = 1$ by the $\frac{1}{2}$ coefficient alone — again a boundary value inside the correspondence, exported to the physical $y_t(M_{\rm Pl})$ only by the declared attachment marked above (W01). The top quark is the maximal projection onto the highest-weight generator j_3, with no cross-block mixing suppression. The prediction $m_t = v_{\rm EW}/\sqrt{2} \approx 174{,}100\,\text{MeV}$ is $+0.87\%$ from the PDG 2026 direct-measurement mass (cross-scheme, declared — Appendix X; an earlier $+0.77\%$ printed here was arithmetically wrong); both the coefficient and the $\sqrt{2}$ are structurally identical to the off-diagonal norm factor in $J_2(\mathbb{O}_s)$* [A632]*. The complement rule $f_k \circ x = 0$ for $x \in P_{ij}$ $(k \neq i, j)$ implies that y_b and y_c are* exactly zero *at tree level from the bare Jordan product; their physical values are generated by the off-diagonal Peirce closure $P_{ij} \circ P_{jk} \subset P_{ik}$ acting as an algebraic mixing insertion* [A633]*.*

A.7 Flat Direction Lemma and tree-level potential

The Session 24 Lagrangian analysis establishes the exact tree-level obstruction to deriving the golden-ratio hierarchy from a polynomial F_4 potential alone.

Novelty label: new specialization proved here.

Lemma 0.5 (Flat Direction Lemma)**.** *Let the F_4-invariant renormalizable potential on $J_3(\mathbb{O}_s)$ be built from the ring generators*

$$T = \mathrm{Tr}(X), \qquad Q = \tfrac{1}{2}\mathrm{Tr}(X^2), \qquad \mathrm{Det}(X) = N(X). \tag{33}$$

On the traceless sector $T = 0$, the most general renormalizable potential is

$$V = \mu_2 Q + \lambda_3 \mathrm{Det} + \lambda_4 Q^2 + \lambda_5 Q\mathrm{Det} + \lambda_6 \mathrm{Det}^2, \tag{34}$$

or, if an external Z_2 is imposed, $V = \mu_2 Q + \lambda_4 Q^2 + \lambda_6 \mathrm{Det}^2$. At a general eigenvalue-space point $(a, b, 0)$ the Hessian in the (d_1, d_2) block has rank one, hence one exactly flat direction. Therefore the ratio d_1/d_2 is unlifted by any polynomial F_4-invariant tree-level potential.

With the traceless constraint understood, the stationarity conditions at $(a, b, 0)$ are

$$\mu_2 = -2\lambda_4 Q_0, \qquad \lambda_3 = -\lambda_5 Q_0, \tag{35}$$

and the explicit Hessian entries are

$$H_{11} = 8\lambda_4 a^2, \qquad H_{12} = 8\lambda_4 ab, \qquad H_{22} = 8\lambda_4 b^2, \qquad H_{33} = 2\lambda_6 a^2 b^2. \tag{36}$$

The upper 2×2 block is an outer product and therefore has eigenvalues $\{8\lambda_4(a^2 + b^2), 0\}$. The zero mode is tangent to the constant-Q_0 curve.

A second exact consequence is the tree-level Peirce-block cancellation

$$H_u = H_d = 0, \tag{37}$$

which follows from the stationarity relation $\mu_2 + 2\lambda_4 Q_0 = 0$. Hence any nontrivial Peirce-mass splitting used in the CKM or bosonic sector must arise beyond tree level.

A.7.1 Signature decomposition

The quadratic form on $J_3(\mathbb{O}_s)$ has signature $(15, 12)$ on the full 27-dimensional space and $(14, 12)$ on the traceless 26-dimensional space. The diagonal eigenvalue sector itself is positive-definite with signature $(3, 0)$. This is why the physical eigenvalue data can remain ghost-free even though the intermediate D=5 tangent space is pseudo-Riemannian.

A.7.2 Coleman–Weinberg outlook

The exact flat direction is not a defect to be hidden. It is the theorem stating that the golden-ratio hierarchy requires a beyond-tree lifting mechanism. The one-loop Coleman–Weinberg effective potential from the 24 off-diagonal modes is the natural mechanism; the D332–D340 dispatch chain has established the key structural results, and the renormalization-scale question (C6) remains open at the current programme state. A complete account of proved items and outstanding corrections is given in Sect. A.7.3.

A.7.3 Coleman–Weinberg dispatch chain: status and S35–S38 corrections

AX2 status (independent of CW). Theorem A.1 (AX2) is *proved* independently of the Coleman–Weinberg chain via the $\mathrm{PSL}(2,\mathbb{Z})$ fixed-point argument [R106]: the unique $\mathrm{PSL}(2,\mathbb{Z})$-fixed eigenvalue ratio is $r^\star = \phi^2$, which selects $J_{\mathrm{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1})$ without appeal to any renormalization scale. AX2 stands regardless of the resolution of the items below.

CW chain: proved items. The D332–D340 dispatch chain established the following structural results:

(i) $V_{\mathrm{CW}}^{\mathrm{bos}} \equiv 0$ as an *algebraic target-space index* [D332] (see §A.7.4): each 8D Peirce sector carries a $(4, 4)$ split target-space signature, and the graded (Witten-type) sum $\sum(+4-4)\, m^4 \log m^2 = 0$ vanishes mode-by-mode. This is an index identity of the $SO(4, 4)$ grading operator, *not* a claim that the physical bosonic one-loop determinant cancels via negative-norm modes (a ghost-free bosonic sector contributes with a single statistics sign); promoting this index to the physical effective potential requires an independent BRST/measure derivation of the grading weight (open).

(ii) $V_{\mathrm{CW}}^{\mathrm{ferm}} \neq 0$ [D333]. Fermions contribute (-1) per loop universally; three Peirce sectors with r-dependent masses $m_f^2(d_i, d_j) = y^2(d_i^2 + d_j^2)$ yield a non-zero net potential minimized at $r^\star = \phi^2$.

(iii) $y^2 = 2/h^\vee(F_4) = 2/9$ [D337], derived algebraically via the 't Hooft coupling $g_{F_4}^2 = 1/h^\vee(F_4)$, the Dynkin index $\ell = 1$, and the 2-fold Peirce eigenvalue degeneracy per block.

(iv) $Q_0 = 4$ exactly [proved]: the Fibonacci identity $\phi^2 + 1 + \phi^{-2} = 4$.

(v) The value $\kappa_{\mathrm{PMS}} = \log(9/4) \approx 0.8109$, first met as a numerical candidate in the dispatch chain [D334–D340], is fixed exactly by the Path-B identity proved below (Rev32.5 reconciliation of this list with the proof; Paper 4, Path A retracted).

C6: κ_{PMS} path — S35–S37 corrections (fatal errors, A342). The earlier claim that $\kappa_{\mathrm{PMS}} = \log(9/4)$ follows from the Coleman–Weinberg principle of minimal sensitivity (PMS) was subjected to independent verification in A342 (Gemini, Session 37), which identified two fatal errors:

(1) *Calculus failure.* Setting $\partial V_{\rm CW}^{\rm ferm}/\partial\kappa = 0$ gives $\mathrm{STr}[M^2(r^\star)] = 0$, which is the supertrace condition pinning $r^\star$, not κ. When y^2 is κ-independent, the renormalization scale drops out of the PMS condition entirely; there is no κ-equation. *The CW/PMS route cannot determine* $\kappa_{\rm PMS}$.

(2) *Arithmetic error.* The exact Lucas/Fibonacci identity gives $\phi^2 + \phi^{-2} = 3$, hence $\log(\phi^2 + \phi^{-2}) = \log 3 \approx 1.099$. This is strictly unequal to $\log(9/4) \approx 0.811$. The two quantities differ by ≈ 0.288; the claimed equality was erroneous.

Consequently: the Theorem A.7.3 and Remark (PMS) that appeared in pre-S37 versions of this appendix are *retracted*. Items (i)–(iii) and (v) of the old theorem remain correct as independent results; item (iv) (two-loop push toward $\log(9/4)$) and the PMS remark are invalid.

Path B: proved (S39, A343+A344). The mechanism and the exact value are now both established.

Novelty label: physical interpretation not established here.

Theorem 0.6 (C6: Natural RG scale from embedding index)**.** *The canonical field renormalization at the $F_4 \to G_2$ symmetry-breaking threshold forces a unique finite shift in the Coleman–Weinberg potential:*

$$\kappa_{\rm PMS} = \log \iota(G_2 \subset F_4) = \log \frac{h^\vee(F_4)}{h^\vee(G_2)} = \log \frac{9}{4}. \tag{38}$$

Proof sketch. Under the maximal embedding $G_2 \times SU(2) \subset F_4$, the adjoint decomposes as $\mathbf{52} \to (\mathbf{14},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3}) \oplus (\mathbf{7},\mathbf{5})$. For any G_2 generator T, summing traces over F_4:

$$K_{F_4}(T,T) = K_{G_2}(T,T) + 5\,\tfrac{T(\mathbf{7})}{T(\mathbf{14})} K_{G_2}(T,T) = \big(1 + 5\cdot\tfrac{1}{4}\big) K_{G_2}(T,T) = \tfrac{9}{4}\,K_{G_2}(T,T), \tag{39}$$

using Dynkin indices $T(\mathbf{14}) = 8$, $T(\mathbf{7}) = 2$. Hence $\iota = 9/4$ exactly. Since G_2 is simple its adjoint is irreducible; by Schur's Lemma the Killing form is the *unique* $\mathrm{Ad}(G_2)$-invariant quadratic form on $\mathfrak{g}_2$. Therefore the canonical field rescaling by $\iota^{1/2} = 3/2$ is uniquely forced, shifting $\log(M^2/\mu^2) \to \log(M^2/\mu^2) + \log(9/4)$ throughout $V_{\rm CW}^{\rm ferm}$. The PMS parameter absorbs this shift: $\kappa_{\rm PMS} = \log(9/4)$ exactly — an algebraic identity (Dynkin indices and Schur), not a fitted value. [A343, A344] □

C6 status: PROVED ⋆⋆⋆⋆⋆.

$\psi_{\rm pol}$ frontier and AX6$_{\rm pol}$ (S38 updates; symbol repaired Rev32.2). The $\sin^2\theta_W = \phi^{-3}$ target is achieved at $\tan^2\psi_{\rm pol} = (6\phi - 1)/4 \approx 2.177$ [D237] — this is the polarisation object AX6$_{\rm pol}$ of Paper 4, written $\psi_\star$ before Rev32.2; the Det^2 selector $\cos(6\psi) = \frac{1}{2}$ is a different object. Whether the Coleman–Weinberg potential selects $\psi_{\rm pol}$ on the exact-colour $I_4 = \phi^3$ orbit was investigated in the correct 8D Peirce framework in Sessions 37–38. The correct framework uses the 8D $\mathbb{O}_s$ fibre within a single Peirce block, with spectrum $\{+32, 0^6, -32\}$ confirmed for the P_{13} sector [A619]. The electromagnetic generators in this framework are $F_1 = e_0$ (real unit of $\mathbb{O}_s$) and $F_3 = e_7$ (the unique imaginary $G_2/SU(3)_c$ singlet) [A620], and the Peirce eigenvalue at $J_{\rm vac}$ is

$$\lambda_{13} = \frac{\phi + \phi^{-1}}{2} = \frac{\sqrt{5}}{2} \tag{40}$$

(correcting an earlier erroneous value $\lambda_{13} = 1$ that appeared in D617.5). The one-loop fermion CW correction on the ψ-circle is

$$\delta V_{\rm CW}(\psi) = -32\big[f(m_+^2(\psi)) - f(m_-^2(\psi))\big], \quad f(m^2) = m^2\left[\log\frac{m^2}{\mu^2} - \frac{3}{2}\right], \tag{41}$$

with projection ratio $R(\psi) = (1 + \sin 2\psi)/(1 - \sin 2\psi)$, which is *independent of* ϕ [A620]. The only critical point of $\delta V_{\rm CW}$ is $\psi_{\rm crit} = 45\check{\rm r}$ ($\tan^2 \psi_{\rm crit} = 1 \neq 2.177$), confirmed numerically to 60-digit precision [A621]. Three independent negative verdicts (A617, A620, A621) confirm that AX6 is a *genuine independent postulate*, not a consequence of the present CW framework. [P-006] (An S38-era sentence, "the programme stands at 99/100 with AX6 as the one remaining axiom", stood here through Rev32.4 and is retired at Rev32.5: tiers name mathematical provenance, $\text{AX6}_{\rm pol}$ is one independent postulate among the declared inputs, and the open list lives in Appendix X.)

Dispatch	Result	Status	Verified
D332	$V_{\rm CW}^{\rm bos} \equiv 0$ (algebraic index)	STRUCTURAL (index-level; §A.7.4)	A332
D333	$V_{\rm CW}^{\rm ferm} \neq 0$; $r^\star = \phi^2$ minimum	PROVED	A333
D337	$y^2 = 2/h^\vee(F_4) = 2/9$	PROVED	A337
D340	$\kappa_{\rm PMS}$ numerical target $= \log(9/4)$	PLAUSIBLE ⋆⋆⋆⋆	A340
D341/A341	PMS algebraic proof	RETRACTED — A342	
A342	Two fatal errors in D341; Path B identified	SOUND ⋆⋆⋆⋆⋆	Gemini
D343	Path B mechanism: Killing form / field rescaling	PLAUSIBLE ⋆⋆⋆⋆	A343
D344	$\iota(G_2 \subset F_4) = 9/4$ exact; $\kappa_{\rm PMS} = \log(9/4)$	PROVED ⋆⋆⋆⋆⋆	A344
A619	M spectrum $\{+32, 0^6, -32\}$ confirmed (8D Peirce)	PROVED	Gemini
A620	$R(\psi)$ ϕ-independent; $\psi_{\rm crit} = 45\check{\rm r}$	NEGATIVE ⋆⋆⋆⋆	Gemini
A621	$R(\psi_{\rm pol}) = 27.105 \neq 2.177$ (60-digit)	NEGATIVE ⋆⋆⋆⋆	Grok

A.7.4 Topological spin structure and the CW mass sector

Definition 0.7 (Triality automorphism)**.** *The* triality automorphism τ *is the unique order-3 outer automorphism of* $\mathfrak{so}(4,4)$*:*

$$\tau : \ 8_v \ \longrightarrow \ 8_s \ \longrightarrow \ 8_c \ \longrightarrow \ 8_v. \tag{42}$$

It is Clifford-invariant (an automorphism of $\mathrm{Spin}(4,4)$ *up to outer class) but* not *an automorphism of the split-octonion multiplication* $(\mathbb{O}_s, \times)$*. The stabilizer of* τ *in* $SO(4,4)$ *is* $G_2^* = G_{2(2)}$*, the split real form of* G_2*, which plays the role of the band-gap / interface symmetry in the moire* $(4,4)$ *vacuum.*

Novelty label: physical interpretation not established here.

Theorem 0.8 (Möbius spin structure and CW sector)**.** *Placing the fermion field* Ψ *in the* $(1,3)$ *Peirce block of* $J_3(\mathbb{O}_s)$ *is equivalent to choosing a spin structure on the* $(4,4)$ *Clifford bundle compatible with the* $(1,3)$ *Lorentz frame. This choice forces:*

(i) Anti-periodic (Möbius) boundary conditions *on fermion loops:* $\Psi(\tau + \beta) = -\Psi(\tau)$.

(ii) Periodic boundary conditions *on boson loops.*

Consequently:

$$V_{\rm CW}^{\rm bos} \equiv 0 \qquad \textit{(bosons periodic: modes cancel exactly)} \tag{43}$$

$$V_{\rm CW}^{\rm ferm} \neq 0 \qquad \textit{(fermions Möbius: modes do not cancel)} \tag{44}$$

with the fermionic minimum at $r^\star = \varphi^2$ [D333].

Proof sketch. In the $(4,4)$ Clifford bundle the bosonic sector 8_v splits as $(4,1)\oplus(1,4)$ under $SO(4)\times SO(4)$, giving four positive and four negative Hessian eigenvalues per Peirce block. The grading operator $\Gamma = \mathrm{diag}(+1,+1,+1,+1,-1,-1,-1,-1)$ on 8_v gives $V_{\mathrm{CW}}^{\mathrm{bos}} \propto \mathrm{Tr}_{8_v}(\Gamma \mathcal{M}^4 \log \mathcal{M}^2) = 0$ identically (the algebraic target-space index vanishes by $\mathbb{Z}_2$ parity of $SO(4)\times SO(4)\subset SO(4,4)$; cf. the Witten index $\mathrm{Tr}(-1)^F$ of [2]). For the fermionic sector, restricting to the $(1,3)$ sub-bundle breaks the $\mathbb{Z}_2$ parity and removes the cancellation. □

Novelty label: physical interpretation not established here.

Proposition 0.9 (Algebraic index and topological triviality of the bosonic CW sector)**.** *The exact vanishing $V_{\mathrm{CW}}^{\mathrm{bos}} \equiv 0$ is a consequence of the vanishing algebraic target-space index of the $SO(4,4)$ vector bundle. The bosonic modes live in $8_v = (4,1)\oplus(1,4)$ under $SO(4)\times SO(4)$. The $\mathbb{Z}_2$ parity grading operator*

$$\Gamma = \mathrm{diag}(+1,+1,+1,+1,-1,-1,-1,-1) \quad \textit{on } 8_v \tag{45}$$

gives

$$V_{\mathrm{CW}}^{\mathrm{bos}} \;\propto\; \mathrm{Tr}_{8_v}(\Gamma\, \mathcal{M}^4 \log \mathcal{M}^2) = 0. \tag{46}$$

Tier caveat (Rev26). *This Γ-graded trace is an algebraic (Witten-type) index, not the physical one-loop bosonic effective potential, which sums all bosonic modes with the same statistics sign. The identification of the two — and hence the reading of $V_{\mathrm{CW}}^{\mathrm{bos}} \equiv 0$ as a* physical *cancellation — holds only if the path-integral measure/BRST structure independently supplies the $SO(4,4)$ grading weight; absent that derivation the statement is index-level, not a proof that the physical bosonic determinant vanishes. The boson/fermion dichotomy below is stated at this (index) tier.*

The fermionic sector $V_{\mathrm{CW}}^{\mathrm{ferm}} \neq 0$: restriction to the $(1,3)$ sub-bundle (Theorem 0.8) breaks the $\mathbb{Z}_2$ parity, generating the vacuum minimum at $r^\star = \varphi^2$. This is status PROPOSITION ⋆⋆⋆⋆ *(algebraically closed; the compact index-theorem setting does* not *apply here since the base manifold $\mathbb{R}^{4,4}$ is non-compact; the correct framework is the algebraic target-space index analogous to the Witten index [2]).*

Novelty label: physical interpretation not established here.

Theorem 0.10 (Topological winding index $p = 8/3$)**.** *The proved mass-formula exponent*

$$p = \frac{\dim(\mathbb{O}_s)}{\mathrm{rank}(J_3(\mathbb{O}_s))} = \frac{8}{3} \tag{47}$$

is read as the topological winding ratio *(target-first provenance; Paper 7 §4): 8 transition dimensions of the split-octonion Peirce block divided by 3 generation winding levels (Peirce idempotents $\{f_1,f_2,f_3\}$). The three idempotents are the three winding sectors of the $(4,4)$ moire vacuum; $p = 8/3$ counts the octonionic dimension per winding level.* [PROVED, A616; geometric restatement.]

Physical narrative. The $(4,4)$ structure arises as the moire interference pattern of two dual vacuum-collapse cycles, one in each metric signature $(1,3)$ and $(3,1)$. SO(8) triality τ is the twist connecting the two sectors; $G_2^* = \mathrm{stab}(\tau)$ is the band-gap symmetry of the interface. Bosons have zero winding number (modes cancel: $V_{\mathrm{CW}}^{\mathrm{bos}} = 0$); fermions carry a Möbius half-twist (modes survive: $V_{\mathrm{CW}}^{\mathrm{ferm}} \neq 0$). Each Peirce idempotent f_i is a topological winding sector; the three generations are three winding levels of the $(4,4)$ moire. This narrative is *physical intuition* consistent with all proved results; it is not itself a theorem. Its *discrete* branch is built (Rev32.3; A1549: the replication map ι, the discrete Kan extension $T\oplus T\oplus T$, aligned, $V = I$ — Paper 0, moire block), with the noncentral defect as the open object; the number three stays an input. The *continuous* realization of this picture

is *refuted* [A693]: gluing the moire cycle by the order-3 triality τ forces the τ-invariant zero-mode to place 1/3 of its weight in the vector block $8_v = J_{12}$ (destroying the Standard Model charges), and a one-cycle holonomy cannot multiply the 16-dimensional fermionic fiber into the 48 states required. Any viable replication must instead be *discrete*: a finite noncommutative-geometry spectral triple in which $\{f_1, f_2, f_3\}$ form a three-point internal space and the Dirac operator is a finite matrix. In that framework the number three is an input, not yet derived; it is recorded here as the only surviving candidate direction.

A.7.5 Quark Gauge-Protection Theorem

Novelty label: new specialization proved here.

Theorem 0.11 (Quark Gauge-Protection Theorem; algebraic/one-loop form)**.** *Let $J_{\text{vac}} \in J_3(\mathbb{O}_s)$ be the vacuum element. Because J_{vac} lies in the singlet of G_2 (the automorphism group of $\mathbb{O}_s$), the algebraic representation-level second variation of the Coleman–Weinberg potential with respect to colour-triplet fluctuations vanishes in the one-loop Hessian:*

$$\frac{\partial^2 V_{\text{CW}}^{(1)}}{\partial P_{ij}^{\text{triplet}}\, \partial P_{ij}^{\text{triplet}}} = 0 \quad \textit{for all triplet-representation Peirce blocks.} \tag{48}$$

Schur's lemma ensures the block structure *at the algebraic representation level — no singlet–triplet mixing, and scalar action λI on each irreducible triplet block; it does* not *by itself force $\lambda = 0$ (Rev32.1 retype, A1538 F1: the previous statement inferred the vanishing from Schur's lemma alone — a non-sequitur, withdrawn). [LIB2-087] The displayed zero rests on the certified numerical computation [A630], carried with the S290.4 falsifier-control note executed at Rev32.1 (2026-08-14): the block-diagonality statement stands, the Schur gauge-protection inference is withdrawn in print (Rev32.1 changelog), and the certified-numerics scope is unchanged. An all-orders statement would require a separate shift symmetry, Ward identity, BRST, or non-renormalisation argument and is not provided here. [LIB2-087, LIB2-250]*

Proof sketch. The CW potential is a functional of traces $\text{Tr}(M^{2n})$ of the squared mass matrix M^2. Under the G_2 action, J_{vac} is a singlet, so any variation δJ that transforms as a colour triplet integrates to zero in $\text{Tr}(J_{\text{vac}} \circ \delta J)$ by Schur's Lemma. The Hessian $\delta^2 V_{\text{CW}}$ is therefore block-diagonal in the G_2-representation decomposition. *Rev32.1 withdrawal (A1538 F1):* the previous version of this sketch concluded "zero entries for all triplet components at every loop order" from the block structure; that inference is withdrawn — Schur permits a nonzero scalar on each triplet block, and a permitted invariant term $\mu^2\langle \delta J_R, \delta J_R\rangle$ would preserve the block structure while lifting the zeros. [LIB2-250] The six triplet zeros stand as the certified numerical result of [A630], not as a consequence of this representation-theoretic argument. [LIB2-087, LIB2-250] □

Novelty label: physical interpretation not established here.

Corollary 0.12. *Conditional on the A630 zero modes (a certified computation — not the withdrawn Schur inference; Rev32.1): quark masses must originate from one of two mechanisms: (i) the confinement potential ΔV_{G_2} (for u, d, s constituent masses), or (ii) off-diagonal Peirce closure mixing insertions producing effective Yukawa couplings (for c, b, t current masses). Neither mechanism is a CW radiative mass.*

A.7.6 Hessian block structure of V_{CW}

The second variation of the Coleman–Weinberg potential at J_{vac} takes the block form

$$\delta^2 V_{\mathrm{CW}} = \underbrace{3\times 3}_{\text{Higgs-like}} \oplus \underbrace{8\times 8}_{P_{12}} \oplus \underbrace{8\times 8}_{P_{13}} \oplus \underbrace{8\times 8}_{P_{23}}. \tag{49}$$

Within each 8×8 block the spectrum is (the G_2 branching organizes the block but does not force these values — Rev32.9, propagating §A.7.5)

$$\{+32,\, 0^6,\, -32\}, \tag{50}$$

where the six zero eigenvalues correspond to the six colour-triplet (quark) modes reported as exact zeros by the certified A630 computation (Rev32.1: no longer attributed to the withdrawn Schur inference of Theorem 0.11 — see §A.7.5), and ± 32 correspond to the G_2 singlet modes (colour-singlet leptons).

The 3×3 Higgs-like block governs the diagonal scalar fluctuations δj_i; its eigenvalues scale as $m(\delta j_i) \propto \varphi,\, 1,\, \varphi^{-1}$, matching the Roman surface anchor hierarchy. [A630]

A.7.7 Roman surface model of Jordan closure

The off-diagonal Peirce closure $P_{ij} \circ P_{jk} \subset P_{ik}$ (the mixing topology of $J_3(\mathbb{O}_s)$) admits a concrete geometric realisation as the *Roman surface* (Steiner 1840), a self-intersecting map $\mathbb{RP}^2 \to \mathbb{R}^3$:

$$(x', y', z') \;\longmapsto\; (y'z',\, z'x',\, x'y'). \tag{51}$$

Setting the diagonal anchors as $x' \sim \varphi$ (Gen 3), $y' \sim 1$ (Gen 2), $z' \sim \varphi^{-1}$ (Gen 1), the cross-product hierarchy becomes:

$$x'y' \sim \varphi\ (P_{12}), \qquad z'x' \sim 1\ (P_{13}), \qquad y'z' \sim \varphi^{-1}\ (P_{23}). \tag{52}$$

This *inverts* the anchor hierarchy: the lightest-generation quarks map to the largest Peirce block P_{23} ($\lambda_{23} = \varphi^{1/2}$), producing the heaviest confinement-driven constituent mass [A632].

The three double-point lines of the Roman surface correspond to the three off-diagonal Peirce blocks (the $\mathbb{Z}_2^2$ grading); the six pinch points (at $(\pm\frac{1}{2}, 0, 0)$ and cyclic permutations) are the rank-drop points of the diagonal Jordan adjoint, while the determinant-zero locus maps to the Roman double-line skeleton [A680]. The non-orientability of the Roman surface reflects the Möbius boundary condition on $(1,3)$-representation spinors established in Theorem A.7.4.

Remark 0.13 (Lepton/quark duality)**.** *Leptons acquire mass through direct CW diagonal coupling, following the primary anchor hierarchy ($\varphi > 1 > \varphi^{-1}$: heavier generation, larger coupling). Quarks acquire mass through the* inverted *cross-product hierarchy: lighter generation, larger Peirce block, larger confinement scale. The duality is an exact algebraic consequence of the Roman surface parametrisation and is not imposed by hand.* [A632]

Remark 0.14 (Nomenclature)**.** *All prior references to a informal cube model for generation mixing are superseded by the Roman surface model, which is the correct mathematical object for the Peirce closure structure. The Roman surface is also known as the Steiner surface; see Bryant–Kusner for the modern parametrisation [1].*

A.7.8 Artin's theorem and the QFT loop bridge

A key structural question for the programme is whether the non-associativity of $\mathbb{O}_s$ propagates into quantum loop corrections, requiring the construction of a non-associative perturbation theory. Artin's theorem resolves this:

Novelty label: standard theorem used.

Theorem 0.15 (Artin)**.** *Any subalgebra generated by two elements in an alternative algebra is associative.*

The split octonions $\mathbb{O}_s$ form an alternative algebra (satisfying $x^2y = x(xy)$ and $yx^2 = (yx)x$). In quantum field theory:

(i) **2-point functions** (propagators) involve two field insertions. Two insertions do not by themselves make the internal algebra labels two-generated; Artin applies where that containment holds, which is the same generator census as item (ii) (Rev32.9, A1686 F05).

(ii) **3-point functions** (Yukawa and gauge vertex corrections) involve the Higgs and a fermion bilinear. Where a diagram's internal algebra labels lie in a subalgebra generated by two elements, associativity follows by Artin; for the generic cross-block Yukawa trilinear (three independent split-octonion entries, Paper 6) that containment is *not established* — momentum conservation constrains momenta, not the number of independent internal octonionic directions (Rev32.1 scope, A1538 F5). [LIB2-088, LIB2-251] A diagram-by-diagram generator census is the named outstanding object. [LIB2-088]

(iii) **4-point functions** (box diagrams) are, pending that census, the first place where three fully independent octonionic directions are guaranteed to appear (P_{12}, P_{13}, P_{23}) together. Non-associativity is here in principle, but suppressed by an additional loop factor relative to the Yukawa and gauge corrections.

Novelty label: physical interpretation not established here.

Corollary 0.16. *Standard* $\overline{\text{MS}}$ *renormalisation-group running of Yukawa and gauge couplings from* Λ_{G_2} *to* m_Z *is protected by Artin's theorem* for diagrams whose internal labels are two-generated *(any propagator or vertex correction meeting the containment of items (i)–(ii) above); the blanket "fully protected" claim is scoped Rev32.1 pending the generator census (A1538 F5). [LIB2-251] The* G_2 *automorphism group provides a consistent associative envelope for the leading-order diagrams that pass the generator census (scoped Rev32.9; the census is outstanding). Non-associative QFT is not required to derive the physical c and b quark masses from the skeleton provided by the Jordan geometry.* [A633]

A.8 Open bridges: KK consistency and Grassmann parity

Two older programme items remain explicit.

(1) **PR-C2 (KK consistency).** The KK-70 counting and the $E_{7(7)}/SU(8)$ target are secure, but the explicit consistent-truncation proof remains open.

(2) **PR-C3 (Grassmann parity / spin-statistics).** The local skeleton displayed here does not yet solve the deeper spin-statistics and Grassmann-parity problem from first principles. This remains an open bridge rather than a hidden omission.

A.9 Electron closure and lepton-family validation

The historical instanton baseline is

$$m_e = M_{\rm Pl} \times \exp\left(-\frac{\pi}{8\alpha}\right) \times \phi^4 \times \sqrt{2} = 0.5036\,\text{MeV}, \tag{53}$$

noting that $2\pi/(16\alpha) = \pi/(8\alpha)$ exactly (same formula, cleaner form). In Rev29 this expression is retained as a semiclassical cross-check; the operative closure is the QED-corrected Koide chain.

Sessions 36–38 replaced the Sprint 3 empirical μ/τ formula by an exact algebraic expression. *Rev29*: that expression is re-tiered STRUCTURAL / LOADED-CORRESPONDENCE / REPRODUCED, and the earlier "fully proved / decisive validation closed" language is withdrawn; the provenance chain and the fit gap are printed once, in Appendix X ("Provenance of the charged-lepton ratio"). The exact tree-level value is

$$\frac{m_\mu}{m_\tau} = \left(\frac{\phi}{\sqrt{5}}\right)^{8/3} \times \frac{\sqrt{2}}{10} = 0.059684 \qquad (\text{PDG: } 0.059461), \tag{54}$$

where $p = 8/3 = \dim(\mathbb{O}_s)/\text{rank}(J_3)$ is an exact dimension ratio [A616] and $C = \sqrt{2}/10$ is an exact dimension count in the $J_2(\mathbb{O}_s)$ sub-algebra geometry: $\dim(J_2(\mathbb{O}_s)) = 10$ and off-diagonal trace norm $= \sqrt{2}$ [A618]. [LIB2-049] *Rev29 correction.* Neither exact fact establishes, by itself, that these are *the* mass exponent and *the* normalization: the provenance is target-first (`D616:99-102` asks whether the measured 0.141 is algebraic in φ; `A616:50-63` offers $\sqrt{2}/10$ while saying the algebraic origin is not yet proved; $\dim J_2 = 10$ arrives afterward at `A618:47-51`; the divide-by-dimension step is "argued by analogy" at `A618:134-140`; and `A618:147` lists the formal derivation as BLOCKING, closed nowhere in the corpus). The claim that this is a lepton mass ratio "obtained with zero free parameters from the Jordan algebra alone" is therefore **withdrawn**. The corresponding muon mass is $m_\mu = 106.05\,\text{MeV}$ (+0.37% PDG), superseding the Sprint 3 empirical value of 104.87 MeV which optimised a float; because C enters multiplicatively, that +0.37% *is* the fit gap — the ratio requires $C = 0.140894$ and the offered constant is $\sqrt{2}/10 = 0.141421$. The calibration $\Lambda_{G_2} \approx 260\,\text{MeV}$ ($260 \pm 0.5\,\text{MeV}$, Appendix X) — one object with one type, an external dimensional calibration (Appendix X input-count box), its value taken from hadronic phenomenology and not a measured $\Lambda_{\rm QCD}$ — is not algebraically derivable; here it serves as the dimensional-transmutation anchor for the heavy-lepton sector, and Paper 2 uses the same calibration for the light-quark constituents [A618] (Rev32.9, R84-h). The electron mass is KOIDE-CONSISTENT ⋆⋆⋆⋆ under the external empirical rule $K = 2/3$ (A636, A637): $m_e = 0.5076\,\text{MeV}$ (−0.66% PDG). [P-030] The charged-lepton chain is structurally pinned with mixed provenance — m_τ anchor, m_μ from a STRUCTURAL / LOADED-CORRESPONDENCE ratio (*re-tiered Rev29*), m_e Koide-consistent under the external rule — and the three masses are not jointly derived; the authoritative ledger is Appendix O ("The authoritative charged-lepton ledger").

Novelty label: physical interpretation not established here.

Proposition 0.17 (Geometric instanton interpretation of the electron mass)**.** *In the* $(4,4)$ *Clifford vacuum of* $J_3(\mathbb{O}_s)$*, with fermions assigned to the* $(1,3)$ *Peirce block (Theorem 0.8), the electron mass formula* (53) *admits the following semi-classical instanton interpretation at* PLAUSIBLE ⋆⋆⋆⋆*.*

Exponent. The factor $-\pi/(8\alpha)$ *is the on-shell Euclidean instanton action of a Möbius half-vortex in the* $U(1)$ *gauge background* $T_{\rm em} = [e_0, e_7]$ [A620]*. The chain is:* $S_{2D} = \pi/e^2 = 1/(4\alpha)$ *(Bogomol'nyi-saturated 2D vortex); lifting to 4D with half-twist area factor* $\pi/2$ *averaged over the 4 positive-signature directions of the* $(4,4)$ *metric (mandated by Proposition 0.9):*

$$S_{\rm inst} = \frac{1}{4\alpha} \cdot \frac{\pi/2}{4} = \frac{\pi}{8\alpha}. \tag{55}$$

The Jordan kinetic term $\mathcal{L}_J$ contributes at subleading order only (saddle-point cancellation at $M_{\rm Pl}$ scale).

Prefactor. The factor $\phi^4 = (r^\star)^2$ is the Jacobian of the scale collective coordinate at the proved minimum $r^\star = \phi^2$ of $V_{\rm CW}^{\rm ferm}$ [D333]. *The factor $\sqrt{2}$ is the quadratic-form normalisation of the real idempotent anchors ($\mathrm{Tr}(f_i f_j) = \delta_{ij}$, so $\sqrt{\mathrm{Tr}(f_1^2 + f_2^2)} = \sqrt{2}$), coinciding with the proved normalisation of the two-generation subalgebra $J_2(\mathbb{O}_s)$* [A618].

Status in Rev29. This instanton formula is no longer the operative mass closure. The electron mass is fixed by the QED-corrected Koide chain (under the external empirical rule $K = 2/3$),

$$m_e = 0.5076\,\mathrm{MeV}, \tag{56}$$

using $m_{\mu,\mathrm{phys}} = 105.72\,\mathrm{MeV}$ after the one-loop QED matching correction. The instanton baseline is retained only as a semiclassical cross-check on the scale and topology of the electron sector. [A636, A637 — *tier annotation:* A637 records this chain as "proved at every step"; that internal rating is **not** the tier carried here. The chain misses by -0.66%, and this appendix deliberately retains it only as a semiclassical cross-check. Where the assessment and the paper disagree on tier, **the paper's weaker reading governs**.]

A.10 Parameter bookkeeping and suite integration

Rev29 uses four explicit bookkeeping classes.

(1) **Algebraic quantities:** Peirce eigenvalues, resolvent ratios, tree-level Yukawa eigenvalues, the mixing formulae (internal constructions; physical tiers per Paper 3).

(2) **Measured anchors:** $v = 246\,\mathrm{GeV}$, $m_\tau = 1776.93\,\mathrm{MeV}$ where used as anchor, and standard benchmark quantities used only for comparison.

(3) **Phenomenological ans"atze:** none remaining in the charged-lepton mass sector. The electron mass $m_e = 0.5076\,\mathrm{MeV}$ is KOIDE-CONSISTENT ⋆⋆⋆⋆ via the QED-corrected Koide chain under the external empirical rule $K = 2/3$ [A636, A637] (the external rule keeps this at mixed provenance, not class-1 algebraic). The muon-to-tau ratio constant $C = \sqrt{2}/10$ and exponent $p = 8/3$ are *exact algebraic quantities* whose *physical attachment* to the mass formula is LOADED-CORRESPONDENCE (*re-tiered Rev29*; the S38 "proved" typing is withdrawn — Appendix X, "Provenance of the charged-lepton ratio").

(4) **Deferred items:** exact weak-angle selector, full KK consistency proof, formal heavy-quark S-matrix bridge, and the remaining bosonic extraction problems.

Papers 0–7 use Appendix A in a strict way. No statement about the local Yukawa or mass-operator structure is allowed to float freely in the prose without either tracing back to the derivation recorded here or being labelled as measured, phenomenological, or deferred.

References

[1] R. Bryant and R. Kusner, "The Roman surface and its symmetries," unpublished notes; see also J. Steiner, "Über solche algebraischen Curven,…," *J. reine angew. Math.* **21** (1840) 33–66.

[2] E. Witten, "Constraints on Supersymmetry Breaking," *Nucl. Phys. B* **202** (1982) 253.

Leibniz Quantum Beats Newton

Appendix B — Internal Consistency Checks (established Rev25; suite Rev33.2)

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August 2026

Abstract

Appendix B records the internal consistency checks attached to the Rev29 cubic-invariant Yukawa derivation. It is included to make the verification trail explicit without interrupting the main papers. The appendix does not add new physics claims beyond the Rev29 PMNS closure update. It states which algebraic steps were cross-checked, what has newly closed in the leptonic phase sector, and what remains outside the verified core. These checks are self-audits performed within the programme; independent external replication awaits public release of the computational kernels.

B.1 Verification questions

The Rev29 cubic-invariant Yukawa derivation was checked against five questions:

(1) Does the Freudenthal cross product of two $(1,3)$-block fields project onto the middle diagonal idempotent?

(2) Does contraction with $J_{\text{vac}} = \text{diag}(\phi, 1, \phi^{-1})$ extract only the middle eigenvalue?

(3) Is the resulting tree-level Yukawa eigenvalue exactly $y_t = 1$?

(4) Is the off-diagonal Higgs geometry preserved?

(5) Does the replacement of the linear route by the cubic route remove the need for an extra Higgs-placement axiom?

B.2 Outcome

The outcome of the internal Rev29 verification pass is:

Check	Result
Cross-product algebra	passed
Vacuum contraction	passed
Tree-level $y_t = 1$ extraction	passed
Off-diagonal Higgs geometry	passed
No-extra-axiom closure	passed

B.3 Remaining issue assessment

The verified core is the tree-level algebra. The following items remain outside the scope of the verification note:

(a) the running of $y_t(\mu)$ beyond the tree-level boundary value,

(b) the derivation of the electroweak vacuum expectation value,

(c) the deeper selector for the golden-ratio vacuum.

B.4 Verdict

The verification summary supports the publication-level statement used in Rev29: the cubic-invariant Yukawa derivation is algebraically ready, while the measured electroweak scale remains an open bridge. The vacuum selector is not an open bridge: it is fixed conditionally, by the vacuum-selector theorems of Appendix E (the Paper 0 ledger row; Rev33.0 correction of an under-claim).

B.5 Leptonic CP phase — V_3 rotation proof (Rev29)

Status change. Rev10 carried the leptonic phase as $-\arccos(-1/3) - 2\pi/7 \approx -160.900°$. Rev29 upgrades the result to a theorem:

$$\delta_{CP} = -\frac{2\pi}{\sqrt{5}} \approx -160.997°, \qquad J_{CP} \approx -0.011. \tag{1}$$

No fitted coefficients enter the PMNS sector (three of four entries theorem-tier pre-Rev29, DERIVED-CONDITIONAL after the re-tiering (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`); θ_{13} structural under the named readout rule, §B.5′). **No row may be cited as zero-parameter.**

The V_3 rotation mechanism. Define the three-dimensional subspace

$$V_3 = \mathrm{span}\{J_{\mathrm{vac}},\, J^{\#}_{\mathrm{vac}},\, T_{\mathrm{half}} \cdot J_{\mathrm{vac}}\}. \tag{2}$$

Kernel verification shows that V_3 is invariant under the adjoint action of T_{half}. Inside this subspace, T_{half} acts as a genuine three-dimensional rotation with angular frequency

$$\Omega = \frac{\sqrt{5}}{2}. \tag{3}$$

The canonical half-period is therefore

$$t^* = \frac{2\pi}{\sqrt{5}}, \tag{4}$$

and the defining orbit relation is

$$\exp(t^* T_{\text{half}}) \cdot J_{\text{vac}} = J^{\#}_{\text{vac}}. \tag{5}$$

The leptonic Dirac phase is the negative of this rotation time:

$$\delta_{CP} = -t^* = -\frac{2\pi}{\sqrt{5}}. \tag{6}$$

Structural support. Three earlier theorem blocks feed directly into the result:

(a) **DET-7 theorem.** The Jordan pair $(J_{\text{vac}}, J^{\#}_{\text{vac}})$ has Gram determinant 7, fixing the mismatch geometry and the normalization of the $(1,3)$ sector. [LIB2-048]

(b) **C1 theorem.** The weight $c = \frac{1}{2}$ in $T_{\text{half}} = T_{\text{scalar}} + \frac{1}{2}T_{\text{oct}}$ is exact, so the generator is fixed with no free coefficient. [LIB2-051]

(c) **Uniqueness theorem.** T_{half} is, up to scale, the unique $G_{2(2)}$-invariant generator in the $(1,3)$ Peirce sector of $\mathfrak{f}_{4(4)}$; the scale is the normalisation $T_{\text{half}} = T_{\text{scalar}} + \frac{1}{2}T_{\text{oct}}$ of the item above (Rev32.9). [LIB2-052]

Independent cross-check. The older $-\arccos(-1/3) - 2\pi/7 = -160.900°$ formula remains useful as a structural cross-check, but it is no longer the authoritative statement of the phase law.

PMNS closure. With

$$\tan\theta_{12}^{\text{PMNS}} = \frac{\sqrt{3}}{\phi^2}, \qquad \sin^2\theta_{23} = \frac{7}{16}, \qquad \sin^2\theta_{13} = \sin^4\frac{\pi}{8} = \frac{3-2\sqrt{2}}{8},$$

and

$$\delta_{CP} = -\frac{2\pi}{\sqrt{5}},$$

three of the four PMNS parameters were marked theorems at ⋆100 pre-Rev29; after the re-tiering the two physical rows θ_{12} and δ_{CP} stand at DERIVED-CONDITIONAL and the θ_{23} octant attachment is LOADED-CORRESPONDENCE (A1566) (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`); θ_{13} is STRUCTURAL ⋆⋆⋆ (round-trip framing T^2, exponent-2; forcing pending [s502/A1275]).

Lemma B.5′ (θ_{13} readout normalization; Structural, readout-lemma). The reactor angle follows from a single readout-normalization step rather than from the deprecated "triality maps $\pi/4 \to \pi/8$, then square" chain. Reading the lepton mixing entry off the *unit endpoint ray* of the Route-B Jordan frame, and passing through the spin double cover with the half-angle $\pi/8$ — a *readout normalization choice* that the $T^2 = -1$ moment structure of Appendix H *motivates* but does *not* make canonical (*Rev29*) — gives

$$|U_{e3}| = \sin^2\frac{\pi}{8} \qquad \Longrightarrow \qquad \sin^2\theta_{13} = \sin^4\frac{\pi}{8} = \frac{3-2\sqrt{2}}{8} \approx 0.02145,$$

in agreement with the NuFIT 6.1 global-fit value $0.02248^{+0.00055}_{-0.00059}$ ($\approx 1.8\sigma$). *Status.* This is STRUCTURAL plus a named readout lemma: it is a theorem *given* the unit-endpoint-ray readout rule, but that readout rule is not itself derived from the algebra (the round-trip T^2 squaring is identified (exponent-2, s502/A1275) but its forcing is pending), so θ_{13} is not promoted to PROVED. [LIB2-281, LIB2-282, LIB2-315] It is the same readout-axiom posture as F-δ for the CKM phase.

(*Rev29*) The T-bridge does not canonicalize $\pi/8$. Earlier wording made the half-angle $\pi/8$ *canonical* by appeal to $T^2 = -1$. That inference is withdrawn. [LIB2-252] $T^2 = -1$ is a statement about a symplectic element of $E_{7(7)}$ on the certified **56** (Appendix H, live T-bridge): it fixes the spinorial character of the substrate half-turn, and it is *consistent* with reading a mixing entry at a half-angle, but it does not select *which* angle is halved, does not fix the ray the entry is read off, and does not supply the squaring step. [LIB2-252, LIB2-315] The half-angle $\pi/8$ is therefore *motivated* by the moment structure and *chosen* by the readout rule — which is exactly what the Status note above already concedes; the lemma statement is here brought into line with its own status note. [LIB2-252] Nothing numerical moves: $\sin^2\theta_{13} = \sin^4(\pi/8) = (3-2\sqrt{2})/8 \approx 0.02145$ and its $\approx 1.8\sigma$ standing against NuFIT 6.1 are as printed, and θ_{13} stays STRUCTURAL ⋆⋆⋆ with forcing pending (`s502`/A1275). Recorded explicitly because this retraction has been made in-house three times (killed S188, resurrected S189, re-killed S261) and reached the papers in none of the three; it is printed here so the canonicality claim cannot be re-mined downstream (p0, p3, p5, Appendix C, Appendix X inherited it).

B.6 CKM sector — algebraic construction and the conditional $\delta_{\rm CKM}$ theorem (Sessions 22–23)

Status (S112 claim-hygiene; re-tiered Rev29). The three CKM *magnitudes* ($|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$) were marked PROVED-tier within the Route-B map pre-Rev29 and agree with PDG within 1σ. After the Rev29 row-by-row re-tiering (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`) the CKM *first row* is LOADED — Paper 3 selected Route B over Route A by comparing both to kaon data — and $|V_{cb}|$ is a Structural formula with LOADED-CORRESPONDENCE attachment (A1566). The pre-Rev29 "six of eight" tally in the provenance ledger (Appendix X) is superseded by **two of eight** ($\theta_{12}^{\rm PMNS}$, δ_{CP}; the physical θ_{23} row is LOADED-CORRESPONDENCE since A1566/D1244, Rev32.6, its internal 7/16 identity theorem-grade). The Dirac *phase* $\delta_{\rm CKM}$ is *not* of that grade: it is a *candidate construction under a named readout axiom* (stated immediately below), i.e. a COINCIDENCE-CLASS numeral with an algebraic identity behind it and no selection of G_7 (Rev32.5 retype; the Rev29 "Structural claim with a derivation path" wording is retired). [LIB2-253] The earlier "100/100, all four GREEN, zero free parameters" framing is retired here: it swept the conditional phase into the same bucket as the proved magnitudes, and was inconsistent with this construction's own status note — the chain (C)–(D) was shown (S87–S88, D772/D773) to be an algebraic *identity* of the construction, not an independent selection theorem.

The $G_7 = \phi + 5\phi^{-5}$ candidate construction (A578+A579; E92–E98) — formerly titled "conditional theorem"; Coincidence-class since Rev29.

Named readout axiom (F-δ, Structural). $\delta_{\rm CKM}$ is read out as an *additive bounded-resolvent channel-expectation* over the Route-B rank-5 complement. Under this axiom — and only under it — the construction below is a theorem. The axiom is itself STRUCTURAL (it is the readout rule, not derived from the algebra), so $\delta_{\rm CKM}$ is a *conditional* theorem, not a PROVED-tier result. Determinant / log-determinant readout foils fail in code (D791/D793), which is why the channel-expectation form is named explicitly rather than asserted canonical.

Under F-δ, the CKM Dirac phase follows from the sub-leading eigenvalue correction of the Albert-

algebra vacuum:

$$G_7 = \phi + 5\phi^{-5}, \qquad \delta_{\mathrm{CKM}} = \arctan\sqrt{3(\phi + 5\phi^{-5})} = 68.1297^\circ. \tag{7}$$

(Comparison: 0.8σ from the γ-fit; 1.6σ from the PDG direct value.)

Four-step algebraic proof (A578+A579; complete):

(C) **Peirce-graph loop order** (E96, part C). The minimum closed Peirce-graph path returning to P_{13} requires exactly $n = 5$ applications of T_{half}. The Fibonacci number at $n = 5$ is $F_5 = 5$, giving correction order ϕ^{-5} with coefficient 5.

(B) **Split-octonion coupling-weight theorem** (E96, A579; Part B). The split-octonion $(4, 4)$ signature of $\mathbb{O}_s$ determines sector-dependent Peirce coupling weights in the 27-dimensional representation of $J_3(\mathbb{O}_s)$. Under the W_{13} block decomposition: the 3 compact imaginary directions e_1, e_2, e_3 (norm -1) contribute *zero* net coupling (isotropic cancellation); the scalar e_0 and 4 non-compact directions e_4–e_7 (norm $+1$) contribute fully. Therefore $w_{P_{13},k} = \frac{5}{8}\, w_{\mathrm{total},k}$ exactly. When inserted into the resolvent sums from $m_{12}(x) \in \mathbb{Q}(\sqrt{5})[x]$, the weighted sums equalize: $\delta M_{P_{13}} = \delta M_{P_{22}} = 5\phi^{-4} = \varepsilon$. *Rev14 status correction:* the equal-shift property of this step was shown (S87–S88, D772/D773 adjudication, artifact-backed) to be an algebraic *identity* of the construction rather than an independent derivation — house-proven circular as a selection argument. The chain (C)–(D) is therefore classified *numerology-class pending derivation*: numerically exact and unchanged, but not a theorem of selection. An attempted kill of the integer 5 (the $\alpha(0)/m_\tau$ error budget, D773) does not land ($4.9635 \pm 0.087 \ni 5$ at 1σ), so the construction is candidate-class — neither derived nor dead.

(A) **Equal-shift formula** (E93, A578; Part A). Equal shift $\varepsilon = 5\phi^{-4}$ on both Peirce masses ϕ^2 and 1 gives $G_7 = \phi + \varepsilon/\phi = \phi + 5\phi^{-5}$ exactly.

(D) J_{vac} **deformation uniqueness** (E97, A579; Part D). The perturbation $(\lambda - (\phi^2 + \varepsilon))(\lambda - (1 + \varepsilon))(\lambda - (\phi^{-2} + \varepsilon)) = 0$ is the *unique* $O(\phi^{-4})$ deformation of J_{vac} preserving the Albert-algebra characteristic-polynomial form. Trace shift $= 3\varepsilon = 15\phi^{-4}$; no alternative corrections at this order.

Table reading rule. Rev32.1 reading rule: tiers name mathematical provenance; physical attachments are stated separately; no row may be cited as zero-parameter; displayed σ values are named comparator distances, not profile likelihoods.

CKM sector status (Rev32.2 comparator; provenance per the Rev29 re-tier; A1566 retype, Rev32.6): first row Loaded, $|V_{cb}|$ Loaded-correspondence attachment (physical) / Structural formula, δ_{CKM} Coincidence-class. [LIB2-066, LIB2-285]

Parameter	Algebraic value	PDG 2026 comparator	Comparator distance
$\lvert V_{us}\rvert$	$\sin\theta_{12}^{\mathrm{PMNS}}/\sqrt{6} = 0.225256$ (`s1162`)	0.22431 ± 0.00085	$+1.11\sigma$
$\lvert V_{cb}\rvert$	$1/(9\sqrt{7}) = 0.041996$	0.0407 ± 0.0013	$+1.00\sigma$
$\lvert V_{ub}\rvert$	$\lvert V_{us}\rvert\lvert V_{cb}\rvert/\sqrt{6} = 0.003862$	0.00389 ± 0.00016	-0.18σ
δ_{CKM}	$\arctan\sqrt{3(\phi+5\phi^{-5})} = 68.1297^\circ$	global-fit $\delta = 1.154 \pm 0.025$ rad ($= 66.12^\circ \pm 1.43^\circ$; radians primary, R-29); direct $\gamma = 66.4^{+2.7\circ}_{-2.8}$	$+1.40\sigma$ / $+0.64\sigma$

No fitted CKM coefficient appears inside the selected Route-B map, but this is not a parameter-free theorem of the physical matrix: the first row is loaded and the phase motivation is incomplete. The "7 structure": $|V_{cb}| = 1/(9\sqrt{7})$ and $\sin^2\theta_{23} = 7/16$ share the algebraic integer $n_{23} = 7$ from $\det G = 7$ (Theorem B.5); the physical octant attachment of $7/16$ remains LOADED-CORRESPONDENCE (A1566). **Note:** The authoritative Rev29 CP formula is $\delta_{CP} = -2\pi/\sqrt{5} = -160.997^\circ$ (V_3 rotation theorem); the older $\mathrm{PSL}(2,7)$ formula involving $2\pi/7$ is a deprecated cross-check and is not part of the current "7 structure" claim.

B.7 Spectral equivalence lemma (C2 — representation independence)

Lemma B.7 (Albert–Freudenthal spectral equivalence; kernel E115). *Let $L_{J_{\mathrm{vac}}} : X \mapsto J_{\mathrm{vac}} \circ X$ denote Jordan multiplication by the vacuum element $J_{\mathrm{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1})$ on $J_3(\mathbb{O}_s)$. The eigenspectrum of $L_{J_{\mathrm{vac}}}$ is the same whether computed abstractly from the Peirce decomposition or concretely from the explicit 27×27 real-matrix lift.*

Proof sketch. The Jordan product rule assigns eigenvalue $(\lambda_i + \lambda_j)/2$ to the (i,j) off-diagonal Peirce block, where λ_i are the diagonal entries of J_{vac}. For $J_{\mathrm{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1})$ this gives:

Sector	Eigenvalue (exact)	Multiplicity
$(1,1), (2,2), (3,3)$ diagonal	$\phi,\ 1,\ \phi^{-1}$	1 each
$(1,2)$ off-diagonal	$(\phi+1)/2 = \phi^2/2$	8
$(1,3)$ off-diagonal	$(\phi+\phi^{-1})/2 = \sqrt{5}/2 \equiv \Omega$	8
$(2,3)$ off-diagonal	$(1+\phi^{-1})/2 = \phi/2$	8

Total: $3 + 24 = 27$ eigenvalues. Kernel check E115 constructs the explicit 27×27 matrix of $L_{J_{\mathrm{vac}}}$ and confirms numerical agreement to better than 10^{-12}. The result is a standard consequence of the Albert–Freudenthal theorem for the split form $J_3(\mathbb{O}_s)$; see also McCrimmon [1] for the abstract Peirce rule. □

Physical consequence. The $(1,3)$ Peirce eigenvalue $\Omega = \sqrt{5}/2$ matches the angular frequency used by the V_3 rotation theorem in $V_3 = \mathrm{span}\{J_{\mathrm{vac}}, J^{\#}_{\mathrm{vac}}, T_{\mathrm{half}} \cdot J_{\mathrm{vac}}\}$ (Theorem B.1, §B.5). The leptonic CP phase is therefore consistent with

$$\delta_{CP} = -t^* = -\frac{\pi}{\Omega} = -\frac{\pi}{\sqrt{5}/2} = -\frac{2\pi}{\sqrt{5}} = -160.997^\circ \quad (0.36\sigma \text{ from NuFIT 6.1 } 212^{+26\circ}_{-36}\text{, central-value only}). \tag{8}$$

The spectral structure of $L_{J_{\mathrm{vac}}}$ matches the expected eigenvalues of the derivation-generated CP phase; a formal proof of equivalence (eigenvalue $\leftrightarrow$ angular velocity) is the subject of a forthcoming appendix. Thus the spectral calculation is supporting evidence, not by itself a complete proof that the eigenvalue is the physical angular velocity.

B.8 In-house re-verification of the Koide chain (S210 machinery; Rev25 fold)

The independently-built verification machinery of S210 (kernels `s532`, shipped as `s1347`, and `s535`) re-derived the banked charged-lepton chain from scratch, deterministically, with no reference to the original build:

Quantity	Route	Re-verified agreement	Kernel
m_τ from (m_e, m_μ)	Koide closure	0.006%	`s535`
$K = 2/3$	circulant null condition	6×10^{-6}	`s535`
a_e (anomalous moment machinery)	QED chain rebuild	3 ppb	`s532`/`s1347`

These rows are *verification*, not new results: they document that the banked lepton-sector chain reproduces under independent machinery at the quoted precision. No observable moves.

References

[1] K. McCrimmon, *A Taste of Jordan Algebras*, Springer, 2004.

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Appendix C — δ_{CP} Precision Tests and Falsification Window

Established Rev25; suite Rev33.2

Tom O'Sieg

August 2026

Abstract

Appendix C records the precision-test language for the leptonic CP phase. The V_3 rotation is an exact algebraic construction, while the physical phase attachment is DERIVED-CONDITIONAL under its named readout premise (the pre-Rev29 THEOREM/$\star$100 marking is historical). The target is $\delta_{CP} = -2\pi/\sqrt{5} = -160.997^\circ$ with $\Omega = \sqrt{5}/2$ (A603/E110). Two physical PMNS rows, θ_{12} and δ_{CP}, stand at DERIVED-CONDITIONAL after the Rev29 re-tiering; the physical θ_{23} octant is LOADED-CORRESPONDENCE (its internal 7/16 identity theorem-grade, A1566) and θ_{13} is STRUCTURAL; no row may be cited as zero-parameter.

C.1 Framework target

The Rev29 phase target is

$$\delta_{CP} = -\frac{2\pi}{\sqrt{5}} = -160.997^\circ \qquad (J_{CP} \approx -0.011,\ \text{rephasing-invariant}). \tag{1}$$

Status (current reading): the V_3 rotation theorem supplies the exact algebraic orbit relation, and the physical identification of its half-period with δ_{CP} is DERIVED-CONDITIONAL under the printed readout premise. The pre-Rev29 THEOREM/$\star$100 label is retained only as a historical stamp. NuFIT 6.1 (NH, w/SK) gives a central-value comparator distance of 0.36σ; the full profile is non-Gaussian. Normal ordering is assumed.

C.1a History of the leptonic CP target

The leptonic CP target changed because the algebraic derivation was refined, not because the value was tuned to oscillation data.

Era / value	Formula	Reason for supersession
Early Rev8 value	$\delta_{CP} \approx -133.36^\circ$	Phase-only/non-associator ansatz; retained historically but no longer the operative derivation.
S11 intermediate	$-\arccos(-1/3) - 2\pi/7 \approx -160.900^\circ$	PSL$(2,7)$ / discrete-flavour refinement; useful cross-check, but the $2\pi/7$ term is deprecated.
Current target (entered Rev29; current at Rev29)	$\delta_{CP} = -2\pi/\sqrt{5} = -160.997^\circ$	V_3 rotation theorem with $\Omega = \sqrt{5}/2$ and $t^* = 2\pi/\sqrt{5}$; authoritative suite value.

C.2 Adopted precision scenario

The publication bundle uses a design-level DUNE-era precision scenario in which the Dirac phase is measured to a characteristic uncertainty of order $\pm 15^\circ$. This is used as a falsification scenario, not as a substitute for future experimental publications.

C.3 Comparison table

Quantity	Framework target	J_{CP}/notes	DUNE precision window	Current reading
δ_{CP}	$-2\pi/\sqrt{5} = -160.997^\circ$ [DERIVED-CONDITIONAL; A603/E110; read-out premise printed]	$J_{CP} \approx -0.011$	$O(10^\circ)$ at 1σ, exposure-dependent	NuFIT 6.1 NH: 0.36σ; DUNE Phase II will test
θ_{23}	41.41° ($\sin^2\theta_{23} = 7/16$; physical octant LOADED-CORRESPONDENCE, internal identity theorem-grade — A1566)	mismatch geometry	few-degree precision	NuFIT 6.1 NH global best fit lower-octant (0.470), 2.3σ away; upper-octant comparator ≈ 0.561 at $\approx 9.5\sigma$

C.4 Interpretation

The logic is intentionally sharp. If the future measured phase sits far outside the adopted band around the framework target -160.997°, the V_3 rotation mechanism ($T_{\text{half}}|_{V_3}$, $\Omega = \sqrt{5}/2$) is falsified. The suite does not reserve an unbounded freedom to repackage the prediction after the fact. The prior ansatz value -133.36° and the intermediate PSL$(2,7)$ value -160.900° are both superseded; the V_3 formula is authoritative.

C.5 V_3 rotation proof of δ_{CP} (exact algebraic layer; physical attachment Derived-conditional)

The proof used by the Rev29 suite is algebraic rather than phenomenological. Define

$$V_3 = \mathrm{span}\{J_{\mathrm{vac}},\, J^{\#}_{\mathrm{vac}},\, T_{\mathrm{half}} \cdot J_{\mathrm{vac}}\}. \tag{2}$$

Kernel verification shows that V_3 is invariant under the action of T_{half} and that the restriction $T_{\mathrm{half}}|_{V_3}$ is a genuine three-dimensional rotation. The rotation frequency is

$$\Omega = \frac{\sqrt{5}}{2}, \tag{3}$$

which coincides with the $(1,3)$-sector eigenvalue of $L_{J_{\mathrm{vac}}}$ in the Albert-Freudenthal spectrum. The canonical half-period is therefore

$$t^* = \frac{2\pi}{\sqrt{5}}. \tag{4}$$

At this time one has the exact orbit relation

$$\exp(t^* T_{\mathrm{half}}) \cdot J_{\mathrm{vac}} = J^{\#}_{\mathrm{vac}}. \tag{5}$$

The leptonic Dirac phase is identified with the negative of this rotation time,

$$\delta_{CP} = -t^* = -\frac{2\pi}{\sqrt{5}} = -160.997^\circ. \tag{6}$$

This proof contains no fitted parameter. The only algebraic inputs are the working vacuum $J_{\mathrm{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1})$, the exact C1 coefficient $c = \frac{1}{2}$ in $T_{\mathrm{half}} = T_{\mathrm{scalar}} + \frac{1}{2} T_{\mathrm{oct}}$, and the kernel-verified V_3 invariance. The older $-\arccos(-1/3) - 2\pi/7 = -160.900^\circ$ expression remains as a useful internal cross-check, but the suite takes the V_3 rotation law as authoritative.

Derived structural CP seed (s502/A1208). The commutator invariant $\mathrm{Im\,Tr}([\Phi_u, \Phi_d]^3) = \varepsilon\, \Delta^3 \cdot (-243\sqrt{15}/256)$ is recorded as a *derived structural quantity*, not a prediction: Parseval/reciprocity fix a single $\mathbb{Z}_2$ sign ε, while the magnitude remains a frozen section (A1208). It is carried for provenance only and moves no observable.

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Appendix D — RG and Threshold Reference Notes (Rev7-era reference; suite Rev33.2)

Legacy Reference — Rev7 era. Superseded by Rev29 for all threshold calculations.

Tom O'Sieg

August 2026

Abstract

Legacy reference (Rev7 era). This appendix is superseded by Rev29 for all threshold calculations. It is retained as a historical record only.

Appendix D is the rev7 reference note for the weak-angle renormalization-group bridge. It is intentionally a sketch. The full threshold derivation is reserved for a later revision. The present appendix records only the algebraic input, the standard running formulas, and the threshold logic adopted by the bundle.

D.1 Tree-level input

The exceptional-Jordan sector supplies a tree-level weak-angle ratio. Rev7 does not identify that tree-level quantity with the measured electroweak weak angle without additional running and threshold effects.

D.2 Standard running

The one-loop Standard-Model running is

$$\mu \frac{dg_i}{d\mu} = \frac{b_i}{16\pi^2} g_i^3, \qquad (b_1, b_2, b_3) = \left(\frac{41}{10}, -\frac{19}{6}, -7\right), \tag{1}$$

— the Standard-Model one-loop coefficients, imported physics (tier INPUT; Rev32.9) — or equivalently

$$\frac{d}{d\ln\mu}\,\alpha_i^{-1} = -\frac{b_i}{2\pi}. \tag{2}$$

D.3 Threshold sector

The rev7 threshold sketch introduces heavy electroweakly charged Peirce-sector modes at a scale of order

$$M_{\rm th} \sim 5 \text{ TeV}. \tag{3}$$

These modes contribute threshold shifts to the electroweak couplings of the order required to reconcile the tree-level algebraic ratio with the measured weak-angle benchmark.

D.4 Status

The threshold bridge is structural but incomplete. The sign of the correction, its multi-TeV scale, and its origin in heavy Peirce-sector doublets are part of the rev7 picture. The full explicit threshold integral is deferred to rev7.

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Appendix E — The Vacuum-Selector Theorem (Rev33.2)

Tom O'Sieg

August 2026

Abstract

This appendix discharges the one item carried by the Rev29 bundle as "theorem carried; proof to be reproduced" [A641/Q5A]: that the working vacuum $J_{\rm vac} = \mathrm{diag}(\phi, 1, \phi^{-1})$ is not chosen but *uniquely forced*. We give two independent, elementary, fully explicit proofs. Route A uses only the two structural conditions the suite already imposes — unit Jordan determinant together with the DET-7 invariant and invariance of the vacuum spectrum under Jordan inversion (the "palindromic" condition) — and selects $J_{\rm vac}$ with *no* reference to any number field. Route B re-derives the same vacuum arithmetically as the unique golden-field unit triple compatible with DET-7. Both routes remove all continuous moduli; the residual input is named explicitly in each case. The spectral corollary $\phi^2 + \phi^{-2} = 3$ is exactly the $\sqrt{3}$ that enters the θ_{12} theorem $\tan\theta_{12} = \sqrt{3}/\phi^2$.

1 Setup and hypotheses

Let $J_{\rm vac} = \mathrm{diag}(a, b, c)$ be a real diagonal element of $J_3(\mathbb{O}_s)$ with $a, b, c > 0$. Two structural conditions are carried throughout the suite and are taken as the hypotheses of the selector:

(H0) **Unit determinant:** $\mathrm{Det}(J_{\rm vac}) = abc = 1$. (The vacuum is a norm-one idempotent-frame element; equivalently the cubic Jordan norm $N(J_{\rm vac}) = 1$.)

(H1) **Inversion symmetry (palindromic spectrum):** the characteristic polynomial of $J_{\rm vac}$ is palindromic, i.e. $J_{\rm vac}$ is isospectral to its Jordan adjoint $J^{\#}_{\rm vac}$. Under (H0) the adjoint equals the inverse, $J^{\#}_{\rm vac} = \mathrm{Det}(J_{\rm vac})\, J^{-1}_{\rm vac} = J^{-1}_{\rm vac}$ [A680], so (H1) states that the vacuum spectrum is invariant under $\lambda \mapsto \lambda^{-1}$ — the Weyl/inversion symmetry of the Peirce ϕ-ladder.

(H2) **DET-7 invariant:** $\det \mathrm{Gram}(J_{\rm vac}, J^{\#}_{\rm vac}) = 7$, the $n_{23} = 7$ "seven-unification" invariant used throughout the mixing and heavy-quark sectors.

Writing $e_1 = a+b+c$, $e_2 = ab+bc+ca$, $e_3 = abc$, the characteristic polynomial is $t^3 - e_1 t^2 + e_2 t - e_3$.

2 The DET-7 invariant in elementary form

Novelty label: new specialization proved here.

Lemma 1. *For $J_{\mathrm{vac}} = \mathrm{diag}(a, b, c)$ with $abc = 1$,*

$$\det \mathrm{Gram}(J_{\mathrm{vac}}, J_{\mathrm{vac}}^{\#}) = (a^2 + b^2 + c^2)(a^{-2} + b^{-2} + c^{-2}) - 9.$$

Hence (H2) is equivalent to

$$(a^2 + b^2 + c^2)(a^{-2} + b^{-2} + c^{-2}) = 16. \tag{1}$$

Proof. With $abc = 1$ one has $J_{\mathrm{vac}}^{\#} = J_{\mathrm{vac}}^{-1} = \mathrm{diag}(a^{-1}, b^{-1}, c^{-1})$. The Gram matrix of the pair $(J_{\mathrm{vac}}, J_{\mathrm{vac}}^{\#})$ in the trace form $\langle X, Y\rangle = \mathrm{Tr}(XY)$ has diagonal blocks $\mathrm{Tr}(J_{\mathrm{vac}}^2) = a^2 + b^2 + c^2$ and $\mathrm{Tr}(J_{\mathrm{vac}}^{-2}) = a^{-2} + b^{-2} + c^{-2}$, with off-diagonal $\mathrm{Tr}(J_{\mathrm{vac}}\, J_{\mathrm{vac}}^{-1}) = \mathrm{Tr}(I) = 3$. Its determinant is $(a^2 + b^2 + c^2)(a^{-2} + b^{-2} + c^{-2}) - 9$. Setting this equal to 7 gives (1). □

3 Route A: symmetry + DET-7 (field-free)

Novelty label: new specialization proved here.

Lemma 2. *Under (H0), condition (H1) holds iff $e_1 = e_2$, and then the spectrum is $\{\rho, 1, \rho^{-1}\}$ for some $\rho > 0$; i.e. one eigenvalue equals 1 and the other two are mutually reciprocal.*

Proof. With $e_3 = 1$ the characteristic polynomial is $t^3 - e_1 t^2 + e_2 t - 1$. Reversing its coefficients gives $-(t^3 - e_2 t^2 + e_1 t - 1)$, so the polynomial is palindromic iff $e_1 = e_2$. In that case $p(t) = t^3 - e_1 t^2 + e_1 t - 1$ and $p(1) = 1 - e_1 + e_1 - 1 = 0$, so $t = 1$ is a root; say $b = 1$. Then $ac = 1$ by (H0), so $c = a^{-1} =: \rho^{-1}$ with $a = \rho$. □

Novelty label: new specialization proved here.

Theorem 1 (Vacuum-selector, symmetry form)**.** *A real positive diagonal $J_{\mathrm{vac}} \in J_3(\mathbb{O}_s)$ satisfying* (H0)–(H2) *is unique up to permutation:*

$$\boxed{J_{\mathrm{vac}} = \mathrm{diag}(\phi,\, 1,\, \phi^{-1}), \qquad \phi = \tfrac{1+\sqrt{5}}{2}.}$$

Proof. By Lemma 2 the spectrum is $\{\rho, 1, \rho^{-1}\}$. Substituting into (1),

$$(\rho^2 + 1 + \rho^{-2})(\rho^{-2} + 1 + \rho^2) = (\rho^2 + 1 + \rho^{-2})^2 = 16,$$

so $\rho^2 + 1 + \rho^{-2} = 4$ (the positive root), i.e. $\rho^2 + \rho^{-2} = 3$. Writing $u = \rho^2$ gives $u^2 - 3u + 1 = 0$, so $u = \frac{3 \pm \sqrt{5}}{2}$. The two roots are reciprocal ($u_+ u_- = 1$) and correspond to interchanging $\rho \leftrightarrow \rho^{-1}$ — a permutation of the spectrum. Taking $\rho > 1$, $\rho^2 = \frac{3+\sqrt{5}}{2} = \phi^2$, hence $\rho = \phi$. Thus the spectrum is $\{\phi, 1, \phi^{-1}\}$, unique up to permutation. [LIB2-019] □

Remark 1. Route A uses no number-theoretic input. The only structural hypotheses are (H1) inversion symmetry and (H2) DET-7; (H0) is the normalization. Both (H1) and (H2) are already imposed elsewhere in the suite — (H1) is the ladder inversion/Weyl symmetry, (H2) is the seven-unification invariant — so the theorem shows these two discrete conditions leave *zero continuous moduli.*

Remark 2 (Status of (H2) — honest label, S64)**.** DET-7 enters this appendix as a structural *postulate*, not a derived necessity; the result is therefore a *conditional* selector theorem. [LIB2-018, LIB2-336, LIB2-337, P-013] Its motivation is threefold. (i) $\det \mathrm{Gram}$ is discrete — there is no continuous dial to fit — once the spectrum is restricted to the integral golden-unit lattice of Route B; on an unrestricted spectrum it varies continuously ($\det \mathrm{Gram}(\mathrm{diag}(t, 1, t^{-1})) = (t^2 + 1 + t^{-2})^2 - 9$; Rev32.9, A1632). (ii) The single integer $n_{23} = 7$ simultaneously fixes three independent observables — $\sin^2 \theta_{23} = 7/16$, $|V_{cb}| = 1/(9\sqrt{7})$, and the 7/3 lepton–quark bridge — plus the mismatch normalization: one discrete choice, multiple independent payoffs, which is compression rather than curve-fitting. (iii) Route B selects the same vacuum by golden-field unit arithmetic in which 7 appears as the *output* of the DET-7 evaluation on the selected triple, not as an input to the field choice. A first-principles derivation of DET-7 itself remains open and is carried as such.

Remark 3 (Origin of ϕ; S66)**.** The golden ratio enters solely through the selector theorem above (unit determinant + inversion/palindromic symmetry + DET-7), which is field-free and therefore *signature-independent*. Two richer candidate origins were examined and *excluded* (D732, three-way). (i) A golden-quasicrystal $\mathbb{Z}[\phi]$ Fourier module via the $\mathbb{O} \to E_8 \to$ icosian cut-and-project chain is *obstructed by the split signature*: it requires a positive-definite metric, whereas the split forms give an indefinite lattice with null cones and no quasicrystal. (ii) An icosahedral (A_5) Galois origin from the resolvent quintics fails because the within-sector quintics are binomial ($\mu^5 - R_d$, dihedral D_5), not icosahedral. Thus ϕ is principled at the elementary selector level; the grander constructions are not its source.

4 Route B: golden-field units + DET-7 (arithmetic)

Novelty label: new specialization proved here.

Theorem 2 (Vacuum-selector, arithmetic form)**.** *Let $J_{\mathrm{vac}} = \mathrm{diag}(a, b, c)$ with $a, b, c > 0$ algebraic integers in $\mathbb{Q}(\sqrt{5})$ and $abc = 1$. [P-024] Then* (H2) *forces $J_{\mathrm{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1})$ up to permutation. [LIB2-020]*

Proof. The ring of integers of $\mathbb{Q}(\sqrt{5})$ is $\mathbb{Z}[\phi]$, whose unit group is $\{\pm\phi^n : n \in \mathbb{Z}\}$ with ϕ the fundamental unit. From $abc = 1$ each of a, b, c is invertible in $\mathbb{Z}[\phi]$, hence a unit; positivity removes the sign, so $a = \phi^{n_1}$, $b = \phi^{n_2}$, $c = \phi^{n_3}$. Then $abc = 1$ gives $n_1 + n_2 + n_3 = 0$. By Lemma 1, (H2) reads

$$\left(\textstyle\sum_i \phi^{2n_i}\right)\left(\textstyle\sum_i \phi^{-2n_i}\right) = 16.$$

Order $n_1 \geq n_2 \geq n_3$ and write the gaps $p = n_1 - n_2 \geq 0$, $q = n_2 - n_3 \geq 0$. The left side depends only on the gaps:

$$F(p, q) \;=\; 3 + A_p + A_q + A_{p+q}, \qquad A_k := \phi^{2k} + \phi^{-2k} = L_{2k} \in \{2, 3, 7, 18, 47, \dots\},$$

with A_k the (strictly increasing, even-index) Lucas numbers, and the zero-sum condition admits a gap pair iff $p + 2q \equiv 0 \pmod 3$. *(An earlier version of this proof asserted strict monotonicity of F in $\max_i |n_i|$; that is false — $F(5, 0, -5) = 15376 > 11561 = F(6, -3, -3)$ — and the step is replaced, Rev32, by the exact case count below; the conclusion is unchanged [A1519 F1; exhaustive $|n| \leq 9$ scan house-run].)* The admissible cases are: $(p, q) = (0, 0)$ gives $F = 3 + 2 + 2 + 2 = 9$; $(p, q) = (1, 1)$ — the triple $\{1, 0, -1\}$ — gives $F = 3 + 3 + 3 + 7 = 16$; no other pair with $p + q \leq 2$ is admissible ($p + 2q \equiv 0 \pmod 3$ excludes $(1, 0), (0, 1), (2, 0), (0, 2)$); for $p + q = 3$ the admissible pairs are $(3, 0)$

and $(0,3)$, each giving $F = 3+18+2+18 = 41$ (attained at $\{2,-1,-1\}$); and for $p+q \geq 4$, $A_{p+q} \geq 47$ gives $F \geq 3+47+2+2 = 54$ (the next value is 64 at $\{2,0,-2\}$). (Rev32.9, A1632: the termwise bound printed through Rev32.8 used $A_p + A_q \geq 20$ for every $p+q \geq 3$, which does not hold — the admissible $(2,2)$ has $A_p + A_q = 14$.) Hence $F = 16$ occurs *only* at $\{n_1, n_2, n_3\} = \{1, 0, -1\}$, i.e. $J_{\text{vac}} = \text{diag}(\phi, 1, \phi^{-1})$. □

Remark 4. Routes A and B are independent and corroborating. Route A's residual input is the inversion symmetry (H1); Route B's residual input is the golden-field hypothesis $a, b, c \in \mathbb{Z}[\phi]$. Each, together with (H0) and DET-7, removes all continuous freedom. [LIB2-020] That ϕ is precisely the fundamental unit of the golden field is the arithmetic reason the same vacuum appears from both directions.

5 Spectral corollary and the solar-angle theorem

Novelty label: physical interpretation not established here.

Corollary 1. *The selected vacuum satisfies the exact spectral identities*

$$\phi^2 + \phi^{-2} = 3, \qquad e_1 = e_2 = \phi + 1 + \phi^{-1} = 1 + \sqrt{5}.$$

Consequently the $(2,3)$ *Peirce spectral ratio yields the parameter-free solar angle*

$$\tan\theta_{12} = \frac{\sqrt{3}}{\phi^2} = 0.66158, \qquad \theta_{12} = 33.49^\circ,$$

compared with NuFIT 6.1 (NH) $33.76^{+0.42\circ}_{-0.41}$ *at* 0.66σ *(Rev32.1 refresh: the* $33.44^\circ/0.06\sigma$ *previously printed here used a pre-6.0-era value — Paper 5 correction) [A537].*

Proof. From $\rho^2 + \rho^{-2} = 3$ at $\rho = \phi$ (Theorem 1). The $\sqrt{3}$ in the solar ratio is exactly $\sqrt{\phi^2 + \phi^{-2}}$; the denominator ϕ^2 is the squared dominant eigenvalue of J_{vac}. The numerical value follows. □

6 Seven declared-class negatives on selector laws (Rev30–Rev31)

The freeze-first arc spent several rounds asking what a selection law *cannot* be. Six results survive as durable negatives (A1483, A1488, A1492, A1493, A1495); a seventh, proved post-closure, is item (vii) below (A1512). Each is scoped to a class that was declared *before* testing, and in each case the scope is the result: stated without its class, every one of them becomes a false universal claim.

(i) The Gram-dark fibre is exactly two-dimensional (`s993`–`s994`). House-verified: $\dim \mathfrak{p} = 70$ and $\text{rank}\,(X \mapsto X\, i_B) = 68$, so the fibre invisible to $G_B(M) = i_B^{\mathsf{T}} M i_B$ has dimension exactly 2. Consequently **no action factoring only through G_B can uniquely select a coset point** — not obstructed at some order, but dead at every order. [LIB2-036] The qualifier "factoring only through G_B" is load-bearing: the all-order no-go is proved for the G_B-factoring subclass, and the wider class of parent actions and boundary laws is explicitly left open. [LIB2-036] Two associated facts are weaker than they look and are recorded as such: the recovery of all 16 horizontal coordinates at second order is a *rank* statement, not a construction, and the independent quadratic-trace no-go offered alongside was not verified in house.

(ii) The frame-free diagonal-invariant class contains no law with a nonzero dark response (`s1002`–`s1003`; Rev32.9: constants belong to the class, so it is not empty). For the class $\mathcal{C}_{\mathrm{ff}}$ of C^2 scalars on $(E_{7(7)}/SU(8)) \times \mathcal{O}_B$ that are invariant under the *diagonal* $E_{7(7)}$ action, with no spurion and a point source space — declared before testing — there is no law with a nonzero dark response: for Z in the dark fibre, e^{tZ} fixes the loaded plane exactly while moving the coset point, so every diagonal invariant is constant along the dark orbit. [LIB2-037] A basepointed full-carrier law does exist and does fire on that fibre, with response exactly t^2, and the rank-two dark projector is minimal; but its basepoint is a convention rather than a consequence, and it carries no Hamiltonian, no action and no observable. [LIB2-037] It is certified mathematics occupying an uninhabited physical slot.

(iii) The transported class consumes a source but cannot select one (`s1011`–`s1012`). The transported functional co-moves the loaded plane to zero residual — the co-moving residual is below 10^{-13} while the fixed-frame shear remains exactly $\frac{3}{4}|j|$ as the tangent displacement of a co-moving plane. But the action is exactly isotropic on sources: equal-norm sources share the minimum value, the transport price and the local spectrum. **The transported quadratic/current class consumes j and cannot select its ray.** [LIB2-038] Two cautions travel with this. Its constants enter under a declared conditioning–minimax principle — a named mathematical principle, not a measured number — and must never be read as fitted parameters; and the object is a transported *selector* functional, not a parent action and not a Lagrangian, a distinction the originating round itself was careful to preserve. [LIB2-038] *(That distinction was later discharged rather than dissolved: the transported selector was extended to a full conditional parent action and closed at the selector tier in S274–S275 — §7.)* [LIB2-038]

(iv) The fixed-grade cubic/current class is closed negative for rank-two detection (`s1005`–`s1006`). [LIB2-039] Restricting the dark fibre to the fixed $+2$ grade, $\operatorname{rank}(\mathfrak{z}_2 \ni Z \mapsto P_{27} Z P_{27}) = 1$: one of the two dark directions moves the carrier *out* of the fixed grade entirely, with $P_{27} Z P_{27} = 0$ while $[D_{56}, Z] \neq 0$, and the witness is exhibited explicitly. [LIB2-039] A class needing rank two therefore cannot be detected inside a fixed grade, and the fixed-grade cubic/current route closes negative. The scope is exactly that: rank-two detection inside a fixed grade. The embedding-tensor class is *scoped out* rather than decided, because no convention-certified branching artifact for it was supplied. [LIB2-039]

(v) An even action cannot resolve the sign (`s1005`–`s1006`). For X in the horizontal 16-dimensional sector, $R_B e^{2tX} R_B = e^{-2tX}$ (house-verified numerically), so every R_B-conjugation-invariant scalar satisfies $\mathcal{S}(e^{2tX}) = \mathcal{S}(e^{-2tX})$. [LIB2-040] **An R_B-even action cannot resolve the sign** — the parity of the law, not its strength, is what fails. [LIB2-040] Any candidate must therefore be odd under R_B somewhere, which is a usable design constraint rather than a mere obstruction.

The representation supplies tensors; it does not select one. The branching $\mathbf{27}_{+2} \downarrow \mathrm{Spin}(5,5) \times SO(1,1)_Y = \mathbf{16}_{+1} \oplus \mathbf{10}_{-2} \oplus \mathbf{1}_{+4}$ is computed, and it delivers the carriers the construction needs. But an *inequivalent* irreducible branch passes the same tests: **uniqueness fails**, and one discrete choice is priced. The name "Higgs" attached to the $\mathbf{10}_{-2}$ carrier is an *imported label*, not a derived object — no electroweak doublet is derived anywhere in this construction, and the label must not be read as if one were. The honest one-line summary of this whole family of results is that *the tensor exists and the action does not.*

(vi) A frame-free F_4-invariant cannot reach the ordered frame (`s978/s981`). The stabilizer of the ordered triple inside the automorphism group leaves a 24-dimensional orbit, hence, at a stationary point on that orbit, at least 24 Hessian zero modes (flat directions) for *any* frame-free invariant (Rev32.9: a statement about stationary orbits). [LIB2-041] The consequence is sharper than the

obstruction it replaces: a frame-free F_4-invariant can at best select the *spectral orbit* $[J_{\mathrm{vac}}]_{F_4}$; **it can never isolate the ordered frame** $\mathrm{diag}(\varphi, 1, \varphi^{-1})$. [LIB2-041] Selecting the ordered frame requires a genuine loading or symmetry-breaking tensor — which is to say the ordering is an input to this class, not an output of it. A companion norm-only statement runs the same way: for any C^2 action $\mathcal{S} = F(\alpha, N(X))$, *ambient* stationarity in X at a regular norm-one point forces $F_N = 0$ and hence $\mathrm{Hess}\,\mathcal{S} \equiv 0$ on $\ker \mathrm{d}N$, giving at least 26 flats structurally. [LIB2-042] (Rev32.9, A1637: under stationarity *constrained* to $N = 1$, F_N need not vanish; the same count then holds for the constrained Hessian $\mathrm{Hess}\,\mathcal{S} - F_N\,\mathrm{Hess}\,N = F_{NN}\,\mathrm{d}N \otimes \mathrm{d}N$, which vanishes on $\ker \mathrm{d}N$.) *Attribution and scope:* this is the lane's theorem, not the house's. The house verified it at the level of the dimension counts; the claim that a frame-free invariant is constant on the orbit is accepted as standard rather than proved here, and the norm-only computation was checked on a generic symmetric cubic in $\mathbb{R}^{27}$ rather than on $J_3(\mathbb{O}_s)$ itself. [LIB2-042]

(vii) The unshifted zero-level quadratic-carrier Kempf–Ness class cannot carry the selector (post-closure; Rev31, S276; A1512 Ask-2). For the class of Kempf–Ness potentials built on an unshifted zero-level quadratic carrier — declared *before* the run (class sha `6ece8dc9...`) — two independent no-gos close it: $\mathrm{rank}\,\mathrm{Hess}_0\|\mu_K\|^2 \le \dim K \le 63 < 69 = \mathrm{rank}\,Q_T$ (the rank no-go), and $Q_T = \sin^2$ along the beat orbit while no beat-invariant KN potential isolates $[W_0]$ (the beat-transitivity no-go). [LIB2-043, LIB2-044] The escapes are named and priced rather than left vague: a central shift is available on proper subgroups only; a beat-breaking spurion, dropping the clock circle, or leaving the KN family altogether each exits the declared class. Scope is exactly the declared class; this negative postdates the closure of §7 and prunes its surroundings, not its content. [LIB2-044]

> **Type signature of anything that could still work.** What the six negatives leave is a shape rather than an object: a law on $(E_{7(7)}/SU(8)) \times \mathrm{Gr}_{16}(V_{56}) \times \mathcal{D}_{\mathrm{phys}}$ possessing a plane-preserving stationary point, a positive horizontal Hessian, a detector for the exact two-dimensional dark fibre, and a full-carrier or downstream tensor that does *not* factor through $i_B^{\mathsf{T}} M i_B$. Two candidate tensors would satisfy the type. They have not been supplied. Nothing in this section promotes an observable, moves a tier, or touches the engine.

(Supersession note (Rev31; S274–S275). The type signature above has since been inhabited, adjudicated under precommitted seals, and closed at the selector tier — §7. The six negatives stand unchanged as the walls the closure was built against; their scopes are unaffected.) [LIB2-036]

7 The parent-action closure: complete-conditional at the selector tier (Rev31; S274–S278, A1506–A1515)

The string that ends here ran four adversarial rounds across three sealed dispatches (D1196–D1198, plus a cold two-referee confirmation D1199), was adjudicated against precommitted house tables (seals `s1043`, `s1045` spent; `s1041` retired scope-limited), and was verified two-host with byte-identical reruns at every step. Its outcome is stated exactly, because its exactness is the result.

What was assembled (A1506, conditional tier)

The first full conditional parent action in the programme's history: a minimal-parent type theorem (any homogeneous $SU(8)/H$ mapping equivariantly to both targets forces $H \subseteq K_{\mathrm{dyn}}$, hence $\dim \ge 53$

with equality iff $H = K_{\rm dyn}$); the forced $+\frac{3}{4}$ source-backreaction coefficient; an all-parameter Hessian positivity theorem; the source-ray selection mechanism — $\mathrm{spec}(Q_L) = \{0^1, 2^{10}, 4^5\}$ computed from the banked artifact with kernel dimension exactly 1, the kernel ray equal to the certified source unit at overlap 1.0, so stationarity selects *one* projective source ray at the witness — and a local Hamiltonian/Legendre attachment, all sector-assembled into $S_{\rm parent}$ with an itemized open list and nothing promoted. The Hamiltonian/Legendre material enters here, and only here: it is a *local attachment* at the conditional tier, not a continuum Hamiltonian and not an LSZ-grade dynamics. *Corrected typing (Rev32; the D1212/A1525 catch, kernel `s1072`):* the Hessian that enters the Legendre step is the *velocity* Hessian I_{32}/U — a different object from the A1506 *configuration* Hessian, which is a stability record only; conflating the two mis-types the attachment. [LIB2-021] The full typing ledger is `Trackers/PARENT_ACTION_HAMILTONIAN_LEDGER_S284.md`.

The boundary datum: one ten-dimensional object, three exact faces (A1508)

The datum the action is conditional on is single: $[j] \iff L_\partial \iff E^* = \mathrm{span}\{i_B j,\ \Omega i_B j\}$ — three faces of *one* ten-continuous boundary object, priced once. [LIB2-022, P-010] E^* is the unique $(53, 10, 0; K_{\rm dyn})$ pair, certified NON-BLIND after a post-target discovery (the 16-member Ω-pair family) forced a scope hold and a PI-ruled non-blind rerun; the certification is consistent-and-redundant with $[j] \iff L_\partial$. The physical selection of this datum remains OPEN/LOADED (register clause: Appendix X, §"The Rev31 register clause"). [LIB2-022]

The obstruction theorem (A1507, A1509 Thm. A, sharpened A1510)

At theorem strength, $(F_B^{16})^{K_{\rm sel}} = 0$: the priced ten-continuous datum has no invariant inhabitant, and the pure-spinor point is exactly that datum (A1507's honest loss, with the owed Fierz/moment-map gates exact: $m_A(j) = 0$, $x^{\#} = 0$, $N_{27} = 0$, $\mathrm{Ann}(j) = L_\partial$). The grammar acts on the 32-dimensional braid-fixed carrier $\mathrm{im}(i_B) \oplus \Omega \cdot \mathrm{im}(i_B)$ as scalars/zero; $U_g = P_{\rm act} + i_B\, g\, i_B^{\mathsf{T}} + (\Omega i_B)\, g\, (\Omega i_B)^{\mathsf{T}}$ is symplectic and commutes with every grammar generator (house-checked, commutators $\sim 10^{-13}$, W_0 included). $O(16)$ acts transitively on source rays — and $SO(16)$ alone is already transitive on S^{15} (the A1510 sharpening, closing the connectedness loophole) — so **no construction internal to the braid/trace grammar can select** $[j]$, L_∂, or E^*. [LIB2-045] The boundary price is irreducible within the grammar. Scope: genuinely external categories are not excluded. [LIB2-045]

The dark attachment (A1509 Thm. B): the grammar owns the ray it was never given

The S_1 fibre law is exact in closed form,

$$S_1(e^Y) \;=\; 54\cos^2\theta\left[\cosh(r/\sqrt{6}) - 1\right],$$

with W_1 zero *at every order* (the S274 residual -3.6×10^{-4} fully demystified). *Provenance upgrade (Rev31; S276, A1512).* Every constant of this law is now *proved-with-forced-constants* rather than house-numeric: the active carrier is a $\mathrm{Cl}(2,1)$ module in exactly 12 copies; $W_h = Q + \frac{3}{2}P + \frac{3}{2}\gamma_1 - \gamma_2$; the constants 54, $\sqrt{6}$, 24, 9/2 are each forced by the module structure; and $\kappa_{\rm law} = \kappa_{\rm clock}$ is a *theorem* — one structure constant appearing simultaneously as an eigenvalue difference and as the commutator coefficient in $[K_0, W_0] = \frac{1}{\sqrt{6}} W_1$, exact at the full 56×56 level (house 8×10^{-16}; kernels

`s1047`–`s1048`, lane proof A1512, house-verified). [LIB2-014, LIB2-024] *Corroboration link (Rev32):* this golden bridge independently re-derives a face of the S205 dynamics seed's golden Cartan flow (L-DynamicsSeed, A1268–A1270, MOTIVATED tier; the $\exp(2\ln\varphi\, H)$ ladder) — printed as corroboration in both directions, neither result derives the other (Appendix X, clause-27 reconciliation). The naive trap — the ground valley of $+\varepsilon S_1$ is the W_1 ray — computed and fenced. The crown is prefrozen: the grammar's own detector quadratic form on the 70-dimensional $\mathfrak{p}$-sector has spectrum $0^1 \oplus (5/3)^{10} \oplus 2^{26} \oplus (5/2)^{32} \oplus (9/2)^1$ (house-recomputed exact) with a *unique* top eigenspace, frozen pre-target and shown post-freeze to be exactly $\mathbb{R}W_0$ (overlap 1.000000000000). [LIB2-023] The dark axis is therefore **grammar-derived**, not dark-axis-labelled, and the selector

$$T \;=\; \|\log M\|^2 - \tfrac{2}{9}\, Q_{S_1}$$

has *forced* coefficients — A1511 proved they follow from the top eigenvalue alone, leaving a parameter-free selector. On the fibre $T = r^2 \sin^2\theta$; the horizontal Hessian is $\frac{8}{9}I$; the total is $(\frac{2}{3} + \frac{8}{9}\kappa_d)I > 0$; K_{dyn} preserves the dark axes, so no new stabilizer datum is owed. The two faces of the selection wall are now *asymmetric, and priced as such*: the grammar cannot see the source ray (obstruction above) but intrinsically owns the dark ray. [LIB2-023]

The dark half twice over, and the forced interior split (Rev31; S276/S278, A1512–A1515)

Two post-closure rounds sharpened the dark attachment without moving its tier. *First (A1513):* the golden Lorentzian form $\eta = \Omega G$ — grammar-derived, zero new data — *independently* produces the selector kernel ray $[W_0]$ **and** the exact dark-fibre law $r^2 \sin^2\theta$: the detector quadratic form of η has spectrum $0^1 \oplus (13/36)^{32} \oplus (13/27)^{36} \oplus 1^1$ with kernel overlap 1.0 against $[W_0]$. The full spectrum is closed *negative* in the declared two-parameter class $\Sigma = a\eta + bP_B$: the interior 36-block resists with a gap of exactly 2/27. The honest premise for that gap (corrected in-round, A1514: the house's "scalar-blind on the entire 36-block" was too strong): the banked Q_{P_B} record $0^{28} \oplus (1/2)^{32} \oplus (2/3)^{10}$ *already* separated the 10-block; what no construction had produced was the exact 15 : 17 interior ratio. So the dark half of the selector is grammar-derived *twice over* — spectrum (A1509) and potential (A1513) — while the interior split carried the remaining loaded information. *Then (A1514, Tier-B unconditional, zero price):* that last loaded split is forced too. P_B forces the exact $26 \oplus 10$ eigenspace split ($Q_P|_{V_{36}} = 0^{26} \oplus (2/3)^{10}$), its symplectic transport defect $\Delta_T = [T_\phi, P_B] = [J_0, P_B]$ forces the complement, and the commuting joint pair (Q_P, Q_Δ) with joint spectrum $(0,0)^2\,(\frac{1}{2},1)^{32}\,(0,\frac{8}{3})^{26}\,(\frac{2}{3},0)^{10}$ — four joint eigenvalues, and polynomials in the pair are scalar on the two-dimensional common kernel — produces **all five** Q_T eigenspaces as subspaces once that kernel is split by the top projector of the Hessian of the banked golden form η, which `s1051` adds at gate G8 ($H_{70} = P_\eta^{\mathrm{top}} + \frac{24}{54}P_{32} + H_{36}$; projector distances $\le 5.1 \times 10^{-14}$; "word degree ≤ 2" counts words in Q_P and Q_Δ; no new datum beyond the banked η — Rev32.9, A1632). [LIB2-025] The eigenvalue *weights* on those subspaces are the subject of Appendix S's production arc (A1515–A1516), where they are derived in-class and their action shape is priced. [LIB2-025]

The closure declaration (A1509 Thm. C; referees A1510/A1511)

> **Complete-conditional, selector tier.** Within the frozen split-real 56-frame and the declared finite/internal loaded-profile class: the selector parent action V_{total} has its global minimum at $(j^*,\, q^* = \frac{3}{2}j^*,\, M^* = I)$ with combined Hessian positive for all $\rho, u_0, \kappa_d > 0$ on the directions transverse to $[W_0]$, the exact projective soft ray at zero inserted dark amplitude, along which it vanishes (scope stated Rev32.9, A1632). [LIB2-021] The incremental price is $10 + 1$ **continuous** — the boundary object (three faces, priced once) and one positive dark coupling κ_d — with **zero discrete data consumed** (Z_2 lifts left explicitly as degeneracies) and no stabilizer owed ($T \geq 0$ globally). [LIB2-022] No further selector datum is hidden. Two cold referees audited the closure volumes independently (A1510 Gemini 6/6; A1511 Grok 6/6) with *zero* disagreements. **Fences:** this closes the finite/internal loaded-profile *selector* only — continuum, pole, and LSZ remain a separately typed frontier; ρ, κ_d, U enter by positivity only, no numeric values claimed; nothing here promotes an observable, moves a tier, or touches the engine.

[LIB2-021]

Typing (Rev33.2). The soft ray $[W_0]$ lies entirely in the span of the twelve noncompact gauge generators of the registered dyonic $SO(6,2)$ gauging (projection 1.0000; A1755 house checks): the grammar-derived soft ray is a would-be Goldstone — a gauge — direction, with stabiliser in the maximal compact, and the adopted selector T is not gauge-invariant (it moves by 0.118–0.605 under the other gauge boosts). The soft ray is a gauge direction; the selector is frame-anchored. The closure statement above is unchanged as a statement about the frozen frame.

8 Rev32 addendum: rulings, prices, and the selection frontier (S284–S288)

The S284 rulings, printed beside the closure (S284.1)

Three PI rulings attach to the closure section above. *(i)* The boundary-condition-family question is ruled: **CLASS-ρ is primary**; CLASS-W applies under $C_4 = 0$; CLASS-H is deferred. [LIB2-323] *(ii)* The `s1073` amendment A1–A3 is adopted (verifier of record and battery filed, byte-certified). *(iii)* The H2 minimal-class rank is **predeclared: rank 2**. The H2 leg's standing price is stated exactly: **six continuous C_0 coefficients plus one-map-for-all-η naturality** — and target-triviality of the action clause is *forced*, not assumed, by the D1214 forced-triviality theorem (house-verified to the digit, A1527). [LIB2-030, LIB2-031]

Coefficient prices have an adjudicated precedent class (S285.1)

The clause-27 price above is not the first priced cross-layer coefficient in this suite: the **AX-MG coefficient-one clause** was proved a genuine independent axiom by exact countermodel (A1435, Appendix S) and stands as the adjudicated model for how such a price is typed. The C_0 six-coefficient price is the same wall type, now at the same evidential grade (below).

The C_0 independence theorem (S285.3; D1217→A1530)

At the frozen H2 tier the six-coefficient freedom is **countermodel-grade**: two *disjoint* symbolic rank-2 candidates pass all 13 gates and 30 controls identically, and no banked selection polynomial separates them (lane ×2 and house ×2 byte-identical; independent house verifier `s1077`, 14/14, ×2). [LIB2-032] This supersedes the earlier census-negative wording *in grade only* — the scope clause is mandatory: a higher-tier S_{joint}, should one be banked, may still derive the coefficients. [LIB2-032]

First finite OS subgate pass, at its tier (S284.3)

The continuum frontier moved by one typed step: the **first finite Gaussian OS subgate pass** and a toy resolvent, both two-host byte-certified (A1529), are recorded at *conditional/finite* tier with their full non-claim fences: no continuum limit, no reflection-positive continuum measure, no pole, no LSZ; the 17-stage continuum staging map (`Trackers/CONTINUUM_STAGING_MAP_S284.md`) locates the step. [LIB2-165, LIB2-166, LIB2-300]

The selection frontier: F-SEL(c) closes negative, scoped (S287.9–.11; D1220→A1533)

The Rev31 arc left one unexplored selection direction: could a *refined* condition family derive, rather than adopt, the four Φ_{diag} relations $\{v = 0,\ w = 0,\ u = t,\ 27z = t\}$? The Rev32 answer is a scoped negative, in three parts.

(a) The house's universal cone claim is withdrawn. An earlier banked argument held that no positivity- or stability-type refinement could answer the question because inequality conditions cannot reduce dimension. [LIB2-254] That claim is **refuted** ($x \geq 0$ with $-x \geq 0$ has feasible set $\{0\}$: degenerate inequality systems can encode equalities). The replacement statement is scoped and stronger where it applies: the presently banked *non-degenerate* PSD/stability conditions $u \geq 0$, $\frac{4t}{9} + v \geq 0$, $\frac{16t}{27} + w - z \geq 0$, $\frac{16t}{27} + w + z \geq 0$, $t_2 \geq 0$ have non-empty interior — $(t, t_2, u, v, w, z) = (1, 1, 1, 1, 1, 0)$ satisfies all five strictly — so the feasible set is 6-dimensional before the scale quotient and 5-dimensional after it, and cannot select a ray. [LIB2-035] Degenerate systems can lower dimension in principle, so any proposed selector must derive its everywhere-active equality locus independently and clear the circularity bar. [LIB2-254] F_{char} is interior only among the four minimal-word inequalities: in the five-inequality cone it lies on the face $t_2 = 0$ (slacks $1, \frac{4}{9}, \frac{5}{9}, \frac{17}{27}, 0$), which Appendix S requires for Φ_{diag}. That face contains the target without selecting it; any other forced degeneracy face would *exclude* it (Rev32.9, A1637). [LIB2-035]

(b) Casimir-naturality is tested and refuting. The lead candidate — $A = f(C_{K_{\text{dyn}}})$, coefficients a function of the K_{dyn} Casimir — clears the circularity bar (it is stated entirely from banked K_{dyn} data) and *fails on the merits*: the Casimir spectrum on the coefficient carrier $\mathfrak{p}_{70}$ is $0^{(6)} \oplus (1/6)^{(20)} \oplus (1/4)^{(30)} \oplus (5/12)^{(14)}$, with P_{10} pure $(1/6)^{(10)}$ and P_{26} *not Casimir-isotypic at all* ($0^{(2)} \oplus (1/4)^{(10)} \oplus (5/12)^{(14)}$); equal Casimirs recur across blocks, so $A = f(C)$ forces $u = v = w = z = 0$ (literal carrier; cut $5 \to 1$) or $u = v = w,\ z = 0$ (support-69 reading, an extra priced premise; cut $5 \to 2$). [LIB2-034] Φ_{diag} needs $27z = t \neq 0$: **Casimir-naturality excludes Φ_{diag}; it is a refuter, not a selector** (independently re-derived house-side in sympy with a falsifier control — make P_{26} isotypic and z survives — kernel `s1083`, 11/11). [LIB2-034] Price to cite it: $A = f(C)$ is *not* forced by ordinary equivariance (the symmetric commutant has dimension 38; functions of the Casimir form a 4-dimensional centre) — it is a further multiplicity-blind naturality principle. [LIB2-034]

(c) The closure, scoped. Across the filed condition inventory — categorical/invariance conditions, word minimality (a separate, already-adopted class axis), ordinary equivariance (cuts nothing), Casimir centrality (excludes), strict positivity/stability (full-dimensional), degenerate faces (other than $t_2 = 0$, which contains the target without selecting it, exclude it), equal-weight and condition-number minimisation (select a different ray), Ξ-blindness (excludes), trace normalisation (scale only), exact Q_T matching (circular, two-dimensional fibre) — **no banked refined condition family selects Φ_{diag} non-circularly. F-SEL(c) closes negative at the current banked class and condition inventory.** [LIB2-028, LIB2-033] Φ_{diag} remains an *adopted structural section* at the printed price: word minimality as a domain/class clause plus the four coefficient relations; without the adoption the enlarged class carries five projective continuous moduli. **Claim boundary, binding:** this is *not* a universal impossibility theorem and must never be printed as one; the reopen condition is a genuinely new, independently derived invariant or condition family entering the banked corpus. [LIB2-033] The open-selection set accordingly drops to G_7 **motivation and SEL-27**, and SEL-27 is a theorem awaiting a PI ruling rather than a research gap — the live selection frontier is one item. [LIB2-324]

9 Status

Theorems 1 and 2 discharge the carried item [A641/Q5A]: the working vacuum *spectrum* $\{\phi, 1, \phi^{-1}\}$ is forced up to permutation, conditional on the named hypotheses (unit norm; inversion symmetry, or the golden-field restriction; DET-7), with the residual input named in each route. [LIB2-018, LIB2-046] The *ordered* physical Peirce frame remains loaded: a frame-free F_4-invariant selects only the spectral orbit (item (vi), `s978`/`s981`), so ordering requires a loading or symmetry-breaking tensor. [LIB2-001, LIB2-019, LIB2-020] The two structural conditions DET-7 and (H1) are not free parameters but invariants used elsewhere in the suite, so the selector relocates no hidden freedom into the vacuum. What remains genuinely external is upstream of this appendix: the choice of the cubic Jordan algebra $J_3(\mathbb{O}_s)$ and its trace form, within which DET-7 and the inversion symmetry are then natural.

This appendix is self-contained and elementary; both routes are verifiable by direct computation (the admissible gap-pair case split and Lucas lower bound in Theorem 2 — Rev32.1: the earlier "spread monotonicity" phrasing here cited the step retracted inside that theorem (A1538 F6); kernel `s1085` verifies the replacement and proves the stronger variational form — and the quadratic $u^2 - 3u + 1 = 0$ in Theorem 1). [LIB2-001]

A third, independent corroboration — *variational* rather than algebraic — is recorded as a speculative probe in Appendix I: a grading-coupled cone potential whose unique positive-definite minimum at unit coupling is J_{vac}, with ϕ emerging from $c^2 - c - 1 = 0$. It shares DET-7 / unit-coupling as structural inputs and does *not* upgrade the tier of this selector; it is reported only because it makes the golden vacuum *overdetermined* (three independent routes pin the same point), which is the substance of the answer to the "numerology" charge. *(Tracker-tier note (Rev25 fold; `s654`+, S221): in the dynamics-lane machinery the house Cartan satisfies $u \cdot u = 24\,\delta$ with Fano-line support, and the CDIZ locus $x_i = 1$ is* forced *rather than chosen there — a fourth, independent appearance of the same vacuum point, recorded as support only (the dynamics tier itself remains held; Appendix Q posture).)*

Leibniz Quantum Beats Newton

Appendix F — The Gauge Boundary: Obstruction, Uniqueness of Completion, and Dynamical Shadow Selection (Rev33.2)

Tom O'Sieg

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Abstract

This appendix settles the status of the gauge sector with three results, obtained in Sessions 74–80 on a certified $E_{7(7)}$ **56/133** embedding-tensor substrate. (i) *The two-theorem gauge boundary*: the *branching wall* (no gauging of maximal N=8, D=4 supergravity, in any symplectic frame, has a gauge algebra containing a compactly embedded $\mathfrak{su}(3) \oplus \mathfrak{su}(2)$ — a fortiori none realizes $\mathfrak{g}_{\rm SM}$) and the *charge wall* (no gauging's algebra contains a *unitary-class* $\mathfrak{su}(3)$, the matter-colour embedding $\mathbf{8}{\downarrow}\mathfrak{su}(3) = \mathbf{3} \oplus 5 \cdot \mathbf{1}$; only the vector and adjoint $\mathfrak{su}(3)$ classes are ever gauging-realizable). The right algebra can appear, but provably never in the right representation. Gauge-group *emergence as a gauging* is therefore closed negative, by theorem rather than by failure-to-find. (ii) *Uniqueness of completion*: among compact reductive low-energy completions that gauge the full derived colour + weak action on exactly the sixteen derived Weyl fermions with the derived hypercharges and 10_H-only scalar closure, $G_{\rm SM}$ is unique up to $U(1)_{B-L}$ (whose extra Cartan is computed unique, $= 3(B{-}L)$) and up to global structure $\Gamma \in \{1, \mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_6\}$. (iii) *Dynamical shadow selection*: on the scalar potential of the gauged theory — built here at the published normalization, calibration constant exactly 1, and certified against the published $SO(8)$ and $SO^*(8)$ spectra — vacuum stability of the missing-16 scalar sector, combined with the shadow-hosting condition, selects $SO(6,2)$ uniquely within the scanned electric family; its dyonic completion is a published Minkowski vacuum with residual $SO(6) \times SO(2) \supset SU(3) \times U(1) \times U(1)$. The net position is stated without inflation: the gauge group remains an INPUT at the group level, now bounded on both sides — it could not have arisen as a gauging (obstruction), and — within the enumerated compact-reductive completion class and the fixed $16 + 10_H$ package — it could not have been anything else (uniqueness up to $U(1)_{B-L}$ and global form) — while the maximal derivable remnant, the shadow $\mathfrak{su}(3) \oplus \mathfrak{u}(1) \oplus \mathfrak{u}(1)$, is selected dynamically rather than by hand. "Dynamically" is relative to the four-dimensional gauged theory: the constraints and the stationarity of the selected point make it a consistent vacuum, not a microscopic origin of the gauging, and no such origin is claimed (Remark 5.2, fence (i)).

Notation (σ). In this appendix σ denotes the $\mathbf{SO}^*(8)$ **time-mirror** $\sigma = \Omega$, under which $\sigma D_{KK} \sigma^{-1} = -D_{KK}$; the terms *$\sigma$-fixed* and *$\sigma$-closure* below refer to it. It is *not* Appendix I's Jordan frame involution $\sigma_{\rm frame}$ ($\rho \mapsto 1/\rho$), *not* Appendix H's throat involution σ_C, and *not* the split-octonion conjugation $\sigma_{\rm oct}$ ($e_7 \mapsto -e_7$) of the A689 dictionary, which does not appear in the typeset suite at all.

1 Scope and provenance

All computations referenced here are reproducible from the project's `e7_substrate` module (Sessions S71–S77): the certified ambient $E_{7(7)}$ substrate (V1–V6: 133 generators on the **56**, signature +7, symplectic, quartic-invariant stabilizer of dimension exactly 133), the embedding-tensor layer (V7/V8: linear **912** constraint and quadratic closure, with the de Wit–Nicolai $SO(8)$ regression at residuals $\sim 10^{-16}$), the Jordan anchor (V9: the missing-16 pinned as the identity-anchored complement $(b) = 42 \ominus 26$), and the potential layer (S77). [LIB2-006] External conventions are taken from de Wit–Samtleben–Trigiante (arXiv:0705.2101) as transmitted by Dall'Agata–Inverso (arXiv:1112.3345), whose equation (3.1) fixes the scalar potential

$$V \;=\; \frac{g^2}{672}\Big(X_{MN}{}^{R}\,X_{PQ}{}^{S}\,\mathcal{M}^{MP}\mathcal{M}^{NQ}\mathcal{M}_{RS} \;+\; 7\,X_{MN}{}^{Q}\,X_{PQ}{}^{N}\,\mathcal{M}^{MP}\Big), \qquad X_{MN}{}^{P} = \Theta_{M}{}^{\alpha}(t_\alpha)_{N}{}^{P}, \tag{1}$$

with $\mathcal{M} = \mathcal{V}\mathcal{V}^T$ the scalar matrix. The 1/672 normalization was citation-checked against the published text this session; the suite's generator normalization reproduces it with calibration constant exactly 1.00000000 (no rescaling of any kind enters the numbers below).

2 The branching-wall theorem (the $\mathfrak{su}(2)_L$ wall, total form)

Novelty label: new specialization proved here.

Theorem 2.1 (Branching wall; S75)**.** *No gauging of maximal N=8, D=4 supergravity, in any symplectic frame, has a gauge algebra containing a compactly embedded $\mathfrak{su}(3) \oplus \mathfrak{su}(2)$. A fortiori no gauging realizes $\mathfrak{g}_{\rm SM}$, and no vacuum of any gauging has residual gauge symmetry containing $\mathfrak{g}_{\rm SM}$. The maximal SM shadow available in this framework is $\mathfrak{su}(3) \oplus \mathfrak{u}(1) \oplus \mathfrak{u}(1)$.*

Proof structure (each step labelled computed or cited). **L1** (cited-standard, with on-substrate positive control): the quadratic constraint implies the embedding tensor Θ is invariant under its own gauge algebra $\mathrm{im}\,\Theta$ [dWST]. Control: the maximal violation over the 28 $\mathfrak{so}(8)$ generators of the de Wit–Nicolai tensor is 1.5×10^{-16} on the substrate. Hence if $\iota = \mathfrak{su}(3) \oplus \mathfrak{su}(2) \subseteq \mathrm{im}\,\Theta$, then Θ is ι-invariant.

L2 (computed): every compactly embedded $\mathfrak{su}(3) \oplus \mathfrak{su}(2)$ in $E_{7(7)}$ conjugates into the fixed maximal compact $\mathfrak{su}(8)$; the faithful embedding classes are exactly the 21 branchings of the **8** (census computed twice, by character arithmetic and by explicit equivariant kernels on the substrate; the two methods agree 21/21).

L3 (new; formal statement): regard the embedding tensor as the linear map $\Theta : \mathbf{56} \to \mathbf{133}$, $e_M \mapsto \Theta_M{}^{\alpha} t_\alpha$. The ι-invariance of L1 says precisely that Θ *intertwines* the ι-action on the **56** with the adjoint ι-action on the **133**: $\Theta(X \cdot e_M) = [X, \Theta(e_M)]$ for all $X \in \iota$. By the first isomorphism theorem for ι-modules, $\mathrm{rowspace}(\Theta) = \mathrm{im}\,\Theta \cong \mathbf{56}/\ker\Theta$ as an ι-module — a *quotient of the* **56**; complete reducibility of the compact ι makes the quotient a direct summand, so every irreducible constituent of $\mathrm{im}\,\Theta$ occurs in $\mathbf{56}{\downarrow}\iota$ with at least its multiplicity. Since $\iota \subseteq \mathrm{im}\,\Theta$ carries its own adjoint isotype $(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3})$, both adjoints must occur in $\mathbf{56}{\downarrow}\iota$.

L4 (new — the wall, exact integer multiplicities): for every one of the 21 faithful classes, $\mathrm{mult}(\mathbf{8},\mathbf{1}) = 0$ or $\mathrm{mult}(\mathbf{1},\mathbf{3}) = 0$ in $\mathbf{56}{\downarrow}\iota = \Lambda^2\mathbf{8} \oplus \Lambda^2\bar{\mathbf{8}}$. [LIB2-076] Twenty classes have $\mathrm{mult}(\mathbf{8},\mathbf{1}) = 0$; the unique $(\mathbf{8},\mathbf{1})$-bearing class, $\mathbf{8} = (\mathbf{1},\mathbf{2}) + (\mathbf{3},\mathbf{1}) + (\bar{\mathbf{3}},\mathbf{1})$, has $\mathrm{mult}(\mathbf{1},\mathbf{3}) = 0$. [LIB2-076] Analytically: an $(\mathbf{8},\mathbf{1})$

in $\Lambda^2\mathbf{8} \oplus \Lambda^2\bar{\mathbf{8}}$ requires $\mathbf{8} \supseteq (\mathbf{3}, m) \oplus (\bar{\mathbf{3}}, m)$ with $m = 1$ forced by slot counting, leaving exactly two slots — a single $(\mathbf{1}, \mathbf{2})$ — whose Λ^2 contains no $(\mathbf{1}, \mathbf{3})$. Contradiction with L3 for every class; the theorem follows. □

Remark 2.2. The failure is *upstream* of the quadratic constraint, in the linear/equivariant structure: no frame enumeration and no classification citation is load-bearing. "Compactly embedded" is the physically correct hypothesis ($\mathfrak{g}_{\mathrm{SM}}$ sits in the maximal compact of any candidate gauge group); noncompact $\mathfrak{sl}(3) \oplus \mathfrak{sl}(2)$-type real forms are not, and need not be, excluded. The one place a referee should push — the self-duality bookkeeping in L3 — carries an on-substrate corroboration: the measured image-deficit of the 11 ι-generators against rowspace(Θ) over each nonzero invariant space equals 1.0000 exactly where both adjoints are absent (full isotypic orthogonality), 0.3805 for the unique $(\mathbf{8}, \mathbf{1})$ class (su(2)-blocked, floor $3/11 \approx 0.27$), and is never near zero. It additionally carries an *independent blind replication* (A759): a second executor, unbriefed on this proof and using a method-disjoint route (exact Gelfand–Tsetlin character arithmetic, no substrate), reproduced the 21-class census, all Inv(912) dimensions, and the full adjoint multiplicity table integer-for-integer — including the fingerprint class $\mathbf{8} = (\mathbf{1}, \mathbf{2}) + (\mathbf{3}, \mathbf{1}) + (\bar{\mathbf{3}}, \mathbf{1})$ with $\mathrm{mult}(\mathbf{8}, \mathbf{1}) = 2$, $\mathrm{mult}(\mathbf{1}, \mathbf{3}) = 0$ — and independently arrived at the same equivariance argument ("no image-compatible candidate for the quadratic constraint to act on").

Consequence for the framework. The Rev29 claim ledger keeps the gauge group as INPUT; Theorem 2.1 upgrades the *reason*: the input label is not "unselected" but *unreachable* — emergence-as-gauging is closed negative at theorem grade. The constructive question moves, with no residue, to the potential layer (Section 5).

3 The charge wall (the second wall: never the right representation)

The branching wall bars the *algebra* $\mathfrak{su}(3) \oplus \mathfrak{su}(2)$. The second wall bars the *representation*: even a lone $\mathfrak{su}(3)$, which gaugings demonstrably can carry, can never be carried in the matter-colour embedding. Among the eight inner embedding classes of a compact $\mathfrak{su}(3) \subset \mathfrak{su}(8)$ (classified by $\mathbf{8}\downarrow\mathfrak{su}(3)$; census computed three ways, exact agreement), matter colour is the *unitary* class $\mathbf{8} = \mathbf{3} \oplus 5 \cdot \mathbf{1}$, with **56**-content $20 \cdot \mathbf{1} \oplus 6 \cdot \mathbf{3} \oplus 6 \cdot \bar{\mathbf{3}}$ and *no octet.* [LIB2-077]

Novelty label: new specialization proved here.

Theorem 3.1 (Charge wall; S80)**.** *No quadratic-constraint-satisfying gauging of maximal N=8, D=4 supergravity contains a unitary-class $\mathfrak{su}(3)$ in its gauge algebra* $\mathrm{im}\,\Theta$*. In particular, no vacuum of any gauging has a residual gauge symmetry containing a unitary-class $\mathfrak{su}(3)$: matter colour is unreachable in any gauging, in any frame, at any vacuum.*

Novelty label: new specialization proved here.

Corollary 3.2 (Realizable colour classes)**.** *The only $\mathfrak{su}(3)$ embedding classes realizable inside the gauge algebra of any QC gauging are the* vector *class ($\mathbf{8} = \mathbf{3} \oplus \bar{\mathbf{3}} \oplus 2 \cdot \mathbf{1}$) and the* adjoint *class ($\mathbf{8} = \mathbf{8}$) — precisely the two classes whose octet occurs in the* **56***. The six remaining classes are barred. This matches every residual observed in the scanned family and in the published vacua (the Warner $SU(3) \times U(1)^2$ point: vector class; $SO(8)$: adjoint class).*

Proof structure (two independent legs, both computed). Suppose $QC(\Theta) = 0$ and a unitary-class $\mathfrak{su}(3) \subseteq \mathrm{im}\,\Theta$. By L1 of Theorem 2.1, $\Theta \in \mathrm{Inv}_{\mathfrak{su}(3)}(\mathbf{912})$.

Leg A (rowspace orthogonality; direct and self-contained): $\mathrm{Inv}_{\mathfrak{su}(3),\mathrm{unitary}}(\mathbf{912})$ was computed explicitly: dimension 140, matching the character prediction $140 = 2\cdot 15\,(\mathbf{36}\oplus\overline{\mathbf{36}}) + 2\cdot 55\,(\mathbf{420}\oplus\overline{\mathbf{420}})$ exactly (singular-value gap 8.9×10^{13}). The union of the rowspaces of *all* invariant Θ has rank 125, and the $\mathfrak{su}(3)$-containment leak equals 1.0000 on all eight generators. [LIB2-077] The 125 has a name: $125 = 35\cdot\mathbf{1}\oplus 15\cdot\mathbf{3}\oplus 15\cdot\bar{\mathbf{3}}$ is exactly the $(\mathbf{1},\mathbf{3},\bar{\mathbf{3}})$ isotype of $\mathbf{133}{\downarrow}\mathfrak{su}(3)_{\mathrm{unitary}}$ — the eight missing directions are precisely the $\mathfrak{su}(3)$ adjoint itself (full isotypic orthogonality). [LIB2-077] Since $\mathrm{im}\,\Theta$ lies inside that union, $\mathfrak{su}(3)\not\subseteq \mathrm{im}\,\Theta$. Contradiction.

Leg B (the octet/module argument; the L3 form): by L3, $\mathrm{im}\,\Theta$ is a quotient of the **56** as a module for $\iota = \mathfrak{su}(3)$; $\iota\subseteq\mathrm{im}\,\Theta$ requires its adjoint octet to occur in $\mathbf{56}\downarrow\iota$. The octet multiplicity in $\mathbf{56}{\downarrow}\mathfrak{su}(3)_{\mathrm{unitary}}$ is *zero* (on-substrate spectrum: 20 singlet states $\oplus$ 36 triplet states, zero octet states; positive control: the vector-class embedding shows exactly 16 octet states in the same pipeline). Contradiction. □

Remark 3.3 (Replication and provenance)**.** Both legs carry an *independent blind replication* (A761h): a second executor, unbriefed on the house numbers and deriving its own character prediction, reproduced the result seven-for-seven — $\dim\mathrm{Inv} = 140$, rowspace rank 125, containment leak 1.0 on all eight generators, octet multiplicity 0, twenty singlets, matched convention pin, control residual 1.5×10^{-16} (house: 1.3×10^{-16}) — and independently identified the 125 as the $(\mathbf{1},\mathbf{3},\bar{\mathbf{3}})$ isotype. This is the same replication pedigree as the branching wall's (A759). A structural companion (self-conjugacy): every $\mathfrak{so}(p,q)$-type residual is an orthogonal algebra, whose **8** is self-conjugate, so only the self-conjugate (vector/adjoint) classes fit residuals — the complex-type unitary class is barred from all orthogonal residuals categorically; the theorem extends the bar from residuals to every gauge algebra.

The two-theorem boundary, stated once. Theorem 2.1 and Theorem 3.1 together close the gauge boundary in both algebra and representation: $\mathfrak{su}(3)\oplus\mathfrak{su}(2)$ never fits (branching wall); a lone $\mathfrak{su}(3)$ fits only in the vector or adjoint class, never the matter-colour unitary class (charge wall). The right algebra can appear — provably never in the right representation. Fences: this is a statement about the supergravity gauging layer, *not* about the derived matter (the 16-C fermions are kernel-side); the vector-class shadow selection of Section 5 stands untouched; $A1$ stays INPUT, now with proven reasons on both sides.

4 Uniqueness of completion (G_2 reframe, D756)

Novelty label: new specialization proved here.

Theorem 4.1 (Uniqueness of the low-energy completion; S76)**.** *Fix the derived one-generation package: exactly the sixteen Weyl fermions of the* **16** *with the derived hypercharges* Y_{SM}*, and the* 10_H*-only scalar sector. [LIB2-081] Among compact reductive low-energy completions* that gauge the full derived colour + weak action *on this package:*

(1) G_{SM} *survives every filter (all twelve-plus anomaly sums vanish exactly, in integer arithmetic; Witten count even);*

(2) the only extension surviving all structural filters is $G_{\mathrm{SM}}\times U(1)_{B-L}$*, and the extra compact Cartan is* unique*: modulo the existing hypercharge direction the extra solution space of the Yukawa-neutrality + anomaly system is one-dimensional, and in the Higgs-doublet-neutral representative*

it is spanned by $(1,-1,-1,-3,3,3) = 3(B-L)$ *(the raw system is two-dimensional,* $X = 2hY + (3q-h)(B-L)$*; Rev32.9, A1647) — an independent, representation-level re-derivation of the* $B-L$ *residue carried elsewhere in the suite; it is removed only by the named no-fifth-force/scalar-closure empirical input;*

(3) every GUT and intermediate foil — $SU(5)$*,* $SO(10)$*, Pati–Salam, left-right, flipped* $SU(5)$ *— fails as a* low-energy *completion (extra massless vectors; breaking scalars absent from* 10_H*); trinification fails the exact-16 matter content itself;* none *fail by anomalies;*

(4) the proper sub-completions $U(1)_Y$*,* $SU(3)\times U(1)_Y$*,* $SU(2)\times U(1)_Y$ *are themselves anomaly-free and are excluded only by the gauge-the-full-action clause — that clause is load-bearing and travels with the theorem;*

(5) the group acting faithfully on the sixteen states is $S(U(3)\times U(2)) = G_{\rm SM}/\mathbb{Z}_6$ *(verified state-by-state); the global structure* $\Gamma \in \{1, \mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_6\}$ *is a line-operator-level choice* not *fixed by the matter spectrum — uniqueness is stated up to* Γ*.*

Enumeration boundary: the foil family and the adversary directions are exhausted; no closed classification over all compact reductive groups on a sixteen-dimensional Weyl space is claimed. [LIB2-081] No second completion was found.

Boundary sentence (mandatory). Theorem 4.1 is uniqueness-of-*completion*, NOT emergence. [LIB2-081] The gauge group remains an INPUT at the group level; Theorem 2.1 is *why* that is so (unreachable as a gauging), and Theorem 4.1 is the strongest honest statement on the other side (the input could not have been otherwise, in the stated class, up to $U(1)_{B-L}$ and Γ). [LIB2-081] The corresponding ledger label is $A1$: [INPUT + STRUCTURAL + empirical uniqueness-of-completion].

4.1 Explicit state-by-state verification of the $\mathbb{Z}_6$ faithful action

The "verified state-by-state" clause of Theorem 4.1(5) is the following one-line integer congruence, displayed here so the derived $\mathbb{Z}_6$ is reproducible rather than merely asserted. The generator of the candidate trivially-acting centre subgroup is

$$g = (\omega \mathbb{1}_3,\ -\mathbb{1}_2,\ e^{2\pi i Y}) \in Z(SU(3)_c)\times Z(SU(2)_L)\times U(1)_Y, \qquad \omega = e^{2\pi i/3}, \tag{2}$$

acting on a Weyl field of colour triality t (with $\mathbf{3}\mapsto 1$, $\overline{\mathbf{3}}\mapsto 2$, singlet$\mapsto 0$), weak duality $d\in\{0,1\}$ (doublet 1), and hypercharge Y in the $Q = T_3 + Y$ normalisation. Then g acts as the phase $\exp(2\pi i\,[\,Y + \frac{t}{3} + \frac{d}{2}\,])$, which is trivial on that field if and only if

$$\boxed{6Y + 2t + 3d \equiv 0 \pmod 6\,.} \tag{3}$$

Evaluated on the entire one-generation **16**:

field	(t,d)	Y	$6Y+2t+3d$	mod 6
Q $(\mathbf{3},\mathbf{2})$	$(1,1)$	$+\frac{1}{6}$	6	0
u^c $(\overline{\mathbf{3}},\mathbf{1})$	$(2,0)$	$-\frac{2}{3}$	0	0
d^c $(\overline{\mathbf{3}},\mathbf{1})$	$(2,0)$	$+\frac{1}{3}$	6	0
L $(\mathbf{1},\mathbf{2})$	$(0,1)$	$-\frac{1}{2}$	0	0
e^c $(\mathbf{1},\mathbf{1})$	$(0,0)$	$+1$	6	0
ν^c $(\mathbf{1},\mathbf{1})$	$(0,0)$	0	0	0

All sixteen states satisfy (3), so g generates a $\mathbb{Z}_6 = \mathbb{Z}_3 \times \mathbb{Z}_2$ that acts trivially on the whole package; since $Z(SU(3))$ alone is non-trivial on Q ($t = 1$) and $Z(SU(2))$ alone is non-trivial on L ($d = 1$), the trivially-acting subgroup is exactly this *correlated* cyclic $\mathbb{Z}_6$, and the group acting faithfully is $S(U(3) \times U(2)) = G_{\rm SM}/\mathbb{Z}_6$. [LIB2-083] This much is DERIVED from the loaded matter content (conditional on the field dictionary). It does *not* fix which $\Gamma \subseteq \mathbb{Z}_6$ is the physical global structure — that remains a line-operator-level choice not seen by the matter spectrum — and it does *not* upgrade the gauge group itself, which stays an INPUT at the group level by Theorem 2.1. [LIB2-083]

5 The potential layer and dynamical shadow selection

5.1 Construction and certification

The scalar potential (1) was implemented on the substrate with $\mathcal{M} = \exp(2\Phi)$ on the K-orthonormal noncompact basis (the **70** of the $E_{7(7)}/SU(8)$ coset, in which the missing-16 projector P_{16} of the V9 anchor acts), with exact polynomial gradients and Hessians at the origin, cross-checked by finite differences row by row. Certification against published values, with the calibration constant exactly 1:

- $V(0) = \frac{1}{4}\mathrm{Tr}\,\theta^2 - \frac{1}{8}(\mathrm{Tr}\,\theta)^2$ reproduced exactly for all seven electric rows: $SO(8) : -6$; $SO(7,1) : -2.5$; $SO(6,2) : 0$; $SO(5,3) : +1.5$; $SO(4,4) : +2$; $CSO(7,0,1) : -35/8$; $CSO(6,0,2) : -3$; and -6 for the dyonic ω-rotated $SO(8)$ (origin ω-independence);
- the N=8 point: $m^2 = -4$, seventy-fold degenerate ($= -\frac{2}{3}|\Lambda|$, the published value);
- the dyonic $SO^*(8)$ row ($\theta = \xi$): a Minkowski vacuum at the origin with $\nabla V = 0$ exactly and mass spectrum $m^2 = \{2^{(\times 2)},\ \frac{1}{2}^{(\times 20)},\ 0^{(\times 48)}\}$ — the published spectrum, multiplicity for multiplicity;
- the zeroth-order tadpole-direction ranks 1/5/8/0 of the earlier engine regression are reproduced unchanged (same P_{16}, same substrate).

Mass units: Hessian eigenvalues in the **56**-trace-orthonormal scalar basis are converted by the factor 12 (the Dynkin index of the **56** of $\mathfrak{su}(8)$ over the fundamental, $\Lambda^2\mathbf{8} \oplus \Lambda^2\bar{\mathbf{8}} = 2(N{-}2)$); the factor is anchored on the $SO(8)$ value and then *predicts* the $SO^*(8)$ spectrum, which lands exactly.

5.2 The selector and the result

Novelty label: new specialization proved here.

Theorem 5.1 (Shadow selection at a stable $SO(6,2)$ point; S77)**.** *Define the selector on a gauging* Θ *at the scalar origin:*

$$S(\Theta) \equiv [P_{16}\,\nabla V = 0] \ \wedge\ [P_{16}\,\mathrm{Hess}\,V\,P_{16} > 0 \ \textit{(Minkowski criterion)}] \\ \wedge\ [\textit{the residual gauge algebra hosts } \mathfrak{su}(3) \oplus \mathfrak{u}(1) \oplus \mathfrak{u}(1)].$$

Over the scanned family (the full electric $SO(p,q)/CSO$ ladder, the dyonic ω-rotated $SO(8)$, and the dyonic $SO^*(8)$):

(i) $SO(6,2)$ *is the* unique *electric row satisfying all three conditions:* $V(0) = 0$*; the entire tadpole* ($|\nabla V| = \sqrt{2}$) *lies along the* D_{KK} *dilaton of the* 6+2 *grading — a* **54***-direction — with the* P_{16} *component at machine zero* (1.5×10^{-15})*; the sixteen eigenvalues of* $P_{16}\,\mathrm{Hess}\,V\,P_{16}$ *are strictly positive* ($m^2 \in [0.171,\ 3.830]$)*; and the residual algebra* $\mathfrak{so}(6) \oplus \mathfrak{so}(2)$ *(dimension 16, compact) contains* $\mathfrak{u}(3) \oplus \mathfrak{so}(2) \supset \mathfrak{su}(3) \oplus \mathfrak{u}(1) \oplus \mathfrak{u}(1)$ *(containment residual* 2×10^{-16}*).*

(ii) Its dyonic completion $SO^*(8) \cong SO(6,2)$ *is a genuine Minkowski vacuum (all seventy tadpoles zero) with residual* $SO(6) \times SO(2)$*, the published spectrum above, and the shadow hosted; the dyonic* ξ *exactly lifts the single* D_{KK} *runaway of the electric row.* [LIB2-079] *Stability extends beyond quadratic order:* 46 *of the* 48 *flat directions stay exactly flat to the probed amplitudes along the probed paths, and the two exceptions — the flat directions intersecting the 16 — lift* positively *at quartic order (Rev32.9, A1647: through Rev32.8 this said "all 48 ... exactly flat" beside the two that lift) (*$V \propto t^4$ *measured); three hundred random flat-space combinations produce no tachyon.*

(iii) Every other row fails: $SO(8)$ *and* $SO(8)_\omega$ *are the AdS N=8 point (*$m^2 = -4$*: BF-allowed in AdS, but failing the Minkowski criterion);* $SO(7,1)$*,* $SO(5,3)$*,* $CSO(7,0,1)$ *have* P_{16}*-tadpoles of order one;* $SO(4,4)$ *passes both stability legs but its residual* $\mathfrak{so}(4) \oplus \mathfrak{so}(4) \cong \mathfrak{su}(2)^{\oplus 4}$ *contains no* $\mathfrak{su}(3)$ *(and the point is de Sitter);* $CSO(6,0,2)$ *— the structural foil, with the same* D_{KK}*-aligned tadpole geometry — fails twice:* P_{16}*-Hessian entirely negative, and residual* $\mathfrak{so}(6)$ *only (no second* $\mathfrak{u}(1)$*).*

Remark 5.2 (Honest fences)**.** (a) Uniqueness in Theorem 5.1 is *family-relative*: the scan covers the stated rows; Theorem 2.1 bounds what any unscanned gauging could deliver (never more than the shadow), and Theorem 3.1 now bounds the *representation* content too (no unscanned gauging can carry matter-colour $\mathfrak{su}(3)$); but the uniqueness of $SO(6,2)$ among *all* gaugings is not claimed. [LIB2-079] (b) "Shadow" is an algebra-type statement: the residual $\mathfrak{so}(6) \oplus \mathfrak{so}(2)$ hosts $\mathfrak{su}(3) \oplus \mathfrak{u}(1) \oplus \mathfrak{u}(1)$; the identification of the two $\mathfrak{u}(1)$ charge patterns on the **56** with the derived Y_{SM} and $3(B-L)$ was the designated next computation. *Executed (S78), with a two-part outcome.* (b1) At the origin of the selected family the match *fails at the representation level*, exactly: the matter **27** (the kernel-locked +2 grade of the D_{KK} grading) branches under the residual as $\mathbf{15}_0 \oplus \mathbf{6}_{+1} \oplus \mathbf{6}_{-1}$, refining under $\mathfrak{su}(3) \oplus \mathfrak{u}(1)_A \oplus \mathfrak{u}(1)_B$ to one octet, *one* singlet, and six triplet-types (Casimir clusters exactly $\{-3^{(8)}, 0^{(1)}\}$ and $-\frac{4}{3}^{(3)}$), whereas the derived matter pattern $16 + 10 + 1$ has *nine* singlets and *no* octet. The residual $\mathfrak{su}(3)$ is the gauging-frame (vector-type) embedding, not conjugate to the matter colour; branching multisets are conjugation invariants, so no $GL(2,\mathbb{Q})$ mixing of the two $\mathfrak{u}(1)$'s and no alternative $\mathfrak{su}(3) \subset$ residual (unique conjugacy class) can repair this. Per the pre-registered Paper 7 fence, "SM-shadow selected" therefore *demotes*, at this point, to "stable $\mathfrak{su}(3) \oplus \mathfrak{u}(1) \oplus \mathfrak{u}(1)$ point selected." (b2) The naive rigidity argument fails by computation (all 36 physical moduli move the D_{KK} grading), so a moduli scan appeared necessary; it is not — the question is settled by *counting* (S78b). The **56** of $E_{7(7)}$ under the residual-class $\mathfrak{su}(3)$ decomposes with Casimir clusters $\{-3^{(16)}, -\frac{4}{3}^{(36)}, 0^{(4)}\}$: *four* $\mathfrak{su}(3)$-singlets in the entire **56**, while the matter pattern requires *nine* inside a 27-plane. [LIB2-080] Conjugation preserves **56**-content, and the faithful $\mathfrak{su}(3)$ class in $\mathfrak{so}(6)$ is unique; hence no point of the 36-parameter vacuum family — indeed no $\mathfrak{su}(3)$ inside any E_7-conjugate of the residual, acting on any 27-plane — can carry the matter multiset. [LIB2-080] *The demotion is final for the entire* $SO(6,2)/SO^*(8)$ *family.* [LIB2-080] The reframed question — matter colour is the *unitary* class in $\mathfrak{su}(8)$ ($\mathbf{8}{\downarrow}\mathfrak{su}(3)_c = \mathbf{3} \oplus 5 \cdot \mathbf{1}$), whereas every $\mathfrak{so}(p,q)$-type residual — including the Warner $SU(3) \times U(1)^2$ point of the $SO(8)$ gauging — supplies only the vector class; can *any* gauging carry the unitary class? — was executed (S80) and is now Theorem 3.1: the global negative holds. The labeled shadow is excluded for *every* gauging, not merely for the scanned family; the demotion

of (b1) is thereby global and final. (c) The stability legs alone do *not* single out $SO(6,2)$ ($SO(4,4)$ survives them); the shadow-hosting leg is load-bearing. (d) The rank-24 mechanism ($12 \times 2 = 24$) remains a conjecture; the 48 flat directions ($= 2 \times 24$) of the $SO^*(8)$ vacuum are consistent with it but do not prove it. (e) The $J_{\rm vac}$-centralizer alternative selector is dead by computation: $\mathrm{stab}_{\mathfrak{e}_{6(6)}}(X)$ has dimension 52 with signature $(28^+, 24^-)$ — an $F_{4(4)}$-conjugate — at *every* $\det = 1$ rank-3 point tested, including $J_{\rm vac} = \mathrm{diag}(\phi, 1, \phi^{-1})$ and degenerate-spectrum points, and its center is zero: it is semisimple and cannot source even one $\mathfrak{u}(1)$, hence neither shadow Cartan. Distinct eigenvalues do not shrink the annihilator. (f) *Cosmological note (S79b).* In full-70 FRW evolution the electric $SO(6,2)$ row is lethal: its single D_{KK} dilaton tadpole drives immediate recollapse (3/3 trajectories, $t \lesssim 1$ — fast collapse, not quintessence), so the dyonic completion is load-bearing *dynamically* as well as statically. (g) *Frame-flip distinction, with granularity stated (S80).* The symplectic form Ω implements the Cartan involution on the substrate ($\mathrm{Ad}\,\Omega = \theta$ exactly: $-X^T$ on all 133 generators, residual zero), flips the D_{KK} grading ($\sigma D_{KK}\sigma^{-1} = -D_{KK}$, spectrum $\{\pm 3{:}1, \pm 1{:}27\}$), and time-mirrors the geodesic-16. Under $\sigma = \Omega$ the purely electric gauging families map *out of frame* (onto magnetic), while the dyonic family maps *into itself*: $\sigma(\Theta_{SO^*(8)}, E{+}M) = +(E{-}M)$, the conjugate dyonic partner (residual 2×10^{-16}). The granularity is part of the statement: no individual Θ is σ-fixed (pointwise stability fails for every gauging); σ-closure holds at *family* level, and the dyonic family is the unique σ-closed attached family. The dyonic completion is thus distinguished a third time — statically (S77), cosmologically (S79b, fence (f)), and by frame-flip closure (S80) — by three independent computations. No dynamical or temporal interpretation of σ is claimed. *Rev14 update (S82):* the $\sigma = \Omega$ family map now carries three-way external provenance (A762b); and three adjacent readings are closed negative at computation grade (D763, 4/4): the dyonic pair-of-pairs is obstructed, flickering rest mass is excluded ($\varepsilon = \mathrm{id}$ exact — shadow mass is σ-permanent), and the σ-turnaround link is null ($H = 0$ smooth, stabilizer dimension constant). (h) *Stability language (Rev14, binding).* All "stable"/"no tachyon" statements in this appendix are *classical, quadratic-order* statements. [LIB2-079] The published one-loop result for this vacuum class ($\mathrm{Str}\,\mathcal{M}^8 > 0$, arXiv:1307.4389) destabilizes $N{=}0$ Minkowski vacua; no quantum-stability claim is made. See Appendix G for the post-Rev13 tower/annulus layer on the same vacuum. (i) *Provenance of the gauging (Rev33.2, house audit S367.9; binding).* Every gauging in this appendix enters as an embedding tensor Θ of the four-dimensional theory. That Θ satisfies the linear **912** and quadratic constraints, and that a point of its potential is stationary and Minkowski-stable, establishes a consistent four-dimensional gauged theory and vacuum; it is *not* a provenance for Θ. No microscopic construction of the $SO(6,2)$ embedding tensor or of its dyonic completion — a higher-dimensional reduction or a generalized-frame realization — is supplied here, and none is claimed. "Dynamically selected" in this appendix means selected by the four-dimensional potential among the scanned gaugings (fence (a)), not derived from a microscopic theory.

5.3 The truncation question settled, and the geometry of the missing 16 (S78b)

The same session closed the long-standing truncation hole (Paper 4, T4-C) in both directions, by the Lie-triple criterion (a subspace $\mathfrak{m} \subset \mathfrak{p}$ of a symmetric space exponentiates to a totally geodesic submanifold — equivalently supports a consistent scalar sigma-model truncation — iff $[[\mathfrak{m},\mathfrak{m}],\mathfrak{m}] \subseteq \mathfrak{m}$):

- **The Jordan-54 is NOT a Lie-triple system**: the projection of $[[\mathbf{54},\mathbf{54}],\mathbf{54}]$ onto the missing-16 has order-one components (max 0.815 over 3×10^4 triples, exhaustive in the leading block). [LIB2-008] No consistent truncation of even the *ungauged* maximal theory retains

exactly the Jordan-visible 54 scalars; the "D=5→D=4 Jordan-54 model" is definitively a counting/structural correspondence. (The gauged-layer observation that no electric row has rank-zero leakage was the shadow of this sharper fact.)

- **The missing-16 IS a Lie-triple system**, to machine precision (4.9×10^{-15} over all 1,920 triples). [LIB2-009] It generates $\mathfrak{g}' = [\mathbf{16}, \mathbf{16}] \oplus \mathbf{16}$ of dimension 28, closing at 7.4×10^{-15}, with compact part of dimension 12, trivial center, rank 4, and Cartan character +4 — a tuple that by itself does not distinguish $\mathfrak{so}(4,4)$ from $\mathfrak{g}_{2(2)} \oplus \mathfrak{g}_{2(2)}$; the identification is certified on the registered plane by `s1167` (End = 1, Killing signature (16, 12), 24 roots of one length; Rev32.9, A1632): $\mathfrak{g}' \cong \mathfrak{so}(4,4)$, $[\mathbf{16}, \mathbf{16}] \cong \mathfrak{so}(4) \oplus \mathfrak{so}(4)$. [LIB2-010] *The missing-16 is, locally, the totally geodesic submanifold* $SO(4,4)/(SO(4) \times SO(4))$ *of* $E_{7(7)}/SU(8)$ (the identification is a local isometry only; Rev32.9, A1632) — the split-octonion (4, 4) norm-form Grassmannian. The framework's foundational split signature reappears at the top of the E_7 stack as exactly the geometry of the "missing" sector, and the geometrically self-consistent sub-sector is precisely the one the potential layer stabilizes (Theorem 5.1).

Scope fence: "consistent truncation" here is the scalar sigma-model criterion; vector-sector statements are downstream and not claimed. [LIB2-008, LIB2-009]

6 Net position of the gauge sector

The gauge boundary, stated once. The gauge group of the Standard Model is an INPUT of this framework, and Rev29 now knows *exactly why and exactly how much*: (1) it could not have arisen as a gauging of the maximal supergravity layer — Theorem 2.1 (no $\mathfrak{su}(3) \oplus \mathfrak{su}(2)$, with $\mathfrak{su}(2)_L$ as precisely the piece that can never fit) and Theorem 3.1 (no matter-colour $\mathfrak{su}(3)$, in any gauging, at any vacuum) — two proven walls: the right algebra, never the right representation; (2) given the derived matter and hypercharges, it could not have been any other group — Theorem 4.1, up to $U(1)_{B-L}$ (itself the unique extra Cartan, $3(B-L)$) and global structure Γ; (3) the maximal derivable remnant, an $\mathfrak{su}(3) \oplus \mathfrak{u}(1) \oplus \mathfrak{u}(1)$ of shadow type, is not inserted but *dynamically selected* by vacuum stability of the missing-16 sector at a stable $SO(6,2)$ point whose dyonic completion is a published Minkowski vacuum — Theorem 5.1, with the fences of Remark 5.2; the SM charge *labels* for the selected $\mathfrak{u}(1)$'s failed at the origin point (S78), were excluded for the whole vacuum family by counting (S78b), and are excluded for *every* gauging by the charge wall (S80) — the shadow is structural, not the Standard Model in disguise. Nothing in the observable map of Papers 1–7 changes: no public number moves. What changes is the epistemic status of the border: the gauge-sector holes are no longer open questions but two proven walls, a uniqueness bound, and a fired selector whose charge labeling is closed negative at theorem grade.

7 The frame-free Higgs line: an $E_{7(7)}$ no-go (S231–S232)

A third wall, of the same character as the two above, closes the question of whether the electroweak Higgs *line* can be selected without a loaded boundary. Working in the Freudenthal system $\mathfrak{F} = \mathbb{R} \oplus \mathbb{R} \oplus J_3(\mathbb{O}_s) \oplus J_3(\mathbb{O}_s)$ with its frame-free automorphism group $G_0 = \mathrm{Aut}(\mathfrak{F}, \omega, Q)^0 = E_{7(7)}$ acting on the real **56**:

Novelty label: standard theorem used.

Theorem 7.1 (No frame-free Higgs line; S231/S232)**.** *There is no G_0-invariant rank-one projector on the* **56***: the only invariant idempotents have rank* 0 *or* 56*. [LIB2-078] Consequently the electroweak vacuum line (LOAD-EW-LINE) cannot be selected frame-free; it is a loaded datum.*

Proof (explicit, octonion-free; S232 kernel `s796`). Present $\mathfrak{e}_{7(7)} = \mathfrak{sl}(8,\mathbb{R}) \oplus \Lambda^4\mathbb{R}^8$ (dimension $63 + 70 = 133$) acting on $\mathbf{56} = \Lambda^2\mathbb{R}^8 \oplus \Lambda^2\mathbb{R}^{8*}$. The 133 generators (i) are independent (raw rank 133), (ii) all lie in $\mathfrak{sp}(56,\mathbb{R})$ (preserve the intrinsic symplectic form), and (iii) close as a Lie algebra of dimension *exactly* 133 — so the generated algebra is $\mathfrak{e}_{7(7)}$, not the larger $\mathfrak{sp}(56)$. The commutant $\{M : [X, M] = 0\ \forall X\}$ then has dimension 1, spanned by I: the **56** is real-irreducible with commutant $\mathbb{R} \cdot I$ (real type). An invariant idempotent lies in the commutant, so its only values are 0 and I, i.e. ranks $\{0, 56\}$; no rank-one invariant line exists. [LIB2-078] □

This upgrades the standing obstruction from a citation (irreducibility of the minuscule ϖ_7) to an in-house construction with nothing assumed. It is the action-side counterpart of the two gauging walls: the boundary the Standard Model needs is *loaded*, not internally selected — and, as elsewhere in this suite, *no public observable moves.* (This is the theorem folded into the interacting-vertex chain of Appendix R as the oldest standing import.)

8 The invariant-connection kill: a holonomy no-go (Rev30, A1496)

The freeze-first arc closed one of the two decidable rows of its plan by a holonomy argument, and the result is a gauge-boundary statement, so it is recorded here. **The declared class is part of the theorem and must travel with it**: *left-$E_{7(7)}$-invariant principal $SU(8)$ connections compatible with Ω and the loaded K_{sel}.* [LIB2-091] Inside that class, and only inside it, the class is empty of anything that could cancel the obstruction.

The argument is two steps, both rebuilt end to end in house (`s1014`, `s1015`) and agreeing with the lane to 9×10^{-15}.

Step 1 (uniqueness, by Schur and Wang). The Casimir acts as an exact scalar on each summand of the symmetric-space splitting — 3/2 on $\mathfrak{p}$ (70-dimensional) and 4/3 on $\mathfrak{k}$ (63-dimensional). The two scalars differ, so $\mathrm{Hom}_{SU(8)}(\mathfrak{p}, \mathfrak{k}) = 0$ by Schur, and Wang's theorem then leaves the canonical symmetric-space connection as the *only* connection in the class.

Step 2 (holonomy, by Ambrose–Singer). Evaluating that connection's curvature on all 2415 noncompact pairs gives coefficient rank 63, so Ambrose–Singer returns $\mathrm{Hol} = SU(8)$ in full. The holonomy therefore escapes K_{sel}, and by Step 1 there is no other connection available to cancel it. [LIB2-091] The frozen kill condition fires.

> **What this is not.** It is *not* a topological no-section theorem — the base is contractible, and no such statement is claimed. What remains inhabited after the kill is a *non-equivariant* K_{sel} reduction σ_{sel} carrying a preserving connection, and it is typed and counted rather than waved at: 43 **continuous fibre coordinates (**53 **once the source is fixed),** 0 **discrete.** [LIB2-091] Stating the kill as "no compensator connection exists" would overstate a class-relative result into a topological no-go this appendix explicitly disclaims.

The parallel dependency survives the kill. The row-2 computation is carried out before any row-1 output exists — only the explicitly labelled $K_{\mathrm{sel},j}$ variant consumes the source ray — and row 1

consumes only frozen-frame tensors together with the boundary datum L_∂. That a later row must import both outputs is composition, not serialization, so the plan's parallel split is intact.

The companion row is a success only relative to a typed input, and is recorded that way. The vector/Higgs **10** carries a *unique* invariant symmetric form η_{10} of split signature $(5,5)$ — house-checked as an exactly one-dimensional invariant-form space, with a random form failing the same test. But the generation of the source is *not* zero-input and was never claimed as such: L_∂ costs 10 continuous parameters, U one inherited slot, ρ one selection-neutral slot, and one orientation bit that exists only after the ray has been fixed. The physical attachment of L_∂ remains INPUT. A kernel coordinate appearing in the construction is a basis witness, not a physics identification. Nothing in this section promotes an observable, moves a tier, or touches the engine.

Provenance

S74: compact wall and first L_Θ table. S75: branching-wall theorem (stages 0–3b, master log; 21/21 kill table). S76: D756 close (uniqueness; three-way, two genuine returns plus house extension; all anomaly sums exact). S77: potential layer (stage-0 cache plus row runner; nine rows; master table), $SO^*(8)$ benchmark, flat-direction probe, J_{vac}-stabilizer run. S78/S78b: charge-pattern test and counting kill; Lie-triple geometry of the missing 16. S79b: full-70 FRW cosmology (electric-row lethality; basin boundary). S80: charge-wall theorem (Inv dimension 140, rowspace 125, octet census 3-way; blind replication A761h, 7/7) and the $\sigma = \Omega$ frame-flip family map (external rerun of the S77/S78 layer: end-to-end identical, A761g). Citation checks performed against arXiv:1112.3345 directly; the Warner-vacuum literature anchor for $SU(3) \times U(1)^2$ at $SO(8)$ complex scalars is Warner '83 and Fischbacher–Pilch–Warner arXiv:1010.4910 (not 1112.3345, whose scan excludes complex vevs).

Rev14 (S82–S88). S82: $\sigma = \Omega$ three-way (A762b); D763 closed negatives; annulus/shell-straddle ledger (D764, blind collision at D761R pedigree). The post-Rev13 tower, annulus geometry, $\mathfrak{e}_{6(6)}$ identification, and Jordan-interface certificates are collected in Appendix G with their own provenance and SHA manifests (S82–S88).

Rev25 (S214 fold). The anomaly filter of this appendix's uniqueness argument is now backed by the complete *exact* audit: kernel `s588` verified that all SM gauge and gravitational anomaly sums cancel exactly from the framework's own derived hypercharges (upgrading the structural `s572` check), formally closing hostile-board item T3-P. Cross-references: Paper 1 (anomaly-cancellation status, Rev25 update) and Paper 7 (Rev25 additions).

Leibniz Quantum Beats Newton

Appendix G — The Kaluza–Klein Annulus: Scherk–Schwarz-Type Mass Levels, $\mathfrak{e}_{6(6)}$ Identification, and the Jordan Interface (Rev33.2)

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Abstract

This appendix records the post-Rev13 layer (Sessions 82–88) of the certified $E_{7(7)}$ **56/133** substrate programme: the discovery, lawful description, documented identification, and finally algebraic *naming* of the vacuum mass tower. (i) *The annulus geometry* (S82): every Dirac pairing connects the **27** to the $\mathbf{27}'$ *through* the (**78**+**1**) annulus; each shell carries one full shadow copy; the grading operator D_{KK} (named for the Kaluza–Klein reading, which is not derived — see the vocabulary note) is a third independent grading. (ii) *The tower law* (S87): on the $SO^*(8)$ Minkowski vacuum the scalar Hessian satisfies the Kaluza–Klein form identity $H = -(\mathrm{ad}\, Z_2)^2$ modulo Goldstone directions; masses obey $m = |q|\, q_0^{\mathrm{KK}}$ with $q_0^{\mathrm{KK}} = 1/(2\sqrt{6})$ exact (the tower quantum; distinct from the clock anchor $q_0^{\mathrm{clk}} = m_e$ of Appendix H — two constants, two names from Rev32.2); the twelve Goldstones are exactly the charged content of the massless 48. (iii) *The documented identification* (S87): the vacuum spectrum coincides point-by-point with the $\mathrm{CSO}(2,0,6) \equiv \mathrm{CSS}_{N=0}$ model of Catino–Dall'Agata–Inverso–Zwirner (arXiv:1307.4389, Table 2) — a twisted reduction on a *spacelike* circle — including the published "12 Goldstones + 36 moduli"; at gauging grade the identification is obstructed for the full gauging, each oriented half being a twisted reduction (Rev33.2); the timelike alternative is excluded by document (Hull–Julia, hep-th/9803239: a timelike circle forces the non-compact $SU^*(8)$ denominator). (iv) *The naming theorem* (S88): the annulus *is* $\mathfrak{e}_{6(6)} \oplus \mathfrak{so}(1,1)_{KK}$ — the five-dimensional U-duality algebra — and the shells are its $\mathbf{27}/\overline{\mathbf{27}}$; equivalently, the tower lives on the canonical minuscule 3-grading $\mathfrak{e}_{7(7)} = \mathbf{27}' \oplus (\mathfrak{e}_{6(6)} \oplus \mathfrak{so}(1,1)) \oplus \mathbf{27}$, i.e. the $4D \leftarrow 5D$ Kaluza–Klein decomposition, with the Jordan **27** realized as the cubic boundary slice of the Freudenthal **56**. The clock is independent of all of it: the certified A_1 block is exactly $\mathfrak{so}(2)$-neutral — tick and tower are separate exact structures on one vacuum. All results are classical/quadratic-order statements on the certified substrate; no public observable moves.

Vocabulary (Rev32.9, ruling R-19; A1644 E-7/E-9). None of the three graded objects in this appendix — the $\mathfrak{so}(2)$ charge q of Theorem 3.1, the $\mathbb{Z}_4$ tick of Appendix P, the grading operator D_{KK} — is exhibited as the fifth momentum p_5 of a Kaluza–Klein reduction: no compact coordinate and no conjugate-momentum map is registered. "Kaluza–Klein" and "Scherk–Schwarz" here name the *documented identification* with the $\mathbb{Z}_2$-twisted 5D reduction of arXiv:1307.4389 at the level of the published spectrum — a finite three-level scalar Hessian with $m = |q|\, q_0^{\mathrm{KK}}$ — not a derived tower. Momentum vocabulary is not used for q, the tick, or D_{KK}.

Notation (σ). Where this appendix cites σ as an involution it means Appendix F's $\mathbf{SO}^*(8)$ **time-mirror** $\sigma = \Omega$. It is *not* Appendix I's Jordan frame involution σ_{frame} ($\rho \mapsto 1/\rho$), *not* Appendix H's

throat involution σ_C, and *not* the split-octonion conjugation σ_{oct} of the A689 dictionary.

1 Scope and provenance

Everything in this appendix is a finite, machine-verifiable statement on the certified $E_{7(7)}$ substrate of Appendix F, obtained in Sessions 82–88 (2026-06-10/11) under pre-committed gates, with SHA-256–manifested artifacts (`_S82`–`_S88` manifests; house scripts `s82*`–`s88*`). External returns used here (D763/D764 annulus computations; D772/D773 scrutiny items; D774 interface lemmas) were re-run or cross-certified in house before banking. Conventions: $D_{KK} = \rho_{\mathfrak{sl}_8}\,\mathrm{diag}(-3,-3,1,1,1,1,1,1)$ (vacuum-aligned frame of S82). The index-permuted form $\mathrm{diag}(1^6,-3,-3)$ is $E_{7(7)}$-conjugate to it, but the two are *not* interchangeable at a fixed gauging: in the frozen gauging frame of the later suite the grading aligned with the centre is $\pm D_{56}$, the $\mathrm{diag}(1^6,-3,-3)$ ordering, whose KK-vector anchor is $-Z/\sqrt{2}$; the solder grading D_{sol} (D1410) is not aligned, and the two have different five-dimensional parent real forms (§3; A1774, `s1262`). Z_2 denotes the certified $\mathfrak{so}(2)$ center of the residual compact algebra at the $SO^*(8)$ Minkowski origin.

2 The annulus and shell geometry (S82)

Under $\mathrm{ad}\,D_{KK}$ the adjoint grades as $\mathbf{133} = \mathbf{27}_{-4} \oplus (\mathbf{78}{+}\mathbf{1})_0 \oplus \mathbf{27}_{+4}$ and the fundamental as $\mathbf{56} = \mathbf{1}_{+6} \oplus \mathbf{27}_{+2} \oplus \mathbf{27'}_{-2} \oplus \mathbf{1}_{-6}$. The banked S82 ledger (D764 close; blind external collision with the sealed house table, house re-run identical to BLAS noise) establishes:

1. every Dirac pairing connects the $\mathbf{27}$ to the $\mathbf{27'}$ *through* the annulus — "stage in the ring, shells at the rim, cores empty" (Rev32.9, A1642 O8: through Rev32.8 the slogan said "matter in the shells"; the $\mathbf{27}/\mathbf{27'}$ shells are Jordan-cubic labels, flat at quadratic order, and carry no particle-content claim); the geodesic-16 sits at grade 0 with fraction exactly 1.000000 (S82b);
2. each shell carries one full shadow copy; D_{KK} is a third independent charge, commuting with the shadow gradings;
3. three closed negatives (D763, 4/4): the dyonic pair-of-pairs is obstructed ($\mathfrak{su}(2)^4$ certified, the pair irreducible); flickering rest mass is closed negative ($\varepsilon = \mathrm{id}$ exact — shadow mass is σ-permanent in the $E{+}M$ completion); the σ-turnaround link is closed negative ($H = 0$ smooth, stabilizer dimension constant);
4. the pre-registered information-content sweep returned its null (no "annulus number"); the D_{KK} ray is the unique residual-symmetric collapse channel (16-vs-0).

The $\sigma = \Omega$ frame-flip family map of Appendix F fence (g) was upgraded to three-way external provenance en route (A762b).

3 The tower law and the documented identification (S84–S85, S87)

Novelty label: new specialization proved here.

Theorem 3.1 (Kaluza–Klein form of the Hessian; S87)**.** *At the $SO^*(8)$ Minkowski origin, on the orthogonal complement of the twelve Goldstone directions, the scalar Hessian satisfies $H = -(\mathrm{ad}\, Z_2)^2$ (operator residual 4.4×10^{-10} relative); consequently every scalar mass obeys $m = |q|\, q_0^{\mathrm{KK}}$ with q the integer $\mathfrak{so}(2)$ charge and*

$$q_0^{\mathrm{KK}} = \tfrac{1}{2\sqrt{6}} \qquad \textit{(exact; deviation } 5.3 \times 10^{-16}\textit{)}.$$

The twelve Goldstones are exactly *the charged content of the massless 48 (principal cosines* 1.000000000000*); the tower lies entirely in the* (**78**+**1**) *annulus; the* **27**/**27**′ *shells are flat at quadratic order; $[Z_2, D_{KK}] = 0$.*

The supporting spectral layer was completed in S85 (28 vector masses: 16 massless with residual $\mathfrak{so}(6) \oplus \mathfrak{so}(2)$, 12 exactly degenerate massive; scalar Hessian $\{0^{\times 48}, \tfrac{1}{24}{}^{\times 20}, \tfrac{1}{6}{}^{\times 2}\}$, no tachyons, $m^2 \propto$ ($\mathfrak{so}(2)$ charge)^2; massive vectors the $\mathbf{6}_{+1} \oplus \mathbf{6}_{-1}$ coset; shadow $\mathfrak{u}(1)$'s = Cartan of the broken $SO^*(8)$), and the σ-link closed at theorem grade (three lanes, two house runs, one blind collision; σ-closure exactly 0.0 two ways; random-Θ control 7/7 — the flip identity is kinematics, the dyonic content is σ-closure).

Remark 3.2 (The charge and where its law holds)**.** The $\mathfrak{so}(2)$ charge q of Theorem 3.1 is the centre Z of the 16-dimensional unbroken algebra at the origin: $q \in \frac{1}{2}\mathbb{Z}$ with $e^{2\pi i q} = (-1)^F$, and the mass law extends from the scalar Hessian to every physical massive state at the origin, $m^2 \propto q^2$ with one constant (`s1233`). Z is $\pm\sqrt{2}$ times the twist generator of the rank-13 end gaugings (`s1234`). The generator Z_2 of the theorem is this line: $Z_2 = q_0^{\mathrm{KK}}\, Z$ with $Z = \frac{1}{2}\sum_a T_a$, one normalisation reproducing both printed eigenvalues ($\frac{1}{24}$, $\frac{1}{6}$) (`s1250`); it is neither a KK translation nor the clock of Appendix H. The single-charge law holds on a rank-one sheet — the origin, the D_{56} line and the golden branch, where $m = |\mu \cdot n|$ (`s1235`) — and *fails* at the tree-critical point c_c (A1764, `s1238`). It is a property of that sheet, not of the gauging.

On the torus-fixed branch F_0 (the fixed set of the unbroken four-torus through the origin and c_c; six noncompact directions, three commuting $\mathfrak{sl}(2,\mathbb{R})$ planes) every point is tree-flat and tree-critical, and the whole spectrum is a quadratic form in the four torus charges, $M^2 = \sum_{a,b} B_{ab} \mathsf{Q}_a \mathsf{Q}_b$, with B in closed form (proved on F_0, A1765; house replay `s1242`). The single-charge law above is its rank-one stratum. A generic B has inertia $(3,1)$: a *mass* form whose negative direction is an instability of fully charged scalars — never a time direction (a signature, with no map to a Lorentzian object). The tachyon-free part of F_0 is the closed region where three linear invariants of B are non-negative (A1765, `s1242`). F_0 is one branch, not the Minkowski locus.

Remark 3.3 (Field content on F_0)**.** On F_0 the matter is vectorlike: there is no chiral matter (house and lane independently; `s1241`, A1765), as fence (3) already says for the appendix as a whole. Parity acts as $w \mapsto -w$, and an explicit charge conjugation fixes F_0 pointwise (on states, internal torus charges only; `s1243`). The four massless vectors have kinetic and θ-matrices in closed form; the θ-matrix, re-derived in house in the frame fixed by $\ker X$ and the parity, equals A1766's $-HG^{-1}$ (`s1248`): it is P-odd, C-even and CP-odd, and vanishes on the a-flat; on the D_{56} line its eigenvalues are $\tanh x$ ($\times 3$) and $-\tanh 3x$, $x = s/(3\sqrt{2})$, and every eigen-coupling has unit modulus. On unit weights $m^2 = \frac{1}{2}g^2$ — not a no-force statement. A tree-level point with a massless charged spin-$\frac{1}{2}$ pair and massive gravitini exists; this is an existence result, not a mechanism: on the a-flat the one-loop potential has its global minimum at the origin and flows through the threshold.

Identification. The vacuum spectrum coincides, entry by entry, with $[SO^*(8)]_{c=1} \equiv \mathrm{CSO}(2,0,6) \equiv \mathrm{CSS}_{N=0}$ of arXiv:1307.4389 (Table 2) — the $\mathbb{Z}_2$-twisted reduction of the 5D maximal theory on

a *spacelike* circle — including the published "12 Goldstones + 36 moduli" (our K4 measurement) and the spin-unified rungs (*rung unit*: the published rung is the $\mathfrak{u}(1)$ charge Q of arXiv:1307.4389 Table 2, and $Q = 2|q|$ with $|q|$ the $\mathfrak{so}(2)$ charge of Theorem 3.1. The centre Z_2 acts on the $\mathbf{8}$ of $\mathfrak{su}(8)$ with charges $\pm\frac{1}{2}$ (four each, in units of q_0^{KK}) — the unique solution reproducing its spectra on the $\mathbf{56} = \Lambda^2\mathbf{8} \oplus \overline{\Lambda^2\mathbf{8}}$ and the $\mathbf{70} = \Lambda^4\mathbf{8}$, house `s1201`. Hence gravitini $|q| = \frac{1}{2}$ ($Q = 1$), the 12 massive vectors and the 20 massive scalars $|q| = 1$ ($Q = 2$), the two heaviest scalars $|q| = 2$ ($Q = 4$); in the published table $m^2 = Q^2/8$ holds for the gravitini and all 28 vectors, but not charge-by-charge for spin $\frac{1}{2}$ ($Q = \pm 1$ carries both $m^2 = 0$ and $\frac{1}{8}$). Through Rev32.11 this parenthesis read "gravitini at half-rung; ... at rung 2", one sentence in two units, Rev33.0, A1642 O9). The timelike alternative is not the identified case: Hull–Julia (hep-th/9803239, Table 2, row $D = 4$) shows that a timelike circle replaces the $SU(8)$ denominator by its non-compact real form $SU^*(8)$ (maximal compact $USp(8)$; see also Hull, hep-th/9806146); the spectrum-grade identification of this appendix is to the spacelike table only. (Rev32.6, S302.4: the earlier gloss "with no compact $\mathfrak{u}(1)$" was the house's reading, not the source's, and is withdrawn. The ladder-centralizer census of Appendix N is invariant under $SU(8) \to SU^*(8)$ — `s1160`: $\theta' = \theta\sigma$ fixes $\mathfrak{su}^*(8)$, dimension 63, Killing signature $(27, 36, 0)$; $\theta' = \theta$ on $C(H)$ to principal angle 2.6×10^{-8} — so nothing computed in Appendix N depends on which real form the circle selects.) The identification is established at *spectrum* grade and at *substrate* grade: the canonical CSS flat algebra $F = \mathbb{Z}_2 \ltimes t^{12}$ exists inside $\mathfrak{e}_{7(7)}$ ($\mathbf{27}_{+4} = 15_{\mathrm{neutral}} \oplus 12_{\mathrm{charged}}$ at $|q| = q_0^{\mathrm{KK}}$, deviation 4.4×10^{-16}; abelianness grading-forced). At *gauging* grade the identification is obstructed for the full gauging in the ordinary class (A1774, `s1262`): the gauging fixes no vector of the $\mathbf{56}$ ($\sum_M A_M^{\mathsf{T}} A_M = 12\, I$), $\mathfrak{e}_{7(7)}$ supplies no rescaling generator, and the gauge algebra $\mathfrak{so}(6,2)$ (dimension 28) is not the 13-dimensional CSS algebra. The suite's own gauging is *not* flat (symmetric pair, forced): the charged 12 of each $D_{56} = \pm 4$ shell is spanned by the nilpotent halves of the twelve noncompact gauge generators at the origin, $t^{12} = \mathrm{span}\{x_+^{\mathsf{T}}\}$ ($D_{KK} = -D_{56}$), and the gauge algebra at the origin is $\mathfrak{t}_{16} \oplus \{t - \theta_C(t)\}$ — the symmetric-pair partner of the flat CSS algebra F, not a flat gauging (`s1250`, `s1251`, A1769). What holds is one level down: the full gauging is the O-mirror sum $X^- + O \cdot X^-$ of a single oriented parent-plus-twist half, in both gradings, and each oriented $\mathbf{351} + \mathbf{78}$ half is a valid five-dimensional parent with a twist (parent $\mathfrak{so}(6)$ under D_{56}, $\mathfrak{so}(4,2)$ under the solder grading D_{sol}). The quadratic constraints leave the relative sign of the two halves free: equal signs give $\mathfrak{so}(6,2)$, opposite signs $\mathfrak{so}(8)$ under D_{56} and $\mathfrak{so}(4,4)$ under D_{sol}, and one half alone the CSO contraction (`s1266`, `s1268`). In the $SL(8)$ frame ($\mathbf{56} = \Lambda^2\mathbf{8} \oplus \Lambda^2\mathbf{8}^*$) the gauging is a dyonic $\mathfrak{so}(6,2)$ on $\mathbf{8} = \mathbf{2} + \mathbf{4} + \mathbf{2}$ with odd plane $(7, 8)$. These are algebra-grade statements; no five- or six-dimensional spacetime is asserted. The 5D reading is therefore: same vacuum, same spectrum, same flat algebra in the substrate, each half a twisted reduction — not the same gauging.

Clock ≠ quanta (S87). The certified A_1 block is exactly $\mathfrak{so}(2)$-neutral (residual 1.3×10^{-17}): the $\mathbb{Z}_4$ tick is not a $\mathfrak{u}(1)$ holonomy of the tower charge; (q, d) are independent commuting gradings. The relational tick $\tau = +i$ (S85–S86; rigid, a constant of the motion, $\leq 1.4 \times 10^{-14}$ across static and FRW probes) and the tower are separate exact structures on one vacuum. The suite is exactly orientation-even at the A_1 and full 36-isotypic layers (S87 scan, worst line 3.8×10^{-14} against a 10^{-12} gate; the only odd content is the sign convention of $\mathrm{Im}\,\tau$); pair-conjugacy is certified at the full 36 layer (cosines 0 to 3.5×10^{-16}).

4 The annulus is $\mathfrak{e}_{6(6)}$ (S88)

Novelty label: standard theorem used.

Theorem 4.1 (F-ANNULUS-E6; S88, house)**.** *Let* $\mathfrak{m}$ *be the trace-form–orthogonal complement of* $\mathbb{R}\,D_{KK}$ *in the grade-zero subalgebra* $\mathfrak{g}_0 = \ker(\mathrm{ad}\,D_{KK}) \subset \mathfrak{e}_{7(7)}$*. Then* $\mathfrak{m}$ *is a simple real Lie algebra isomorphic to* $\mathfrak{e}_{6(6)}$*, and the grade-*± 4 *shells are its* $\mathbf{27}/\overline{\mathbf{27}}$*:*

$$\mathfrak{e}_{7(7)} \;=\; \mathbf{27}' \;\oplus\; \left(\mathfrak{e}_{6(6)} \oplus \mathfrak{so}(1,1)_{KK}\right) \;\oplus\; \mathbf{27}.$$

Certificate (all gates pre-committed; script `s88a`, manifest `_S88_outputs.sha256`): closure of $\mathfrak{g}_0$ exact to machine zero and $B(D_{KK}, [\mathfrak{g}_0, \mathfrak{g}_0]) = 0$ exactly; trace-form signature on $\mathfrak{m}$ equals $(42, 36)$ — the $\mathfrak{e}_{6(6)}$ fingerprint, which excludes the real forms $\mathfrak{e}_{6(2)}, \mathfrak{e}_{6(-14)}, \mathfrak{e}_{6(-26)}$; generic-element centralizer dimension 6 (rank); adjoint Casimir $= 2 \not\Vdash$ to 2.3×10^{-15} (simplicity); the action on the grade-$+4$ shell is *absolutely irreducible* (commutant dimension 1, cyclic rank 27/27, traceless), with Casimir ratio

$$\frac{c_{27}}{c_{78}} = \frac{13}{18} \quad \text{exact to twelve digits,}$$

the $\mathfrak{e}_6$ fundamental/adjoint ratio. The dimension-and-rank foils $\mathfrak{so}(13)$ and $\mathfrak{sp}(12, \mathbb{R})$ (both 78-dimensional, rank 6) were declared in advance and are killed by the existence of the 27-dimensional irreducible action, which neither algebra possesses.

Remark 4.2 (Reading)**.** $\mathfrak{e}_{6(6)}$ is the U-duality algebra of maximal supergravity in five dimensions. Theorem 4.1 therefore upgrades the Scherk–Schwarz identification ladder by an *algebra-grade* rung: the suite's own grading reproduces the canonical $4D \leftarrow 5D$ decomposition — spectrum grade (Table-2 match) $\rightarrow$ substrate grade ($F = \mathbb{Z}_2 \ltimes t^{12}$) $\rightarrow$ algebra grade (this theorem). At gauging grade the ladder splits. Each rank-13 end gauging is, at tensor level, an equal-four-parameter CSS twisted circle gauging of the five-dimensional theory (arXiv:1307.4389 eq. (3.5); residual 0, intertwiner exhibited; A1763, `s1234`). The full gauging at the balanced point has no section-obeying generalised Scherk–Schwarz uplift (arXiv:1708.02589 §4.3; A1763) and is obstructed in the ordinary class (§3). Its non-geometry is of M-theory type: against every $SL(8)$ section it occupies a locally non-geometric R slot ($R \in \mathbf{1}_{14}$ or the $\mathbf{7}_6$ of R^i, $R^{ij}{}_k$, in the labels of arXiv:1710.05919), and no E_7 frame places it on geometric slots alone (`s1331`, `s1332`).

Novelty label: standard theorem used.

Lemma 4.3 (Jordan interface; D774, house cross-certified)**.** *In the graded* $\mathbf{56} = \mathbf{1}_{+6} \oplus J_{+2} \oplus J^{\vee}_{-2} \oplus \mathbf{1}_{-6}$ *with* $J_{+2} \cong J_3(\mathbb{O}_s)$*: (i) the symplectic form* Ω *pairs only opposite grades;* J_{+2} *is isotropic and* $L_J = \mathbb{R}e_- \oplus J_{+2}$ *is Lagrangian; (ii)* $\Omega : J_{+2} \times J^{\vee}_{-2} \to \mathbb{R}$ *has full rank* 27*; (iii) the quartic invariant restricts to the Jordan cubic norm by adjoining the opposite singlet,* $I_4(\beta e_- + X) = \beta\, N(X)$ *(*β*-linearity verified at* 1.4×10^{-14} *with non-zero witness), the grading making any other monomial weight-forbidden; (iv) the stabilizer of* e_- *inside* $\mathfrak{g}_0$ *is precisely the* $\mathfrak{e}_{6(6)}$ *of Theorem 4.1 (certified by the same invariant gates, including* $c_{27}/c_{78} = 13/18$ *on the* $\mathbf{56}$*-side Jordan* $\mathbf{27}$*).*

Remark 4.4 (Known mathematics, certified in-substrate)**.** Lemma 4.3 is classical Freudenthal-triple-system / Jordan-pair structure theory (Freudenthal; Günaydin–Koepsell–Nicolai, hep-th/0008063; Krutelevich, math/0411104; see also the exceptional-supergravity literature). Its content here is not novelty but *certification*: these are now finite, re-runnable facts about the same certified substrate that carries the walls of Appendix F and the tower of Theorem 3.1, with the suite's measured objects (annulus, shells, D_{KK}, J_{vac}-aligned frame) appearing as the named classical structures. Two independent same-day routes (house annulus-invariants; external Freudenthal-pairing free hand, D774) converged on the same identification and were cross-certified (script `s88b`).

5 Phase box and universal weight (S85–S86)

For completeness of the post-Rev13 record: the phase-box keystone PH1 is the equivariant L-intertwiner ((z,z,z) block, spec `ikkljl->ij`; equivariance $\leq 2 \times 10^{-15}$; multiplicity-one Schur family; origin bank un-fit 9/9; kill test $\leq 3.3 \times 10^{-15}$ at 28/28 against naive controls at 1.1×10^{-5}–1.3×10^{-2}). The relational tick $\tau = +i$ is rigid (S86; retires the S85 "suggestive" caveat; cite as Q11/Q12 — the tick is phase structure, never a frequency). The certified pair is global-phase conjugate *everywhere* (overlap 1 to 2.5×10^{-16}, 8/8); the Ξ determinant-phase observable is blind at the origin ($8 \times \pi/4 = 2\pi$) and demoted to off-origin, where it is pair-common (forced). The A770 universal-weight lemma fixes every Peirce channel weight at 2/9 exactly, making the 7/3 ratio a *conditional trace theorem* at the finite internal layer — its physical (LSZ/S-matrix) promotion remains open, and the $\mathbb{Z}_{12}$ fused-clock reading is dead ($C \notin N(\mathfrak{e}_{7(7)})$; recorded in the junkyard).

6 Fences and language (Rev14)

Binding language fences for everything in this appendix. (1) *Stability is classical.* Every "stable/no tachyon" statement is a quadratic-order statement on the classical potential. The published one-loop result for this vacuum class has $\mathrm{Str}\,\mathcal{M}^8 > 0$ and destabilizes $N{=}0$ Minkowski vacua (arXiv:1307.4389); no claim of quantum stability is made anywhere in the suite. (2) *Spacelike only — stated as a predicate (Rev32.6).* The KK circle is spacelike: the compact orbit's norm stays positive on the carrier. Timelike-torus/timelike-circle language is excluded by that predicate, not by label (PE2; Hull–Julia hep-th/9803239 Table 2 gives the timelike case its own real form, $SU^*(8)$, which this appendix does not identify); the Gödel null circle, $\sinh r = 1$, is the exhibit of the predicate failing. The annulus reading is (spacelike circle) $\times$ (internal twist $\mathfrak{u}(1) \supset \mathbb{Z}_2$). (3) *No chirality gloss.* The $\mathbf{27}/\mathbf{27}'$ of this appendix are Jordan-cubic versus symplectic-pair structures. They are *not* 4D fermion chirality, and no "vector-like bulk versus chiral boundary" physics claim is made; any such mapping requires an explicit fermion-embedding statement that has not been made. The walls of Appendix F stand on their own proofs; the interface picture is consistent with them, it does not re-derive them. (4) *Numerology class.* The $G_7 = \phi + 5\phi^{-5}$ correction and the related tau-'5' bridge are formally classified numerology-class pending derivation (D772/D773, artifact-backed); an attempted kill of the integer 5 via the $\alpha(0)/m_\tau$ error budget does not land (the required coefficient is 4.9635 ± 0.087, containing 5 at 1σ; the Λ_{G_2} rounding convention dominates), so the bridge is scheme-dependent candidate-class — neither derived nor dead. [LIB2-229] (5) *Tick.* τ-rigidity is cited as Q11/Q12; the tick is phase structure, never frequency; clock $\neq$ quanta is banked — no holonomy gloss. (6) *Scope.* No public observable moved anywhere in S82–S88; the gauging-grade CSS equivalence is obstructed for the full gauging (each end gauging and each oriented half admits the twisted reading, §3), and the physical (LSZ) layer of 7/3 remains open; both are stated as such.

Provenance

S82: annulus/shell-straddle ledger (D763 4/4 closed negatives; D764 3/3 blind collision at D761R pedigree; `s82_annulus_straddle`, `s82b_stage_grade`). S83: the extractor stack (su(8) frame via triality-computed 8_s, Schur intertwiner 3.9×10^{-16}, exact-912 dressing; gravitini $1/4 \times 8$ exact — the

A_1 Takagi values in the de Wit–Samtleben–Trigiante normalization, not a rung label; $E \pm M$ phase conjugates at origin). S84: σ-link at norm level ($15/15 \leq 7.1 \times 10^{-15}$, controls fail 15/15). S85: D767 arc (blind replication A767a point-identical 15/15); spectral completion; relational tick $+i$ exact; Ξ origin-blind. S86: PH1 equivariant intertwiner; τ rigid; Q13 global-phase conjugacy; A770 universal weight 2/9; $\mathbb{Z}_{12}$ killed. S87: orientation-odd null scan; KKT joint grading (Theorem 3.1); W4 document pull (Hull–Julia; 1307.4389); SS gauging grade ($F = \mathbb{Z}_2 \ltimes t^{12}$ in-substrate; suite gauging not flat). S88: F-ANNULUS-E6 (Theorem 4.1, `s88a`); D774 interface lemmas cross-certified (`s88b`); D773 five-kill error budget banked. All house artifacts SHA-manifested (`_S83` 15/15, `_S84` 8/8, `_S85` 6/6+caches, `_S86` 35/35, `_S87` 18/18, `_S88` 6/6); external packs content-verified including renamed-file hash audit (S88 sweep).

Citations: F. Catino, G. Dall'Agata, G. Inverso, F. Zwirner, arXiv:1307.4389; C. M. Hull, B. Julia, hep-th/9803239; C. M. Hull, hep-th/9806146; M. Günaydin, K. Koepsell, H. Nicolai, hep-th/0008063; S. Krutelevich, math/0411104.

Leibniz Quantum Beats Newton

Appendix H — The Time Bridge: the Moment Clock, T-Reversal as a Live $\mathfrak{e}_{7(7)}$ Element, and the One Circle (Rev33.2)

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Abstract

This appendix records the post-Rev14 *time layer* (Sessions 106–108) of the certified $E_{7(7)}$ **56/133** substrate programme. (i) *The flicker frame* (S106): the clock normalisation is $q_0^{\mathrm{clk}} = m_e$ (a unit choice inside the one registered dimensionful class — Appendix X input-count box; through Rev32.8 this read "the single dimensionful anchor") (the clock anchor; not the Kaluza–Klein tower quantum $q_0^{\mathrm{KK}} = 1/(2\sqrt{6})$ of Appendix G — the two shared one symbol before Rev32.2), and the substrate clock frequency ω_0 is *calibrated* against the electron Compton/zitterbewegung rate, $m_e = \hbar\omega_0/c^2$, which fixes c and $\hbar$ in terms of ω_0; that calibration is a declared normalisation of the constructed clock and is *not* a claim that the substrate phase circle *is* the electron's zitterbewegung (*Rev29*); ω_0 is a *declared input*, not a new constant, and the admissible-constant set stays $F_3 = \{q_0^{\mathrm{clk}}, \kappa, \Gamma\}$. (ii) *The live time-reversal bridge* (S107): the past↔future operation T is a *genuine* element of $E_{7(7)}$ on the live **56** — the half-turn of the compact generator of the grading $\mathfrak{sl}(2,\mathbb{R})$ of D_{56} — and the live representation *corrects* the earlier abstract model: T is symplectic ($T \in E_7 \subset Sp(56,\mathbb{R})$) and squares to -1 (a spinor half-turn), so the sign that antiunitary time reversal carries in quantum mechanics is *reproduced* here as $T^2 = -1$ rather than as anti-symplecticity; T is a symplectic element of $E_{7(7)}$, not an antiunitary operator on a physical Hilbert space, and no physical Kramers degeneracy is claimed (*Rev29*). T realises the moment half-turn of the constructed clock ($T^4 = +1$ = two moments) and normalises the E_6 structure algebra, giving a genuine, *projective* $32 \to 64$ doubling. (iii) *One circle* (S108): the gauge-side dial moment-circle and the grading circle are $E_{7(7)}$-conjugate by an explicit $g \in E_{7(7)}$, so the dial "moment" and the grading half-turn T are the same operation, and the two **27**s are the dial's future/past windings. (iv) *Moment span vs. thickness* (S108): the moment span is a single shared envelope ($\frac{1}{2}$ future $+\frac{1}{2}$ past), while each *ladder slot* carries its own clock rate and hence its own model proper-time thickness — "thickness" being a quantity of the construction, not an asserted measured property of a physical particle (*Rev29*). A third split-octonion cell invariant accompanies c and $\hbar$: the entropy-like invariant $S_0 = \ln\varphi^2 = 2\ln\varphi$ on the skew J_4, numerically equal to the model's per-tick proper-time rate — one cell, two cones, one bit [s500]; reading S_0 as a *physical* minimal entropy quantum is not asserted (*Rev29*). All results are classical/finite, machine-verifiable statements on the certified substrate; no public observable moves.

Notation (σ) — three senses in this appendix. (i) σ_C is the **throat involution**, the signature-preserving Jordan automorphism that flips the WH–BH neck; it is always written with its subscript. (ii) Where the **Jordan frame involution** $\rho \mapsto 1/\rho$ is meant, it is Appendix I's σ_{frame}, the frame transposition $(1\,3)$. (iii) $\sqrt{\sigma}$ is the **string tension** and is not an involution at all. Appendix F's

time-mirror $\sigma = \Omega$ and the split-octonion σ_{oct} ($e_7 \mapsto -e_7$) are two further distinct objects, named here so that no bare σ in this suite is read as a single map.

1 Scope and provenance

Everything in this appendix is a finite computation on the certified $E_{7(7)}$ substrate of Appendix F (133 generators on the real **56**), obtained in Sessions 106–108 (2026-06-13) under pre-committed gates, with SHA-256–manifested artifacts (`_S106`, `_S107`, `_S108` manifests; house scripts `s106*`, `s107a`, `s108a-e`; replays byte-exact). External inputs are used only in the companion mass discussion (PDG 2026 charged-lepton pole masses, in Appendix H §9 and the p2 Koide block) and are flagged as external. Conventions: the grading element is $D_{56} = \rho_{\mathfrak{su}_8}\,\mathrm{diag}(1,1,1,1,1,1,-3,-3)$ (vacuum-aligned frame; the index-permuted $\mathrm{diag}(-3,-3,1,1,1,1,1,1)$ is $E_{7(7)}$-conjugate to it but *not* interchangeable at a fixed gauging: in the frozen gauging frame the grading aligned with the centre is $\pm D_{56}$, whose KK-vector anchor is $-Z/\sqrt{2}$; Appendix G conventions, A1774, `s1262`); $H = D_{56}/2$; $\kappa = 1/(3\sqrt{2})$ (the substrate central charge, exact); $\varphi = (1+\sqrt{5})/2$. The scope clause of Appendix G applies verbatim: these are classical, finite statements on the certified substrate; no seconds, dynamics, or public-observable claims are made. Section 10 additionally folds the engine-side winding ladder recorded in the workbook tabs `Electron Shell Levels` (S164), `Winding→Mass (forward)` (S166), and `Exact Windings↔Energies` (`s264`/S167), so that the paper and the engine agree; that material is likewise scale-free structure with all absolute energies/masses α-/anchor FENCED and the occupation selector open.

2 The flicker frame (S106)

The substrate carries one dimensionless clock (the occupation/phase circle) and one dimensionful anchor q_0^{clk}. We set $q_0^{\text{clk}} = m_e$ and *calibrate* the clock frequency against the electron Compton/zitterbewegung rate,

$$\omega_0 = \frac{m_e c^2}{\hbar} \approx 7.7634 \times 10^{20}\ \text{rad/s} \qquad (\text{zitterbewegung } 2\omega_0 \approx 1.5527 \times 10^{21}\ \text{rad/s}), \tag{1}$$

so that $m_e = \hbar\omega_0/c^2$. This is option A of `s106i`: it places the electron at depth 0 of the model ladder (the "bare flicker" *slot*) and fixes c and $\hbar$ *within that frame* from ω_0; ω_0 itself is a declared input, not a new constant. The calibration is a normalisation of the constructed clock: nothing here asserts that the substrate phase circle *is* the electron's physical clock, and no measured quantity is derived from it (*Rev29*). The admissible-constant set is unchanged, $F_3 = \{q_0^{\text{clk}}, \kappa, \Gamma\}$. A *moment* is one half-turn of the base clock carrying the spinor sign $g_{\text{half}} = -1$; the identity ($+1$) return takes *two* moments (a full turn).

Fence F1. The rotation is the substrate's own occupation/phase circle. ω_0 is an input frequency anchor; no statement about external seconds or external-time dynamics is made.

3 The grading circle on the live 56 (S107)

$D_{56} \in \mathfrak{e}_{7(7)}$ (residual $< 10^{-9}$) is symplectic, has real spectrum, and gives the three-grading whose level-zero algebra is its centraliser $\mathfrak{e}_{6(6)} \oplus \mathfrak{so}(1,1)$ (dimension 79, trace-form signature $(43, 36)$; A1786)

$$\mathbf{56} = \mathbf{1}_{-6} \oplus \overline{\mathbf{27}}_{-2} \oplus \mathbf{27}_{+2} \oplus \mathbf{1}_{+6}, \tag{2}$$

with eigenvalue multiset $\{-6:1,\ -2:27,\ +2:27,\ +6:1\}$ (gate a). The two level-± 2 spaces are the two clock orientations — the **27** (future) and $\overline{\mathbf{27}}$ (past). $H = D_{56}/2$ has weights $\{\pm 3, \pm 1\}$.

4 T as a genuine $\mathfrak{e}_{7(7)}$ element (S107a)

Build the grading $\mathfrak{sl}(2,\mathbb{R}) \cong \mathfrak{so}(2,1)$ by Jacobson–Morozov *inside* $\mathfrak{e}_{7(7)}$: $\mathfrak{g}_{\pm 4} = \{x \in \mathfrak{e}_{7(7)} : [D_{56}, x] = \pm 4x\}$ are 27-dimensional; pick a regular $E \in \mathfrak{g}_{+4}$ and linear-solve $F \in \mathfrak{g}_{-4}$ with $[E,F] = H$ (residual $\sim 2 \times 10^{-13}$; $[H,E] = 2E$, $[H,F] = -2F$ automatic). The compact generator is $K = E - F \in \mathfrak{e}_{7(7)}$, with purely imaginary spectrum $i \cdot \{\pm 3\,(\times 1), \pm 1\,(\times 27)\}$ — one spin-$\frac{3}{2}$ plus 26 spin-$\frac{1}{2}$ (K is compact, H and $E + E^{\mathsf{T}}$ non-compact: the trace form on $\langle H, E + E^{\mathsf{T}}, K\rangle$ is $72\,\mathrm{diag}(1,1,-1)$, so the grading algebra is the split $\mathfrak{sl}(2,\mathbb{R})$, not a compact $\mathfrak{su}(2)$; `s1252`). The time bridge is the half-turn

$$T = \exp\Big(\tfrac{\pi}{2} K\Big) \in E_{7(7)}, \tag{3}$$

the Weyl reflection of the grading $\mathfrak{sl}(2,\mathbb{R})$: $THT^{-1} = -H$ (residual $\sim 9 \times 10^{-11}$), so T maps level $k \mapsto -k$ and swaps $\mathbf{27} \leftrightarrow \overline{\mathbf{27}}$ and $\mathbf{1}_{+6} \leftrightarrow \mathbf{1}_{-6}$ exactly (leaks $< 10^{-6}$). This is the live past↔future clock swap.

Novelty label: new specialization proved here.

Theorem 4.1 (live T-bridge)**.** *On the certified* **56** *there is a genuine element $T = \exp(\frac{\pi}{2}K) \in E_{7(7)}$, with K the compact generator of the grading $\mathfrak{sl}(2,\mathbb{R})$, such that $THT^{-1} = -H$; T interchanges the future* **27** *and past* $\overline{\mathbf{27}}$ *(and the two singlets) exactly.*

Remark 4.2 (canonicality, S108d)**.** The Jacobson–Morozov build uses a random regular $E \in \mathfrak{g}_{+4}$; different choices give conjugate triples. Across 6 independent seeds the build closes (residual $< 10^{-11}$) and *every* invariant is reproduced — $T^2 = -1$ (worst 1.0×10^{-10}), symplecticity (3.6×10^{-11}), $THT^{-1} = -H$ (2.3×10^{-10}), the $\mathbf{27} \leftrightarrow \overline{\mathbf{27}}$ swap (2.2×10^{-12}), and the fixed spectrum $i \cdot \{\pm 3 \times 1, \pm 1 \times 27\}$. So T is canonical up to conjugacy in the grading-$\mathfrak{sl}(2,\mathbb{R})$ normaliser: "the" time bridge is well-defined, not a seed artifact.

Remark 4.3 (the bridge on the registered gauging, `s1236`)**.** For charge-neutral triples the bridge $T = \exp(\frac{\pi}{2}(E - F))$ swaps the two ends ($TD_{56}T^{-1} = -D_{56}$), keeps the registered charge Z, and carries the dyonic gauging $X(\omega)$ to $\pm X(-\omega)$; $T^4 = -I_8$ on $\mathbb{C}^8$. The generic (charged) triples of this appendix do not. T is not asserted to be a time reversal on states (fence F4), and $\omega \to -\omega$ is a change of presentation of the gauging.

Remark 4.4 (independent machine re-validation (`s548`, S211; Rev25 fold))**.** The T-bridge was later rebuilt from scratch in an independent in-house kernel: $T = \exp(\frac{\pi}{2}K)$ reconstructed with all gates at the $\sim 10^{-13}$ level, reproducing Theorem 4.1 exactly. With the original live build and the external verification (A1303), the bridge now has a three-way convergence record — live construction, hostile-lane check, and independent kernel rebuild — all agreeing on $T^2 = -1$, symplecticity, $THT^{-1} = -H$, and the sector swap. Recorded as validation only; nothing new is claimed.

Remark 4.5 (neutral-signature anti-isometry as a type model (`s1087`, S291))**.** The strong finite model of a past/future exchange as an η-isometry between signature-$(1,3)$ and signature-$(3,1)$ summands is obstructed by Sylvester inertia. A refined model exists in neutral signature: an explicit integral $T_{4,4} \in GL(8,\mathbb{Z})$ satisfies $T_{4,4}^{\mathsf{T}} \eta T_{4,4} = -\eta$, $T_{4,4}^2 = -I$, and $T_{4,4}^4 = I$, exchanges the two summands, and normalizes $\mathfrak{so}(4,4)$. In the census with $p+q=8$, such an anti-isometry exists iff $p=q$. This is a type-admissibility result only. The live time bridge of Theorem 4.1 is instead a symplectic element of $E_{7(7)}$ on the **56**; the two constructions share the order-four, square-minus-one, and sector-swap fingerprint, but they are not identified. (27 computed gates plus one documentary type-note; kernel `s1087`.)

Remark 4.6 (two-full-turn closure of the spinor dial (`s1088`, S291))**.** On the hand-built $\mathbf{27} \oplus \mathbf{27}$ dial carrier, the plain swap has square $+I$ and vector order two, whereas the spinor dial has square $-I$ and vector order four. Thus the spinor dial closes after four half-turn steps, or two full turns, and it alone has the same square/order fingerprint as the live Appendix-H bridge $T^2 = -I$, $T^4 = I$. This is closure arithmetic of the dial model, not a measured width of the electron and not an equality with the live **56** representation. Appendix H's slot thickness remains a construction quantity under its Rev29 fence. (18 computed gates; kernel `s1088`.)

5 What the live representation corrects (S107a)

The abstract model (the earlier $\mathbf{27} \oplus \overline{\mathbf{27}}$ hand-built swap) made two claims the live representation overturns:

(C1) T is *symplectic*: $T \in E_7 \subset Sp(56,\mathbb{R})$, defect $\|T^{\mathsf{T}}\Omega T - \Omega\| \sim 2\times 10^{-11}$. It is *not* anti-symplectic; the earlier "anti-symplectic" reading was a real-swap-model artifact.

(C2) $T^2 = -1$ on the **56** ($\|T^2 + I\| \sim 7 \times 10^{-11}$), the spinorial/Kramers half-turn; $T^4 = +1$. The abstract plain swap had $T^2 = +1$ and missed it.

Structural reading (*Rev29*; constructed layer). The sign that antiunitary time reversal carries in quantum mechanics — the fermionic $T^2 = -1$ of Kramers — is *reproduced* on the substrate by a symplectic half-turn, *not* by anti-symplecticity. That is a structural match, not an identification: $T \in E_{7(7)} \subset Sp(56,\mathbb{R})$ is a linear symplectic element of the certified substrate; it is not an antiunitary operator on a physical Hilbert space; and no physical Kramers degeneracy, no physical time-reversal selection rule, and *no half-angle readout normalization for any observable* follows from $T^2 = -1$ alone. (This last point is load-bearing downstream: Appendix B Lemma B.5′ has been re-typed in Rev29 so that the $\pi/8$ half-angle is a readout choice this moment structure *motivates*, not one it makes canonical.) Within the model, $T^2 = -1$ is exactly the moment half-turn $g_{\text{half}} = -1$ (S106d): T is the moment operator of the constructed clock — one application = one moment, sliding the future sector onto the past; $T^4 = +1$ = two moments.

6 Energy is T-even: the clock generator (S112, Defect-176 correction)

The half-turn $T = \exp(\frac{\pi}{2}K)$ of §4.1 settles a sign question about *which* substrate generator is the energy, and with which time-reversal parity. Two grading generators must not be confused:

- the *compact* clock generator $K = E - F$ (§4), with purely imaginary spectrum $i\cdot\{\pm 3(\times 1), \pm 1(\times 27)\}$, which generates the bounded unitary clock flow $U(\theta) = \exp(\theta K)$ ($|\operatorname{eig} U| = 1$, symplectic); and
- the *non-compact* grading/boost $H = D_{56}/2$, with *real* spectrum $\{\pm 3, \pm 1\}$, which cannot generate a bounded clock.

The clock generator of the construction is the compact K (the model energies are the eigenvalues of the Hermitian $-iK$, bounded); no physical Hamiltonian is being identified here (*Rev29*). Because T is generated by K, it commutes with it, so on the live **56**

$$T\,K\,T^{-1} = +K \qquad (\text{defect } \lesssim 10^{-6}), \tag{4}$$

i.e. the energy is T**-even**. The bookkeeping depends on the linearity of T. With the Hermitian energy $h = -iK$, an *anti*unitary T obeying $T\,K\,T^{-1} = -K$ still preserves the energy, $T\,h\,T^{-1} = h$, because it conjugates i; it is a complex-*linear* T that must commute with K to preserve h. The T here is complex-linear (unitary) on the polarized **56** (`s1255`, A1772), so $T\,K\,T^{-1} = +K$ is the energy-preserving statement for it. The operator that T genuinely reverses is the boost/grading, $T\,H\,T^{-1} = -H$ (Theorem 4.1), as a boost must be — not the energy.

Novelty label: physical interpretation not established here.

Theorem 6.1 (energy is T-even)**.** *On the certified* **56** *the compact clock generator K satisfies $T\,K\,T^{-1} = +K$ (energy T-even), while the non-compact grading $H = D_{56}/2$ satisfies $T\,H\,T^{-1} = -H$ (boost T-odd). The clock generator is K, and $U(\theta) = \exp(\theta K)$ is a bounded unitary flow. K is* not *a positive Hamiltonian on the polarized* **56***: the form ΩK has inertia* $(26, 30)$*, and the clock frequencies on the* **28** *are* -1 *(*$\times 15$*),* $+1$ *(*$\times 12$*),* $+3$ *(*$\times 1$*) (`s1255`); no positive-energy reading follows.*

Remark 6.2 (coalition record: Defect-176)**.** An earlier coalition proposal (the s109-era "Hamiltonian-bridge" model) promoted the grading to the Hamiltonian and imposed $T\,H\,T^{-1} = -H$ as the energy relation. The in-house substrate audit `s112b_hamiltonian_bridge.py` (run; defects $\sim 10^{-6}$) shows that relation belongs to the boost $D_{56}/2$, not to a clock: the energy is the compact K and is T-even. The proposing model subsequently concurred. This is recorded as the convergent correction, not hidden.

Honest scope of the time bridge (S112–S113), re-framed against Appendix I (*Rev29*). A T-even clock is *necessary* for a stable spectrum but is *not* by itself a proof of stable dynamics. Two different potentials must be kept apart; earlier revisions of this paragraph conflated them and drew a dynamical conclusion from the wrong one.
(i) The ungauged invariant probe potential on the $J_3(\mathbb{O}_s)$ slice. There the golden point $J_{\text{vac}} = \text{diag}(\varphi, 1, \varphi^{-1})$ has Hessian signature $(15+, 12-)$, and the 12 negative directions are the negative-norm directions of the split-octonion norm (sig $= (4, 4)$). Probe `s113a_eta_stabilizer.py` (run) establishes that the unique G_2-invariant quadratic on each octonion block *is* that indefinite norm, so *no* G_2/F_4-covariant scalar-potential term can change that signature: a covariant uplift ηN stays $(15+, 12-)$ for all η. Only a G_2-breaking *compact* (Euclidean) uplift reaches positive-definiteness, at the exact value $\eta_c = 2\varphi^2$ (the Appendix-I "$\sim \varphi^2$" figure, in that probe's $\frac{1}{2}$Tr normalisation). All of this is a fact about the *ungauged invariant toy*, and it is exactly what `s113a` certifies — no more.
(ii) The gauged dynamical statement supersedes the "saddle" reading. Appendix I retires the reading of $(15+, 12-)$ as the *dynamical* verdict on J_{vac}. On the certified gauged $E_{7(7)}/SU(8)$ bed at the dyonic point $\omega = \pi/4$ (Appendix I, Theorem "the golden vacuum is a tachyon-free Minkowski critical point at $\omega = \pi/4$"; kernel `s120c_golden_omega.py`) the split directions are organised by the gauging, and the golden point is a *tachyon-free* $V = 0$ Minkowski critical point of Hessian signature $(0_-, 34_0, 36_+)$ (at a 10^{-6} cut, $(0_-, 30_0, 40_+)$: four of the 34 are light massive modes, Appendix I; `s1222`). There is no tachyon at the golden point on the gauged bed. The Rev16/Rev17 "saddle" language, which this appendix carried forward, is withdrawn as a dynamical claim and retained only as the statics of the invariant toy.
(iii) What does not *depend on the withdrawn reading.* The $(4, 4)$ Lorentzian-signature seed is a statement about the split-octonion norm and the uniqueness of the G_2-invariant quadratic on each octonion block; `s113a` certifies it directly, and it was never a consequence of J_{vac} being a saddle. It survives the withdrawal unchanged. Likewise the missing-**16** portal failure (`s112c`) fails because no G_2/F_4-covariant invariant term can supply a definite quadratic on the octonion blocks — again a statement about the invariant family, not about the critical-point type of J_{vac}. Both conclusions stand as printed.
(iv) Status. The time bridge stands as a clock/kinematic result. Dynamical stabilisation is *not* "open" in the sense this paragraph previously printed (an open item awaiting the gauged sector): the gauged sector has supplied a tachyon-free critical point. What remains open is narrower and is named in Appendix I — the lift of the residual σ-even flats, pending the certified field-dependent tensor C_{iab}, plus a separate no-scale dilaton input. Appendix I's own operative status, *stabilization partial (one named open tensor)*, governs; this appendix makes no independent stabilisation claim.

Remark 6.3 (a split-octonion product on the gauging's three real eights)**.** The commutant of the three $\mathfrak{sl}(2, \mathbb{R})$ planes of the registered gauging in $\mathfrak{e}_{7(7)}$ is $\mathfrak{so}(4, 4)$; the **56** holds three real eights, each with an invariant $(4, 4)$ form, and the $\mathfrak{e}_7$ bracket between them obeys the composition law exactly (constant $\frac{1}{4}$): a product isomorphic to the split Cayley–Dickson octonions (isomorphism exhibited), with derivation algebra $\mathfrak{g}_{2(2)}$. The registered plane cycle (order three, preserving the gauging) reduces $\mathfrak{so}(4, 4)$ to $\mathfrak{g}_{2(2)}$ and selects the unital product up to isomorphism; a rank-two sub-torus of the four charges acts by automorphisms (`s1240`, `s1243`, A1766). This is a structure *on* registered objects: the gauging is not derived from it, and no identification with the split-octonion norm blocks above or with Appendix G's σ_{oct} is established.

7 The $32 \to 64$ doubling, honestly (S107a)

The banked maximal commuting set of in-substrate involutions (the F_4-class flips) has order $2^5 = 32$ (S106k). T normalises the E_6 structure algebra: $T\,D_{56}\,T^{-1} = -D_{56}$, so T maps the centraliser centraliser$(D_{56}) = \mathfrak{e}_{6(6)} \oplus \mathfrak{so}(1,1)$ (dimension 79) to itself (residual $\sim 1 \times 10^{-9}$), and $\mathfrak{e}_{6(6)}$ contains $\mathfrak{f}_4$ and those 5 flips. Hence adjoining T genuinely *doubles* the flip group, $32 \to 64$. Because $T^2 = -1$ is central, the doubling is *projective*: $\langle T \rangle = \mathbb{Z}_4$ (a spinor line), so the strict $(\mathbb{Z}_2)^6 = 64$ of the abstract model becomes a $\mathbb{Z}_4$-refined statement (the sixth/time line is a spinor, not a plain $\mathbb{Z}_2$).

Provenance note (honest scope of §6). The $32 \to 64$ statement here rests on two certified facts: (i) T normalises $\mathfrak{e}_{6(6)}$ on the live **56** (`s107a` gate f, residual $\sim 10^{-9}$), and (ii) the in-substrate involutions have 2-rank 5 ($= 32$), established on the **27** in `s106k`. An *explicit* live-**56** enumeration of the five commuting F_4-class flips with T adjoined — verifying $\langle \text{flips}, T \rangle$ has order 64 directly — has not yet been run; it is recorded as future work. The doubling is therefore stated via normalisation plus the cited 2-rank, and the $\mathbb{Z}_4$ (rather than $\mathbb{Z}_2$) character of the new line is the load-bearing refinement.

8 One circle: the dial *is* the grading circle (S108b)

The gauge-side dial moment-circle $C_n = [\mu_1, \mu_2]/\kappa$ ($\kappa = 1/(3\sqrt{2})$, Sessions 101d/104a) and the grading circle K of §4 have the *identical* spectrum on the **56**, $i \cdot \{\pm 3\,(\times 1), \pm 1\,(\times 27)\}$ (with C_n normalised by $\kappa/2$): the same compact-circle content (one spin-$\frac{3}{2}$ and 26 spin-$\frac{1}{2}$ weights). They are conjugate inside the group — an explicit $g \in E_{7(7)}$ (built by a Lie-algebra Newton flow, every step the exponential of a genuine $\mathfrak{e}_{7(7)}$ element; symplectic defect $\sim 1.7 \times 10^{-11}$, $\det g = +1$) satisfies

$$g\,C_n^{\text{norm}}\,g^{-1} = K \qquad (\text{residual } \sim 8.9 \times 10^{-10}). \tag{5}$$

Consequently g intertwines the whole circles, half-turns included: $g\,(\text{dial half-turn})\,g^{-1} = T$ (residual $\sim 7.2 \times 10^{-9}$), and the half-turn squares to -1 on both. In the grading frame one dial moment carries the future **27** onto the past $\overline{\mathbf{27}}$ (leak $\sim 7 \times 10^{-14}$).

Novelty label: physical interpretation not established here.

Theorem 8.1 (one clock)**.** *The dial moment-circle and the grading circle are $E_{7(7)}$-conjugate; hence the dial "moment" and the time-bridge T are the same half-turn, and the two* **27***s of the live* **56** *are the dial's future and past windings. No new constant is introduced.*

This *settles* the two-**27**s relation: the moment-dial clock and the future/past grading clock are one circle in two frames.

9 Moment span versus per-particle thickness (S108c)

The moment *span* is shared: T sweeps π of the 2π grading circle (span fraction exactly 0.5) and carries the future **27** onto the past $\overline{\mathbf{27}}$, so one moment $= \frac{1}{2}$ future $+\frac{1}{2}$ past, with $T^2 = -1$ the universal "tick" — a single envelope for all 56 states. (The companion "past→past, future→future" four-tick return, $T^4 = +1$, is the -1 holonomy T accrues on reflection at the Det $= 0$ wall of

Appendix I; an external hostile review (S175, D905) confirms this reading is sound *classical* holonomy — a structural clarification of the turnover, not a source of any coupling.) The *thickness* is per-particle, at two layers:

(a) *Grading rate.* Under $\exp(\theta K)$ a state's phase advances at rate $|H\text{-eigenvalue}| = |\text{level}|/2$, so the singlets (level ± 6, rate 3) tick three times as fast as the **27** matter (level ± 2, rate 1) — a 3:1 ratio, not one shared rate.

(b) *Compton rate.* Each fermion *slot* of the ladder is assigned $\omega_f = \omega_0\, \varphi^{n_f}$ with $n_f = \log_\varphi(m_f/m_e)$, so its model proper-time thickness is $\tau_f = 1/\omega_f = (1/\omega_0)\, \varphi^{-n_f}$. Heavier $\Rightarrow$ thinner/faster; e.g. the muon $\sim 1/199$ and the tau $\sim 1/3477$ of the electron thickness; within one electron-moment the slot completes φ^{n_f} internal turns. These are quantities of the construction, computed from external PDG masses; no proper time of a physical particle is being predicted (*Rev29*).

Net: the moment is the shared frame ($\frac{1}{2} + \frac{1}{2}$); the thickness is the particle's own (grading 3:1 times golden depth φ^n). The PDG pole masses entering the muon/tau thickness figures are external inputs.

10 Engine-side capture: the winding ladder and electron-shell closure (Track-1, S164/S166/S167)

The per-particle thickness $\tau_f = (1/\omega_0)\, \varphi^{-n_f}$ of §9 is one face of a single integer ladder that the engine workbook records in three tabs (`Electron Shell Levels`, S164; `Winding→Mass (forward)`, S166; `Exact Windings↔Energies`, `s264`/S167) but that no earlier tex captured. This section folds that content so the paper and the engine agree. All of it is scale-free structure on the certified substrate; the one dimensionful ingredient (an absolute energy or mass) is, as of Rev25, precisely the framework's single *registered* input — the depth-cylinder Bargmann class $L_{\text{depth}}/L_\sigma$, registered with value and theorem chain in Appendix X (consistency: one dimensionful contact point here, one registered input there; they are the same object read in different frames) — and the occupation selector is DERIVED-CONDITIONAL (Appendix M, Rev25 section, $\{31, 39\}$ live falsifier). Nothing here moves a public observable.

(a) Electron-shell closure: the level integer is an orbital winding

On the hydrogen Bohr ladder the de Broglie wavelength $\lambda_n = h/(m_e v_n)$ with $v_n/c = \alpha/n$ obeys, around the n-th shell circumference $C_n = 2\pi r_n$,

$$\#\text{de Broglie wavelengths around shell } n \;=\; \frac{C_n}{\lambda_n} \;=\; n \qquad \text{exactly,} \tag{6}$$

the scale-free residue of the identity $a_0 = \lambda_C/2\pi\alpha$ (CODATA-rounding residual 1.4×10^{-9}). Thus the shell integer n is an *orbital winding number* — how many times the internal phase closes around the orbit — and is firewall-safe: the *ratio* $E_n/E_1 = 1/n^2$ is scale-free, while the absolute E_n (eV) needs α and is FENCED. This is the same closure mechanism as the ± 30 four-part doubling (`s249`) and the golden rungs $u_n = n \ln\varphi$ (`s251`).

(b) The golden-ray boost rungs and the AX1 vacuum

Write the boost frame $J(u) = \mathrm{diag}(e^u, 1, e^{-u})$. The rungs sit at rapidities $u_n = n \ln \varphi$, so $e^{u_n} = \varphi^n$ exactly, and the golden vacuum $J_{\mathrm{vac}} = \mathrm{diag}(\varphi, 1, \varphi^{-1})$ is the rung-1 boost. The single load-bearing identity is the relativistic-boost reading of the golden axiom (`s251`):

$$\cosh(2 \ln \varphi) = \tfrac{3}{2} \quad \Longleftrightarrow \quad \varphi^2 + \varphi^{-2} = 3 \ \text{(the DET-7 condition; equivalent to AX1 for } \rho > 1), \tag{7}$$

i.e. the golden vacuum is exactly the rapidity at which the boost's cosh equals 3/2; "harmony of phases" is this boost, with $v_\phi \, v_g = c^2$ and the rest clock $H_{\mathrm{rest}} = \omega_0 \, P_{\mathrm{code}}$ precessing at ω_0 (its bilinear runs at the zitter rate $2\omega_0$ *inside the model*; no laboratory observable is asserted, *Rev29*; `s252`). No new constant enters: φ is AX1.

n	$u_n = n \ln \varphi$	+mode φ^n	−mode φ^{-n}	product (det)	orbital winding
1	0.48121	1.618034	0.618034	1	1
2	0.96242	2.618034	0.381966	1	2
3	1.44364	4.236068	0.236068	1	3
4	1.92485	6.854102	0.145898	1	4
5	2.40606	11.090170	0.090170	1	5
6	2.88727	17.944272	0.055728	1	6
7	3.36848	29.034442	0.034442	1	7

Table 1: Exact windings↔energies (`s264`): the two null modes of the golden-ray boost $J(u)$ carry $+u$ (explosion) and $-u$ (implosion); $\det = 1$ locks them equal-and-opposite (product $\varphi^n \varphi^{-n} = 1$), and T swaps them.

(c) Rest mass is a T-conjugation invariant

The two columns $\varphi^{\pm n}$ are the two metric-signature null modes of $J(u)$; $\det = 1$ keeps them synchronised and the live half-turn T (Theorem 4.1) maps one onto the other. Because $T \in Sp(56, \mathbb{R})$ with $T^2 = -1$ is a *conjugation*, the loaded operator's eigenvalues are invariant under it.

Novelty label: physical interpretation not established here.

Theorem 10.1 (the rest-mass *rung* is invariant under winding, `s264`)**.** *In this construction the rest-mass rung is carried by the T-conjugation–invariant spectral eigenvalue of the loaded clock operator (*Rev29*: a representation of rest mass inside the model, not an identification of physical rest mass with a substrate eigenvalue). The orbital winding n (number of de Broglie waves closing the orbit) and the forward/backward density may vary across shells, but the rest-mass rung (beat-depth) is fixed — for the electron it is the anchor $n_f = 0$ at every shell. "Wound up more/less" is orbital n, not the rest-mass rung.*

The elementary winding cell is the four-element set $\{\pm\} \times \{\varphi, \varphi^{-1}\}$ read off the $\mathbb{Z}/60$ closure dial as golden harmonics $2\cos(2\pi k/60)$: the combinations $(+, \varphi), (+, \varphi^{-1}), (-, \varphi^{-1}), (-, \varphi)$ sit at $k = 6, 12, 18, 24$ with values 1.618034, 0.618034, −0.618034, −1.618034. The $\det = 1$ sync collapses the four lattice combos to two T-conjugate pairs, $[(+, \varphi) \sim (-, \varphi^{-1})]$ and $[(+, \varphi^{-1}) \sim (-, \varphi)]$.

(d) The forward winding→mass map (descriptive; selector open)

Read forward, the same ladder is the one-parameter-free mass estimate $m_{\text{pred}} = m_e\,\varphi^N$, using only φ, the electron anchor m_e, and an *integer* winding N. The fit is clean for leptons and MS-bar-fuzzy for quarks (Table 2); the beat-depth $n_f = \log_\varphi(m/m_e)$ is read backward from PDG and is *descriptive*. Crucially, the integer winding N — equivalently which rungs $\{0, 3, 11, 17, \dots\}$ are occupied — is the **selector**: it is not forced *here*; its status since Rev25 is DERIVED-CONDITIONAL (Appendix M, three-layer selector; live falsifier $\{31, 39\}$ — one status, stated Rev32.9). Electron 0, muon 11, tau 17 are near-integer "clean" rungs; the up quark sits alone at $N = 3$; the remaining quarks carry genuine MS-bar tension.

particle	q	m_{PDG} (MeV)	$n_f = \log_\varphi \frac{m}{m_e}$	N [open]	$m_{\text{pred}} = m_e\varphi^N$	residual %
electron e^-	-1	0.51100	0.00	0	0.51100	0.0
muon μ^-	-1	105.658	11.08	11	101.691	-3.75
tau τ^-	-1	1776.93	16.94	17	1824.78	$+2.69$
up u	$+\frac{2}{3}$	2.16	3.00	3	2.165	$+0.21$
down d	$-\frac{1}{3}$	4.70	4.61	5	5.667	$+20.58$
strange s	$-\frac{1}{3}$	93.5	10.83	11	101.691	$+8.76$
charm c	$+\frac{2}{3}$	1272.9	16.25	16	1127.77	-11.40
bottom b	$-\frac{1}{3}$	4186.0	18.73	19	4777.33	$+14.13$
top t	$+\frac{2}{3}$	172600	26.45	26	138707	-19.64

Table 2: Forward winding→mass (S166), with the Rev32.2 comparator refresh. Charge q is the raw (DIAL-1) winding; mass is golden beat-depth. The heavy-quark inputs are PDG 2026 ($m_c(m_c)$ and $m_b(m_b)$ in $\overline{\text{MS}}$, and the top direct/MC pole proxy), while m_{pred} is the winding-model value. The displayed residuals are signed percentage offsets, not same-scheme tensions; N remains the OPEN selector.

Reconciliation (Track-1↔Track-2, `s247`–`s254`). The charge-spurion is orthogonal to the diagonal involution (`s247`); the de Broglie clock $H_{\text{rest}} = \omega_0 P_{\text{code}}$ and the golden-ray boost $\cosh(2\ln\varphi) = 3/2$ are the kinematic content of the electron-shell tab (`s251`/`s252`/`s254`). What this section does *not* claim: no absolute mass or energy is derived (all α-/anchor FENCED, A962); the integer winding/occupation selector is OPEN (`s263`); the confinement string tension $\sqrt{\sigma}$ and related hadronic numbers are out of scope and live fenced in Paper 8. Engine = paper after this fold; no public observable moves.

11 The golden bridge: the braid word's symplectic square root (Rev31; S276, A1512–A1513)

The winding ladder of §10 carried its golden content numerically; this section records its algebraic root, proved in the post-closure clock round (A1512 ASK-3, house-verified `s1047`–`s1050`) and placed here because it lives *next to* the `s264` winding table: it is the same two-mode null-pair sector, now produced inside the frozen braid representation with **no numerical φ input anywhere**.

Novelty label: new specialization proved here.

Theorem 11.1 (the golden bridge, A1512)**.** *Within the frozen* 56*-frame let* $W_h = Q + \frac{3}{2}P + \frac{3}{2}\gamma_1 - \gamma_2$

be the braid hyperbolic word and set $A = W_h - Q$*, so that* $A^2 - 3A + P = 0$*. [LIB2-015] Then*

$$H_\phi \;=\; Q + \frac{A+P}{\sqrt{5}}$$

is an exact symplectic unimodular square root *of the braid word,* $H_\phi^2 = W_h$*, with spectrum*

$$\operatorname{spec} H_\phi \;=\; \varphi^{-1\,(12)} \;\oplus\; 1^{(32)} \;\oplus\; \varphi^{+1\,(12)},$$

boost generator $G^2 = P$*, Lorentzian form* $\eta = \Omega G$ *of inertia* $(12_+, 12_-, 32_0)$*, and a swap operator* T_ϕ *with* $T_\phi^2 = -I$ *exchanging* $n \leftrightarrow -n$*. The twelve* $\pm$ *pairs are* ***golden-ray winding doublets****: the algebraic realization, inside the frozen braid representation, of the* `s264` *two-mode null-pair sector with its* det *lock* $\varphi^n\varphi^{-n} = 1$*.*

Remark 11.2 (the golden bridge's class)**.** $H_\phi = \exp(\ln\varphi \cdot G)$ with $G = (3\gamma_1 - 2\gamma_2)/\sqrt{5} \in \mathfrak{e}_7$ of the frozen frame, and $H_\phi^2 = W_h$ exactly (A1785). Over $\mathbb{C}$, G is $E_7(\mathbb{C})$-conjugate to iZ and to each iT_a (conjugators exhibited); over $\mathbb{R}$ it is not $E_{7(7)}$-conjugate to them (hyperbolic against elliptic). The half-step T_ϕ is symplectic but not in E_7, and "unimodular" here means determinant one: H_ϕ preserves no integral lattice ($\operatorname{tr} H_\phi = 32 + 12\sqrt{5}$; `s1251`, A1769).

The $\sqrt{5}$ is not inserted: it is the discriminant of $A^2 - 3A + P = 0$, i.e. of the trace-3 identity $\varphi^2 + \varphi^{-2} = 3$ (the DET-7 condition, equivalent to AX1 for $\rho > 1$) that Appendix E obtains from its conditional selector theorem, in which DET-7 is a structural postulate (Rev32.9). The ladder's golden rungs and the braid grammar are therefore *one* object: iterating the bridge is the ladder, and the half-step (the spinor double cover) is the φ^n column of the `s264` table.

Fence (carried verbatim from the round; binding). The occupation rule, the middle Peirce mode, the physical clock, and the energy spectrum are **not** derived here. *And the occupation fence is now at theorem grade (A1513):* the grammar-natural braid occupation class is CLOSED NEGATIVE — the algebra acts *identically* on all 12 active doublets (commutant dimension 144) and on all 16 middle pairs (256); only whole multiplicity blocks are selectable. An occupation law therefore *requires* a Hamiltonian- or boundary-typed selector; nothing internal to the braid grammar can pick individual rungs. This fence travels with the bridge, never separately: the integer winding N of Table 2 remains the OPEN selector, exactly as before this fold. No public observable moves.

12 The two-sheet wormhole–black-hole throat (S230–S233)

The split-octonion algebra $\mathbb{O}_s$ carries *both* quaternion types: a division $(4,0)$ and a split $(2,2)$ subalgebra, giving two (1+3) sheets that share a common 3-plane and are exchanged by the Möbius $\mathbb{Z}_2$ (the second lattice). The physical metric is then *forced per real-form sheet*: the compact-$\mathbb{H}$ sheet selects the determinant (spin-factor) metric of signature $(1,3)$, while the split-$\mathbb{H}'$ sheet selects the Koecher–Vinberg Hessian metric of signature $(3,1)$. That these two intrinsic metrics select *opposite* $\mathbb{C}$-lines is the honest obstruction, named rather than hidden.

The whole object is a $\mathbb{Z}_2 \times \mathbb{Z}_2$: the throat involution σ_C (a signature-*preserving* Jordan automorphism, the WH–BH neck flip) and the real-form involution W (the signature-*changing* sheet flip) are distinct axes. This $\mathbb{Z}_2 \times \mathbb{Z}_2$ is not the $\{O, C\}$ of the registered gauging: from its own definitions σ_C has

signature $(11, 16)$ and the sheet involution $(18, 9)$, the latter is not a similitude, and neither is conjugate to any element of the gauging's involution table (signature $(12, 15)$; `s1237`). The real-form datum of the gauging is $\eta = \exp(\pi Z)$, a half-turn of the registered charge. The associated Wick rotation is the real-form flip $E_{7(-25)} \leftrightarrow E_{7(7)}$, i.e. $SO(8) \leftrightarrow SO(4,4)$ (this corrects an earlier $SO(6,2)$ reading of the *throat* axis; the $SO(6,2)$ of the gauge-boundary shadow selection in Appendix F is a *separate* object and stands). No public observable moves; this is structural geometry at conditional tier.

13 Fences

F1. The rotation is the substrate clock (occupation/phase); ω_0 is an input frequency anchor; no seconds and no external-time dynamics are asserted.

F2. No Standard-Model particle is identified; the $\mathbf{27}/\overline{\mathbf{27}}$ are substrate sectors.

F3. No new constant: π, φ, $\kappa = 1/(3\sqrt{2})$, the grading, and ω_0 ($= q_0^{\mathrm{clk}}$ in the Compton frame) are all intrinsic or already-declared inputs.

F4. **Interface fence (*Rev29*).** Every object in this appendix — the clock circle, the moment, the half-turn T, the thickness τ_f, the winding rungs — is a *constructed-layer* object on the certified substrate. Constructed-layer objects are not asserted to *be* physical-layer objects: T *models* time reversal, the flicker frame *calibrates* against the electron Compton rate, and a thickness *represents* a proper time. Sentences of the form "X *is* the physical Y" are retired throughout; where they survived earlier revisions they have been re-typed rather than deleted, and every number they carried is unchanged.

14 A clock candidate, killed: the $\mathcal{D}_J$ operator carries no golden content (Rev30, A1479/A1480)

An operator $\mathcal{D}_J$ built from $\log P(J)$ was proposed as a golden clock and adjudicated directly against the fences of this appendix. It fails, and the reason is worth printing because it is a normalisation artifact rather than a subtle dynamical obstruction.

The computation. For $J = t_1 e_1 + t_2 e_2 + t_3 e_3$ with all $t_i > 0$, the operator $\log P(J)$ has eigenvalues $\log t_i + \log t_j$ — multiplicity 1 for $i = j$ and 8 for $i < j$. So $\log P(J)$ is **linear in** $\log t$: it is the grading derivation of the frame and nothing more. In closed form $\mathcal{D}_J = L_{D_\varphi}$ with $D_\varphi = \mathrm{diag}(1, 0, -1)$.

The kill. Normalising $J = t\, e_1 + e_2 + t^{-1} e_3$ by $2 \log t$ makes the t-dependence cancel identically. φ **does not survive its own normalisation.** $\mathcal{D}_J$ is the Peirce grading operator of the frame, carrying *zero golden content and zero vacuum content*: it cannot distinguish J_{vac} from any other regular element sharing that frame and ordering. A companion transfer-operator reading collapses the same way — the associated $\mathbb{T}_J$ is exactly diagonal, so its "theorems" (strict positivity, reflection positivity, self-adjointness, Gram exactness) are *identities for a positive diagonal matrix* and not theorems at all. All three objects offered are affine in the Peirce grading; they are one result presented three times.

What this does and does not disturb. The time fences of this appendix are untouched and the adjudication was made against them: the tick remains *phase structure*, never frequency or time, and the B2-time no-go stands. Nothing here reopens a dynamical reading. **One caution on provenance:** the first refutation was made by type analysis — argument, not measurement — because no banked clock matrix could be located on disk; it was subsequently confirmed by direct computation of the closed form above, and it is the confirmation, not the type argument alone, that carries the result. Nothing in this section promotes an observable, moves a tier, or touches the engine.

15 A second clock candidate, killed: the dilaton / $SO(1,1)$ grading clock (S283–S284, D737c)

The $\mathcal{D}_J$ kill above has a sibling, adjudicated in the same programme of work against the same fences, and it fails for three independent reasons rather than one. It is printed here for the first time; the result has stood in the house census since S284 and reached no paper until Rev33.0.

The candidate. The dilaton direction of the $D = 5 \to D = 4$ reduction, with its $SO(1,1)$ grading, was offered as the generator of a physical clock — the most natural candidate on the list, because the grading already flows.

The kill, in three parts, any one of which is sufficient.

1. **The candidate quadratic form is sign-indefinite**, with inertia $(28, 28, 0)$. It is therefore not a positive generator and cannot serve as a Hamiltonian on a physical state space: what it generates is the noncompact grading overlay, not a reduced positive flow.
2. **It is decoupled from the mass proxies**, so even granted a flow it would not move the objects this suite reads.
3. **It presupposes its own conclusion.** The candidate is born inside the $D = 5 \to D = 4$ spacetime reduction — that is, inside the very structure it was asked to *derive*. A clock recovered from a reduction that already assumes spacetime has derived nothing.

Type. The route is **closed as a derivation**. Its geometric and intuitive use survives at *interpretation tier only*, and nothing in this suite may cite it above that.

Why this kill is printed, and what it does not disturb. The fences of this appendix are untouched: the tick remains phase structure, the B2-time no-go stands, and no observable moves, no tier moves, and the engine is not touched. The point of printing a closed negative is that the closure is a *result* — the most natural clock candidate in the reduction is dead, and dead for a reason that generalises: **a grading is not a clock**, and neither is a coordinate. The full standing requirement for any future promotion — a positive reduced generator, a relational clock observable, covariance, a continuum reconstruction, unit registration, a readout theorem, and compatibility controls — is recorded in the house census (`Trackers/PHYSICAL_TIME_WALL_CENSUS_S284.md`); none of the eight is supplied by the parent Legendre transform, the compact K algebra, the page picture, the AdS seed or the finite OS control, alone or together. **Provenance, stated because the delay is part of the record.** The kill was derived and typed at S283–S284 and recorded in a tracker; it reached no component of this suite for sixty-nine sessions, and was recovered by the S264–S352 master sweep at S353 (`Trackers/LOST_SHEEP_MASTER_SWEEP_S353.md`, row S353-12). Its same-session twin, the continuum census, was cited by path in Appendix E at the time. **One reached the papers and one did not, and nothing in the house noticed the difference for sixty-nine sessions.**

16 Interior consolidation (Sessions 89–105)

The time layer sits atop a certified interior built in Sessions 89–105: the exact dial $\mathfrak{su}(2,\mathbb{R})$ algebra with $\kappa = 1/(3\sqrt{2})$ (S101d), the winding-lattice identity $\exp(T\cdot C_n) = 1$ (S102b), the clock form $\frac{\sqrt{3}}{2}(B-L) + \frac{1}{2}\hat{L}$ (S102e), the Hilbert/occupation completion (S104a), and the shadow-stratum centraliser (S100h). These are recorded in the `_S89`–`_S105` manifests and are summarised here rather than reproduced.

Provenance footer. `_S106_manifest`, `_S107_manifest` (10/10), and `_S108_manifest` (10/10) all verify; replays byte-exact; `state_check` green; zero tex was written during Sessions 106–108 — this appendix is the first paper capture of the time layer. Posture: Definition A (derivation program in progress); the time layer is structural and moves no public observable.

Leibniz Quantum Beats Newton

Appendix I — Dynamics of the Golden Vacuum: the Derived Dynamical Selection on the $E_{7(7)}$ Substrate, the σ-Odd Defect Mechanism, the Geometric Bounce, and the $E_{7(7)}$ Clock (Rev33.2)

Derived σ-odd-defect selection (modulo the AX1 unit); Coleman–Weinberg cannot uniquely select golden in the tested σ-equivariant layers; stabilization partial (one named open tensor). Definition A — no public observable moves.

Tom O'Sieg

August 2026

Abstract

The three external hostile reviews of the Rev14/Rev15 suite converged on one gap: the framework had no *dynamics* — it presented a fixed algebraic correspondence map with the golden vacuum $J_{\text{vac}} = \text{diag}(\phi, 1, \phi^{-1})$ inserted by hand, inviting the "numerology" charge. This appendix folds the in-house dynamics layer (S120–S124) that answers the gap at the level it can now honestly reach, on the certified $E_{7(7)}$ **56/133** substrate. The headline is a *derived dynamical vacuum*: scanning the dyonic ω-deformation $\Theta(\omega)$ of the shadow-selected $SO(6,2)$ gauging ($= SO^*(8)$), the electric runaway present at the origin cancels against its magnetic completion at exactly $\omega = \pi/4$, where the golden vacuum J_{vac} is a *tachyon-free Minkowski* ($V = 0$) critical point with Hessian signature $(0_-, 34_0, 36_+)$ (§1; this *supersedes* the earlier compact-slice "saddle" reading, which is now understood as the statics of the ungauged invariant potential). On this dynamical bed the *selection* of ϕ is then *derived*, modulo one named unit: the one-loop Coleman–Weinberg (CW) supertrace $\text{Str}\, M^4$ is *σ-even* under the Jordan involution $\sigma : \rho \mapsto 1/\rho$ — bosonic (§2) *and* indexed-fermionic, the latter *bein-independent* — so CW is extremized at the symmetric point $\rho = 1$, *never* at golden, in every tested σ-equivariant layer; the *σ-odd* unit defect-source B is the sole selector, with its defect equation $F(\rho) = \rho - \rho^{-1} = 1$ *identical* to the golden axiom AX1 $\rho^2 - \rho - 1 = 0$. A BPS / source-completion analysis (§3) derives the direction of B (the σ-odd grading K) and the completion $+\frac{1}{2}\|B\|^2$ (a manifest-square constant; the bare linear spurion alone gives anti-de Sitter $V = -\frac{1}{2}$), with a physical radial mode of curvature $\gamma^2 + 4$, i.e. $m^2 = 7/4$ at the unit point; the unit magnitude itself is *not* independently BPS-forced — it is equivalent to AX1 — so selection is derived *modulo the AX1 unit* (the framework's core axiom), not "from nothing." Stabilization is *partial*, and is now stated on the certified spectrum. At the golden point the 34 near-flat directions of the tree Hessian are 24 gauge Goldstones, 6 strict moduli and 4 quasi-flat light modes (`s1222`, reproducing the S220 adjudication); the strict moduli span an exactly flat Minkowski branch on which the golden point is a non-isolated (Morse–Bott) tree-level minimum, joined at zero cost by the $V \equiv 0$ golden ray to the $SO(6,2)$ origin, which is not a tree-level minimum (`s1283`, `s1361`, `s1249`). At one loop the Coleman–Weinberg inertia on the ten near-flat directions is $(9, 0, 1)$ — five strict moduli and all four quasi modes lifted, one flat direction left (Appendix Q, Gate H) — but the golden point is not one-loop stationary and lies above the origin on the ray (`s1228`); no source-free vacuum is certified. The generation count appears as a clean Morse index $\chi(\mathbb{O}P^2) = \sum(-1)^{\text{idx}} = 3$ over the cells $\{0, 8, 16\}$ (§5; a correspondence with the input N_g, not a derivation; the generation↔cell

assignment is the non-finite A1 datum, fenced open). The earlier probe layers are retained: the overdetermined invariant routes to ϕ (§8), the classical geometric bounce (§9), the $E_{7(7)}$ clock connection with $T^2 = -1$ (§10), and the moment–annulus synthesis (§11). Everything is fenced and Definition A: no public observable moves, and the dimensionful mass scale remains an INPUT (the Honest Boundary).

Binding fence (read first). This appendix folds a *derived selection* result, not a completed dynamics. Concretely: (a) the dynamical vacuum (§1) is computed on the certified $N = 8$ $E_{7(7)}/SU(8)$ potential at the dyonic point $\omega = \pi/4$ — real and tachyon-free — but the transverse **54** runaway of the *non-compact* gauging is a generic property of $SO(6,2)$, present already at the origin, fenced separately from golden; (b) the *selection* of ϕ is derived *modulo the AX1 unit*: the direction of the defect source and its completion are derived, the *unit magnitude* $\gamma = 1$ is equivalent to AX1 $\rho^2 - \rho - 1 = 0$ (the framework's banked golden axiom), *not* derived here from nothing; (c) *stabilization is partial* — the certified one-loop inertia on the ten near-flat directions is $(9, 0, 1)$ (Appendix Q, Gate H), the golden point is not one-loop stationary, and no source-free vacuum is certified (§4); (d) the bounce is a *classical* metric-completeness statement; (e) the dimensionful mass scale is not derived (Appendix X, Honest Boundary). Nothing here is PROVED-tier and no public observable moves. The open next computation is a certified one-loop stationary-point classification on the flat branch (§4, §14).

Label convention. The **Theorem/Proposition/Lemma** environments in this appendix denote *certified in-house computations* — a Definition-A *certified-computation* tier: machine-verified on the certified $E_{7(7)}$ substrate and reproducible from the bundled kernels — *not* PROVED-tier analytic theorems in the suite's claim-language sense. No claim here is PROVED-tier.

Notation (σ) — four senses meet in this appendix, and this is the one that matters most. Throughout, σ **means the Jordan frame involution** $\sigma_{\text{frame}} : \rho \mapsto 1/\rho$, the Peirce-frame transposition $(1\,3)$ with odd grading $K = \text{diag}(-1, 0, 1)$ — and in particular *every* use of σ*-even* and σ*-odd* below refers to it. Three other objects share the letter and are *not* it: Appendix H's throat involution σ_C; Appendix F's SO$^*(8)$ time-mirror $\sigma = \Omega$; and the split-octonion conjugation σ_{oct} ($e_7 \mapsto -e_7$), the only registered up/down-type exchange, which is computed in the kernels and *never appears in the typeset suite*. A fourth, $\sqrt{\sigma}$, is the string tension and not an involution. $\sigma_{\text{frame}} \neq \sigma_{\text{oct}}$ is computed, not assumed (house kernel `s1183`).

1 The derived dynamical vacuum: the dyonic $\omega = \pi/4$ point

The first dynamical content is a vacuum that is *selected by a potential*, not posited. We work on the certified $N = 8$ gauged-supergravity bed: the shadow-selection analysis of Appendix F fixes the gauge group to the non-compact $SO(6,2)$ (the unique stable completion of the missing-**16**), and the golden direction sits in the scalar **27** of the E_6 branching $\mathbf{70} = \mathbf{42} \oplus \mathbf{27} \oplus \mathbf{1}$ (kernel `s120a`, 13/13; the three primitive idempotents of the Peirce frame realize $\chi(\mathbb{O}P^2) = 3$ explicitly). Evaluating the certified Dall'Agata–Inverso $N = 8$ potential at the certified golden vacuum reproduces the published origin values $\{SO(8) : -6,\ SO(6,2) : 0,\ SO(4,4) : +2\}$ and the $SO(6,2)$ shadow tadpole; the golden Jordan-**27** direction is an *exactly flat, $V = 0$ Minkowski valley* of $SO(6,2)$ (ray scan $V \equiv 0$ to 2×10^{-15}, $\mathrm{d}V/\mathrm{d}t \equiv 0$ to 10^{-11}), unique to that gauging (kernel `s120b`, sha `88c010a6`).

Novelty label: new specialization proved here.

Theorem 1.1 (the golden vacuum is a tachyon-free Minkowski critical point at $\omega = \pi/4$)**.** *Let* $\Theta(\omega) = \cos\omega\, \mathfrak{so}_e(\eta) + \sin\omega\, \mathfrak{so}_{mA}(\eta)$ *be the dyonic deformation of the* $SO(6,2)$ *embedding tensor* $(= SO^*(8))$*, consistent for all* ω*. The golden gradient is symmetric in* ω *and vanishes* exactly *at* $\omega = \pi/4$*: the magnetic completion cancels the electric* **54** *runaway. At* $\omega = \pi/4$ *the exact golden vacuum* $J_{\rm vac} = \mathrm{diag}(\phi, 1, \phi^{-1})$ *is a critical point in all three orientations (*$\|\nabla V\| \sim 3 \times 10^{-11}$*, portal tadpole* $\sim 10^{-11}$*,* $V = 0$ *Minkowski) with Hessian signature*

$$(0_-,\, 34_0,\, 36_+) \qquad \text{— } \textit{tachyon-free.}$$

(Kernel `s120c_golden_omega.py`*,* sha `54b2be21`*, reproducible; cross-checked against the external probe* `d806` *modulo a grading-sign and a factor-2* η *normalization; literature control Warner 1983 / Dall'Agata–Inverso confirms the non-compact origin runaway and the* $V \equiv 0$ *no-scale ray. A887. At a* 10^{-6} *cut the signature is* $(0_-, 30_0, 40_+)$*:* 30 *exact zeros —* 24 *gauge Goldstones and* 6 *strict moduli — and* 4 *quasi-flat light modes of mass*^{2} 3.4×10^{-5} *and* 9.9×10^{-5} *in these units, which the* 10^{-4} *cut of* `s121a` *counted as flat;* `s1222`*.)*

This is the first vacuum in the suite that is a genuine *critical point of a certified dynamical potential* rather than an algebraically posited point, and it is the beachhead answer to the "no dynamics" killer.

This supersedes the old "saddle" reading. Earlier drafts (Rev16/Rev17) reported $J_{\rm vac}$ as a *saddle* of signature $(15_+, 12_-)$. That computation was correct but for the *ungauged invariant potential on the* $J_3(\mathbb{O}_s)$ *slice* (the 12 negative modes are the split $(4,4)$ octonion directions, G_2-protected — the Lorentzian-signature seed, not a removable instability; §14, Q1). The dynamical statement is Theorem 1.1: on the *gauged* $E_{7(7)}/SU(8)$ bed at the dyonic point the split directions are organized by the gauging and the golden point is tachyon-free Minkowski. The slice saddle is now read as the statics of the invariant toy, retained in §8–§14 for its kinematic content (the moment-graded dual collapse, A861).

Fence (binding). The transverse **54** runaway of the non-compact $SO(6,2)$ is generic (present at the origin), so $J_{\rm vac}$ is a critical point but not a global minimum of the full scalar manifold (at tree level it is a non-isolated local minimum whose flat branch reaches the origin, where the potential runs away; `s1361`, `s1249`); full stabilization is treated in §4. Of the 34 near-flat directions, 30 are exact — 24 gauge Goldstones (the orbit of all 28 gauge generators; the earlier "16" counted only the compact ones) and 6 strict moduli of an exactly flat branch — and 4 are light massive modes (`s1222`, `s1283`); their one-loop fate is the subject of §4. *Provenance (Rev33.2, house audit S367.9).* "Selected by a potential, not posited" is relative to the chosen four-dimensional gauged theory: $\Theta(\omega)$ is a consistent embedding tensor for every ω, and $\omega = \pi/4$ is fixed by stationarity at the golden point. Neither statement is a microscopic origin for $\Theta(\pi/4)$ — no higher-dimensional or generalized-frame construction of it is supplied here, and none is claimed (Appendix F, Remark "Honest fences", fence (i)).

Remark 1.2 (the dyonic vacuum *validates* the dual-collapse intuition; it does not re-open Wyler/α (S175))**.** The dyonic $\omega = \pi/4$ vacuum of Theorem 1.1 is exactly the electric/magnetic (collapse $\leftrightarrow$ expand) cancellation that the dual-orientation intuition predicts: two dual orientations meeting at $\pi/4$ kill the runaway along the dyonic direction and leave a tachyon-free Minkowski point —

at tree level a non-isolated (Morse–Bott) local minimum on a six-dimensional exactly flat branch (`s1283`, `s1361`), not an absolutely stable one: the golden ray, along which $V \equiv 0$, joins it at zero cost to the $SO(6,2)$ origin, which is not a tree-level local minimum (`s1249`), and one loop lifts it (`s1228`) — with the Freudenthal round-trip $(J^{\#}_{\rm vac})^{\#} = J_{\rm vac}$ closing the dual loop. This *validates* the dual-collapse picture at tree level, not a stable vacuum. It does *not* re-open Wyler or derive α: the tree-level point fixes the *modulus* φ (via the AX1 self-duality $F(\rho) = \rho - \rho^{-1} = 1$), not the $U(1)_{\rm EM}$ *fibre* coupling. Wyler's number stays falsified ($1/\alpha_W = 137.0360824$ vs CODATA 137.035999, gap $+6 \times 10^{-7}$; App. N, Leg 1) and α stays fenced. Validation of the mechanism, not a derivation of α.

2 Why golden, derived: the σ-even/σ-odd selection dichotomy

Given the dynamical bed, *why* the vacuum is golden and not symmetric is a selection problem. We resolve it into one structural dichotomy organized by the Jordan involution σ (the time-reversal $1 \leftrightarrow 3$ of Appendix H), which acts on the radial coordinate as inversion $\sigma : \rho \mapsto 1/\rho$ and fixes the symmetric point $\rho = 1$.

First, golden depth is not manufactured by the discrete scaffold. An explicit number-field analysis (kernel `s123a`, 10/10) shows $\phi \notin \mathbb{Q}(i, \sqrt{6})$ — the field generated by the pixel/$\mathbb{Z}_4$-clock/$\chi = 3$ data (with $q_0^{\rm KK} \to \sqrt{6}$) excludes $\sqrt{5}$, since $x^2 - x - 1$ is irreducible over that field and $\mathbb{Q}(\sqrt{5}) \cap \mathbb{Q}(i, \sqrt{6}) = \mathbb{Q}$. The golden depth $R_d = 1/(\phi^{2d} - 1)$ therefore reduces to the single irreducible axiom AX1, hosted (not derived) by the scaffold (3-way confirmed, A919–A922).

Novelty label: new specialization proved here.

Proposition 2.1 (Coleman–Weinberg is σ-even and cannot uniquely select golden in the tested σ-equivariant layers)**.** *The one-loop supertrace* $\mathrm{Str}\, M^4(\rho)$ *along the J_3-slice line $J(\rho) = \mathrm{diag}(\rho, 1, \rho^{-1})$ (the object of `s123b`/`s124b`) is invariant under $\sigma : \rho \mapsto 1/\rho$, hence extremized at the σ-fixed point $\rho = 1$, not at golden. This line shares the length* $\ln \varphi$ *with the coset golden ray* $\exp(t\Phi) \cdot o$ *of §1 but is not its image under any isometry of $E_{7(7)}/SU(8)$ or any cubic-compatible linear map (the slice generator has* 20 *zero eigenvalues on the* **56***, the ray's none; `s1230`). This holds (i)* bosonically *(kernel `s123b`,* 8/8*: $\partial_\rho \mathrm{Str}\, M^4|_\phi \neq 0$, single-well argmin $\rho \approx 1$); and (ii) for the* indexed fermionic *supertrace (kernel `s124b`,* 12/12*): σ is a verified J_3 automorphism (*$\det, \mathrm{Tr}$ *preserved, $A \to \bar{C}$, $B \to \bar{B}$), so any σ-equivariant mass set gives* $\mathrm{Str}\, M^4$ *σ-even and stationary at $\rho = 1$ — a* bein-independent *no-go (*20 *arbitrary equivariant masses all σ-even). Coleman–Weinberg is thus structurally unable to* uniquely *select golden in the tested σ-equivariant layers (bosonic, fermionic-norm, indexed-fermionic): the $Aut(J_3)$-equivariance premise is structurally* forced *(the A_1/A_2 shift tensors transform equivariantly, so the no-go does not use the complex* **56***-bein, which is now registered (`s545`)), while σ-evenness fixes $\rho = 1$ as a stationary point but does not by itself exclude σ-*breaking *field-dependent terms — which is where the field-dependent tensor C_{iab} enters; its evaluation is Appendix Q's Gate H (§4).* Scope qualifier (*Rev29*). *This proposition is a no-go* within the tested σ-equivariant layers*, stated against the supertrace-closure bookkeeping as it stood when the kernels cited above were run. That bookkeeping has since been reopened in-house as* UN-EVALUATED*, and a separate mechanism result bearing on the CW lift has been recorded but not identified against it (status box immediately below). The proposition is therefore not to be read as a closed statement about the CW lift of the residual flats, in either direction.*

(A925, A934; corroborated 3-way A926–A929, and by the self-contained external run A944 $\max |\mathrm{Str}\, M^4(\rho) - \mathrm{Str}\, M^4(1/\rho)| = 7 \times 10^{-15}$*.)*

(*Rev30*) Y5/Y6 CW-lift: the three-way state resolves into TWO scoped rows — they were never the same object. Rev29 printed this as a three-way inconsistency whose third item was the open question *"whether (1) and (2) name the same object"*. Kernel `s1035` (S269, 9/9 gates, payload `9111d769...e165`) answered it: **different objects**. The S215 reopening does *not* contradict the S207 closure — different mass matrices, different theorems, and disjoint parameter dependence. The item is therefore split, not resolved by fiat.

Row (i) — free-multiplet tier: closed, and it stands. The SS/Möbius-twisted one-loop supertrace of the *free* $N = 8$ multiplet (`s508`/`s509`, S207). Rebuilt independently at `s1035` from multiplet degrees of freedom and the twist *alone*, with no reference to any vacuum spectrum: on-shell $n_B = n_F = 128$ from $(1, 8, 28, 56, 70)$ with $\mathrm{Str}\, M^0 = 0$; the finite Hosotani potential $V'' = 256(2\pi)^2 \cdot \frac{7}{8}\,\zeta(3)$ reproduced by independent series summation to $< 10^{-3}$; twist typing $q_{\mathrm{fermion}} = \frac{1}{2}$, $q_{\mathrm{boson}} = 0$.

Row (ii) — interacting golden-vacuum tier: open, and it stands as open. The Kallosh–Karlsson supertrace of the *interacting* de Wit–Nicolai spectrum at the golden vacuum (`s578`–`s605`, S214–S215), rebuilt at `s1035` from vacuum spectra *alone*, with no reference to the twist. It does not close, and the obstruction is localized rather than diffuse: per-moment rescalings s_k have spread $\approx$1.185 against a 1.05 tolerance, so **no single rescaling closes it**. The vacuum itself is clean — `s605` finds 0 tachyons and 32 zero modes with the 24 Goldstone rank exact, so the point is Minkowski-stationary and the Kallosh–Karlsson theorem genuinely applies. The remaining walls are the vector shape (`s578`), the κ^2 pin, and the scalar-metric convention immediately below.

> **Name the scalar-metric convention wherever moments are printed.** The two scalar conventions in circulation differ by an exact factor 4^k at moment order k — verified at `s1035` to 10^{-3} relative, giving ratios 4 and 16 at $k = 1, 2$. A moment quoted without its convention is therefore ambiguous by a factor of four or sixteen, and that ambiguity is exactly what kept row (ii) looking like a physics discrepancy when it is a normalization one. **Every supertrace moment in this suite is to be read in the convention declared at its point of use**; where no convention is declared, the moment is not yet a claim.

Consequently: row (i) may be cited as closed *at the free-multiplet tier and only there*; row (ii) remains open and pre-registers the C_{iab} build (§4, §14) together with the convention declaration above. Neither row promotes anything, and Prop. 2.1 is unaffected. The blanket string "Y5/Y6 CLOSED" must still not be printed anywhere in this suite — unqualified, it now conflates two objects that `s1035` separated.

The complementary *σ-odd* channel is the unit defect source. Its defect equation *is* the golden axiom:

Novelty label: new specialization proved here.

Lemma 2.2 (the σ-odd defect pins golden $\Leftrightarrow$ AX1)**.** *The σ-odd cotangent source B on the moment-graded slice has defect function $F(\rho) = \rho - \rho^{-1}$. At unit weight $F(\rho) = 1$, which is $\rho^2 - \rho - 1 = 0$, i.e. AX1, with positive root $\rho = \phi$; the BPS completion keeps $V = 0$ and gives a physical radial mode $m^2 = 7/4$ (kernel `s123c`; A929). Hence the σ-odd defect is the* sole *viable golden selector, and it selects golden exactly when the framework's golden axiom holds.*

Proposition 2.1 and Lemma 2.2 together are the selection mechanism: the symmetric (σ-even) loop physics cannot break to golden, and the only object that can — the σ-odd unit defect — breaks to golden precisely through AX1.

3 The BPS / source-completion theorem

We now make the defect source quantitative on the J_3 arena and state precisely what is derived and what is modular.

Novelty label: new specialization proved here.

Theorem 3.1 (derived defect direction, completion, and radial mode — modulo the AX1 unit)**.** *On the J_3-slice line with bulk barrier $-\ln\mathrm{Det}$, the aligned source whose components are $b_i = 1/x_i - x_i$ equals the σ-odd grading $K = \mathrm{diag}(-1, 0, 1)$ at golden, and its direction is* forced *by the bulk barrier gradient together with the BPS condition $V = 0$. The completed potential is the manifest square*

$$V \;=\; \tfrac{1}{2}\,\big\|G(J) - B\big\|^2, \qquad G(J_{\mathrm{golden}}) = \textit{unit } \sigma\textit{-odd } K,$$

so the completion term $+\frac{1}{2}\|B\|^2$ is a genuine constant (a Fayet–Iliopoulos / brane-defect self-energy); the bare linear spurion alone *gives $V = -\frac{1}{2}\|B\|^2$, an anti-de Sitter value. The radial mode is the physical R-ii fluctuation with raw curvature $\gamma^2 + 4$, normalized by $K_{\mathrm{radial}} = 20/7$, giving*

$$m^2 \;=\; \frac{\gamma^2 + 4}{20/7} \;\xrightarrow{\gamma=1}\; \frac{7}{4},$$

*and a rank-*8 *source lifts* 8 *directions of the J_3-slice model (the "*18 *physical moduli" count of earlier revisions is withdrawn, §4). The* unit magnitude $\gamma = 1$ *is* not *independently BPS-forced: $\rho_\star(\gamma) = (\gamma + \sqrt{\gamma^2 + 4})/2$ selects a different point for every γ, and $\gamma = 1$ is the unique value for which $\rho_\star = \phi$, i.e. $\gamma = 1 \Leftrightarrow F = 1 \Leftrightarrow$ AX1.*

(Kernel `s124a`, 17/17*,* sha `5f737000`*; 3-way panel A931 (Grok)/A932 (Gemini)/A933 (ChatGPT), all reproduced in-house. The $\propto \|B\|^2$ form named by the text-only reviewers is the asymptotic limit of the exact law $m^2 = (\gamma^2 + 4)/(20/7)$; the $+4$ floor is the bulk-barrier curvature their leading order omits. The unit-magnitude correction is ChatGPT A933, verified in-house.)*

The honest end-state of the selection problem is therefore: *derived* are the σ-odd alignment, the $+\frac{1}{2}\|B\|^2$ completion (linear-alone $\to$ AdS), the physical radial mode $m^2 = (\gamma^2 + 4)/(20/7)$ ($= 7/4$ at the unit), and the rank-8 $\to$ 8-of-18 lift; *not* derived are the unit magnitude (equivalent to AX1), the specific $N = 8$ Fayet–Iliopoulos/Gukov–Vafa–Witten/brane identity of B, and the disposition of the residual flats (§4). *Golden selection is a derived σ-odd-defect mechanism modulo the AX1 unit*; stabilization stays partial — and stays partial for the reason recorded in §4 and in the Rev29 status box of §2: the certified field-dependent tensor C_{iab} has not been built, and the one in-house mechanism result that would bear on it is itself under an unresolved identification (*Rev29*).

One operator carries charge and the golden mass ladder (banked, kernel-verified). The hypercharge operator $Y = L_{\mathrm{diag}(1,1,-2)}$ and the mass operator $L_{J_{\mathrm{vac}}}$ *commute*, $[Y, L_{J_{\mathrm{vac}}}] = 0$ (kernel `s105b`; commutator norm $< 10^{-10}$, with a live non-commuting control). Hence the structure-group charges $\{J_{12} : +1,\ J_{13}, J_{23} : -\frac{1}{2}\}$ and the golden mass ladder $\{\phi^2/2, \sqrt{5}/2, \phi/2\}$ are read off the *same* $L_{J_{\mathrm{vac}}}$ that fixes the vacuum. (Fences: Higgs-*class* P/CP order parameter, not the SM weak doublet; the ladder is dimensionless; a dimensionful lift needs a dimensionful input — the registered class $L_{\mathrm{depth}}/L_\sigma$ (Appendix O; Appendix X input-count box) — *and* a tower normalisation, the pure number $q_0^{\mathrm{KK}} = 1/(2\sqrt{6})$ of Appendix G; no $q_0^{\mathrm{KK}} \mapsto$ GeV map exists (Rev32.9, A1642 O7).)

One number printed three ways (Rev32.6; A1567 §2.2, `s1158`). The middle rung $\sqrt{5}/2$ of the ladder above, the $(1,3)$ Peirce eigenvalue of Appendix B, and the V_3 rotation frequency Ω of Paper 5 are one number by the golden polynomial $2\phi - 1 = \sqrt{5}$, i.e. $\Omega = \phi - \frac{1}{2}$; the identity is specific to the golden point ($\lambda_{13}(r) = r - \frac{1}{2} \iff r = \phi$; the `s1158` controls at $r = 3/2$ and $r = \phi + 1/10$ fail), and no registered map turns the rung into the rotation (Appendix N, Part N.E).

4 Residual moduli and the stabilization status

Rev33.2 — reconciled with Appendix Q. The counting that stood here through Rev33.1 ("18 physical moduli grading as 9 σ-odd +9 σ-even, 8 CW-liftable flats +1 dilaton") rested on a compact-only gauge count and on a σ-grade defined on the J_3 model, not on the certified 70; it is withdrawn. The certified anatomy at the golden point is: 24 gauge Goldstones, 6 strict moduli, 4 quasi-flat light modes, 36 heavier positive modes, no negative mode (`s1222`, reproducing `s647`). The 6 strict moduli are *exactly* flat critical directions — the potential and its gradient vanish along them, with no cubic or quartic term — so the golden point lies on a six-dimensional flat Minkowski branch and is a non-isolated (Morse–Bott) tree-level minimum (`s1283`, `s1361`). The branch is not isolated from instability: the $V \equiv 0$ golden ray joins the golden point to the $SO(6,2)$ origin, where 36 physical directions are massless and the quartic is indefinite, so the origin is not a tree-level minimum and the potential runs away from it (`s1249`).

At one loop the decisive object — the field-dependent tensor $C_{iab} = \frac{1}{2}\partial_a\partial_b m_i^2$ contracted with

$$H_{ab}^{\mathrm{CW}} = \frac{1}{32\pi^2}\sum_i (-1)^F\, n_i\, [2u_i \ln(u_i/\mu^2) + u_i]\, C_{iab}, \qquad u_i = m_i^2,$$

— has been evaluated in the certified $E_{7(7)}/SU(8)$ frame with the complex **56**-bein (`s545`): the Coleman–Weinberg inertia on the ten near-flat non-gauge directions is $(9,0,1)$, i.e. $(5,0,1)$ on the strict moduli and $(4,0,0)$ on the quasi modes (Appendix Q, Gate H; the pre-registered blind prediction $(10,0,0)$ was scored a loss). On the golden ray the full one-loop potential is minimised at the origin, not at the golden point, which it lifts; off the ray the origin is a one-loop saddle whose one negative mode is the D_{56} line, along which the one-loop potential runs away with no stationary point (`s1228`, `s1229`, A1760).

Novelty label: new specialization proved here.

Proposition 4.1 (undecidability, re-scoped)**.** *σ-evenness of the J_3 slice together with the point spectrum and the low-order supertrace constraints does not determine the sign of the residual one-loop Hessian; only the field-dependent tensor C_{iab} does (kernel `s124i`; A941). That tensor is now evaluated (Gate H above).*

Honest stabilization summary (Rev33.2). *Selected — yes,* at the level stated in §3 (the σ-odd defect on the J_3 slice, modulo the AX1 unit). *Stabilized — no source-free vacuum is certified:* at tree level the golden point is a Morse–Bott minimum on a flat branch that reaches an unstable origin; at one loop five of six moduli and the four quasi modes are lifted, one flat direction remains, and the golden point is neither stationary nor below the origin. The open object is a certified one-loop stationary-point classification on the flat branch (or a controlled stabilising contribution along D_{56}). The free-multiplet supertrace (row (i)) and the interacting golden-vacuum supertrace (row (ii)) stay **different objects** (R141), as before.

The no-scale direction. The one direction Gate H leaves flat is the no-scale direction; it is not Coleman–Weinberg-fixed (the one-loop minimum $L^\star = A/2B + \frac{1}{4}$ shifts with μ) and remains a separate scale/lift input.

5 The generation count as a Morse index

The generation *count* is a clean topological index on the same Peirce frame.

Novelty label: physical interpretation not established here.

Proposition 5.1 ($N_g = 3$ as a Morse index)**.** *On the Cayley plane $\mathbb{O}P^2$ realized by the three primitive idempotents of the golden frame, the cubic-norm Morse function has critical cells of index $\{0, 8, 16\}$, so*

$$\chi(\mathbb{O}P^2) \;=\; \sum_{\text{cells}} (-1)^{\text{idx}} \;=\; 1 - 0 + 1 - 0 + 1 \;=\; 3 \;=\; N_g,$$

ordering-invariant across all 6 *orderings (kernel* `s124c`*,* 5/5*; A935). This is a* topological index*, not the refuted "*rank $3 \Rightarrow 3$*" route (A691–A694).*

The generation↔cell *assignment*, by contrast, is the non-finite A1 datum: the three idempotents form a single S_3 automorphism orbit with identical intrinsic data, so all algebra-only labels are S_3-symmetric (0 bits) — the assignment needs the golden mass-ordering (dynamics), and is folded here as honest-open (kernel `s124g`, 4/4; A939). $\chi(\mathbb{O}P^2) = 3$ is a theorem about the carrier. That it equals the generation count N_g — an INPUT (Appendix X: algebraic replication is refuted) — is a correspondence, not a derivation; and the generation↔cell map is not available either.

6 Confinement as the spectral geometry of the cubic norm

The same cubic norm $N(J) = \mathrm{Det}J$ whose Morse cells count the generations (§5) also organizes the confinement sector. Three structural facts follow from N alone — its homogeneity, its polarization, and the degeneracy of its Koecher–Vinberg Hessian $G_{MN} = -\partial_M \partial_N \ln N$ (the cone metric of Eq. (1) below) — independently of any particular numerical parametrization of that spectrum.

Novelty label: new specialization proved here.

Proposition 6.1 (a formal scaling identity for the cubic weight)**.** *For the scaling action $S = c\,N(J)$ on $J_3(\mathbb{O}_s)$ ($\dim_{\mathbb{R}} = 27$, $\deg N = 3$) and real $c > 0$ — the sign branch is fixed here, before the identity: for $c < 0$ the real substitution $J \to |c|^{1/3} J$ gives $|c|^{-9}$ instead, and c^{-9} below means the positive branch*

(Rev32.10, A1711-F04) — the substitution $J \to c^{1/3} J$ gives, formally, *$Z(c) = \int e^{c\,N}\, dJ = c^{-9} Z(1)$, hence*

$$c\,\frac{d}{dc}\log Z(c) \;=\; -\,\frac{\dim_{\mathbb{R}} J_3(\mathbb{O}_s)}{\deg N} \;=\; -9,$$

constant in c. The integral does not converge on the real 27-dimensional space for either sign of c (N is odd), so the identity holds only for a regulator or integration domain that respects the scaling (a cone, or a contour); none is fixed in this appendix. The quantity $c\,\partial_c \log Z$ is a scaling exponent, not the renormalization-group β-function, and no statement about the running of physical masses follows from it. The metric $G = -\partial\partial \ln N$ is undefined on $\{N = 0\}$; the nine directions tangent to the null cone (Euler's relation $J\cdot\nabla N = 3N = 0$ there) describe the limit of its degeneracy, not flat directions of G on the cone. (Retyped Rev32.9, A1641 F01: through Rev32.8 this was titled "the overall coupling does not run: $\beta_c = -9$" and concluded that running must come from composite correlators. [LIB2-058] Algebraic identity; `L-BetaCTrivial`*.)*

Novelty label: new specialization proved here.

Proposition 6.2 (Faddeev evasion: the cubic invariant survives where the quadratic vanishes)**.** *At J_{vac} the one-loop correction to β_c from any* bilinear *composite vanishes identically, $g^{\mathsf{T}} H^{+} g = 0$: this is the mesonic, evaluation-type channel $M^{\dagger}M$ built on the quadratic trace form. The baryonic channel is built instead on the* cubic *polarization of N — the $\varepsilon_{abc}\,\varepsilon_{ijk}$ contraction (colour × generation), i.e. the $\Lambda^3 \to$ singlet / determinant trilinear. [LIB2-136] That contraction is antisymmetric within each index triple and therefore symmetric under a paired exchange of one colour and one generation index; the polarization of the cubic form N is a symmetric trilinear. A trilinear is not a quadratic form, so the $g^{\mathsf{T}} H^{+} g = 0$ cancellation does not act on it. [LIB2-136] Whether its residue is nonzero, and its sign, is not computed here. Confinement is proposed to use the invariant of N that the quadratic degeneracy leaves untouched. (Rev32.9, A1671 F05: through Rev32.8 this read "totally antisymmetric" and "its residue is strictly positive"; neither was argued. Tensor-structure observation;* `L-FaddeevEvasion`*.)*

Novelty label: physical interpretation not established here.

Proposition 6.3 (confinement as a spectral singularity, not strong coupling)**.** *Lifting the null-cone eigenspace by a mass ε gives a spectral trace $T(\varepsilon) = A/\varepsilon$, with A the trace of the cone Hessian over the lifted sector. [LIB2-135] At the self-consistent point $\varepsilon = m^2 = \alpha A$ one has $\alpha\, T = \alpha\,(A/\alpha A) = 1$* exactly, independent of α, *fixing a confinement scale $\Lambda^2 = \alpha A$ with the coupling weak throughout. [LIB2-135] The mechanism is a degeneracy-driven pole — closer to spectral localization than to asymptotic freedom — not a strong-coupling limit. It is proposed as the mechanism behind the G_2 confinement scale Λ_{G_2} that anchors the light-quark constituent masses (Paper 2, §3.4); Λ_{G_2} itself is a calibration (App. X).* Scope (Rev32.9, A1671 F06): *the lift and $T(\varepsilon) = A/\varepsilon$ come from the legacy rank-2 vacuum* $\mathrm{diag}(\phi^2, 1, 0)$, *where $N = 0$ and $G = -\partial\partial \ln N$ is undefined; that point is not related to the current vacuum* $\mathrm{diag}(\phi, 1, \phi^{-1})$ *($N = 1$) by a normalization, and the transport of the null-eigenspace lift to the golden point is not supplied. (Mechanism sketch;* `L-HybridMassGap`*.)*

Remark 6.4 (three companion facts about the same Hessian)**.** The cone Hessian G carries three further structural facts from the same analysis, each a property of N rather than of a fitted spectrum. (i) *The vacuum is a spectral background, not a saddle:* J_{vac} is selected by the eigenvalue structure of G, not by $\nabla N = 0$ (indeed $\nabla N \neq 0$ there), which is why the expansion is organized by the Hessian spectrum rather than by a stationary phase. [LIB2-137] (ii) *Colour isotropy:* within each Peirce sector the eight $SU(3)$-colour components of an off-diagonal octonionic entry carry G-eigenvalues of equal magnitude and opposite signs in the raw block ($\pm 2\xi_k$, fourfold each —

Remark 6.5); in the reflected positive metric they carry one common eigenvalue, so it is the reflected metric that is colour-blind at tree level (Rev32.9, A1647 F01) (cf. the Casimir band law of App. X). [LIB2-086] (iii) *Relevance:* the cubic coupling has mass dimension $[c] = (6-d)/2$ for a canonically normalised two-derivative kinetic term (a premise, stated Rev32.9), relevant in $d = 4$ exactly as in ϕ^3 theory — the same homogeneity that fixes the scaling exponent in Prop. 6.1. [LIB2-059] (Library `L-SpectralBackground`, `L-ColourBlindExact`, `L-CubicRelevant`; the legacy vacuum normalization $\mathrm{diag}(\phi^2, 1, 0)$ is superseded by the current $\{\phi, 1, \phi^{-1}\}$ frame, the structural statements unaffected.)

Remark 6.5 (the flux-tube ratio $\sqrt{\sigma} = \Lambda_{G_2}\varphi$ is an in-frame, α-free eigenvalue ratio (`s284`, S175))**.** On the current $N = 1$ cone the off-diagonal octonionic blocks of G at J_{vac} carry eigenvalues $\pm 2\xi_k$ with $\xi = (\varphi, 1, \varphi^{-1})$, so the golden flux-tube relation $\sqrt{\sigma} = \Lambda_{G_2}\,\varphi$ is exactly the eigenvalue ratio $2\xi_1/2\xi_2 = \varphi$ — a clean Hessian-eigenvalue ratio, recovered frame-current and *independent of* α (an upgrade from the thinly-banked legacy fence of Paper 8). It nonetheless merely *restates* the single scale Λ_{G_2} (it reproduces no Λ-independent datum), so it is not an independent hadronic observable and nothing is promoted. The cone yields only this one φ-power family of α-free ratios; a second, genuinely independent in-frame hadronic observable does *not* follow (honest negative, `s284`).

Remark 6.6 (why the confinement *scale* is blocked behind α (`s284`, S175))**.** The confinement scale cannot be derived without first un-fencing α, for two independent reasons. (i) By Prop. 6.1 the overall coupling $\beta_c = -9$ is *constant* (pure homogeneity, $-\dim/\deg = -27/3$). Standard dimensional transmutation requires a *log-running* coupling, $\Lambda = \mu\, e^{-1/(bg^2)}$ with $b \neq 0$; a constant β generates no exponential scale, so G_2-running produces no Λ_{G_2} of its own. The scale is instead fixed by the spectral-singularity self-consistency $\Lambda^2 = \alpha A$ (Prop. 6.3) — i.e. by α (fenced, App. N) times a pure cone trace — so Λ_{G_2} sits *behind the* α *fence*, not upstream of it. (ii) The cone supplies only the one φ-power family of α-free ratios (Remark 6.5), so no second, genuinely independent in-frame hadronic observable follows; the minimal two-observable falsifiable hadronic core is not reached from cone geometry alone. Both are honest negatives: nothing here promotes Λ_{G_2} to a prediction, and α stays fenced.

Hadronic scope fence (binding). The framework models the confinement *scale* Λ_{G_2} (Prop. 6.3; one Λ_{QCD}-like calibration input, App. X) and the confinement *mechanism* (Props. 6.1–6.3, structural). [LIB2-135, LIB2-136] It does *not* attempt hadron spectroscopy: no glueball or full meson/baryon spectrum, no lattice comparison. The numerical values once attached to this sector — the spectral trace $A = 144$, the ambient Morse index 17/27, and the baryon/meson ratio $M_B/M_M \approx 6.7$ — were computed in the legacy ambient-27 Hessian parametrization and predate the current $\{\phi, 1, \phi^{-1}\}/\mathbb{O}P^2$ description. [LIB2-335, LIB2-354] An in-frame recompute on the current $N = 1$ cone (kernel `s284`, S175) is decisive: the ratio $M_B/M_M \approx 6.7$ *does not survive* — the cone supplies only ϕ-power eigenvalue ratios and the PDG $938/140 = 6.72$ is not a clean ϕ-power (nearest $\phi^4 = 6.854$, $+2\%$) — so it is **retired** as a legacy null-cone artifact, not a frame-current relation. (The companion flux-tube relation $\sqrt{\sigma} = \Lambda_{G_2}\varphi$ does survive in-frame and α-free, but only restates the one scale; Remark 6.5.) Both remain recorded, fenced, in Paper 8 (Junkyard, *Fenced: out-of-scope work*) and lie outside the falsifiable core; no public observable depends on them.

7 The probe Lagrangian and its scalar constraint

The preceding dynamical results sit on the certified gauged bed. We retain the earlier *probe* model, whose kinematic content (the bounce, the clock, the moment-graded collapse) remains valid and feeds the synthesis. We work with a probe field $J(t) \in J_3(\mathbb{O}_s)$ with cubic norm $N(J) = \mathrm{Det}J$ and Freudenthal quadratics $\mathrm{Tr}, \mathrm{Tr}^{\#}$, governed by a Wheeler–DeWitt-type scalar constraint

$$\mathcal{H} \;=\; \tfrac{1}{2}\, G_{MN}(J)\, \Pi^M \Pi^N \;+\; \lambda\,(N-1)^2 \;+\; \gamma\,\mathrm{Tr}(K\cdot J) \;\approx\; 0, \tag{1}$$

with kinetic metric $G_{MN} = -\partial_M \partial_N \ln N(J)$ (the Koecher–Vinberg cone metric), $K = \mathrm{diag}(-1, 0, 1)$ the banked Peirce grading (the same K whose half-turn is the time-reversal T of Appendix H; the sign is fixed by stationarity of J_{vac}, so the largest eigenvalue ϕ sits at grading -1), and λ, γ stiffness couplings. Equation (1) is the radial/diagonal restriction of the coset $\frac{M^2}{4}\mathrm{Tr}(P_\mu P^\mu)$ on $E_{7(7)}/SU(8)$, the $(N-1)^2$ term confining to $\mathrm{Det} = 1$ and $\mathrm{Tr}(K\cdot J)$ the symmetry-breaking grading coupling discussed in §8.

Why the grading term and not an invariant potential. A potential built purely from the characteristic-polynomial invariants $(\mathrm{Tr}, \mathrm{Tr}^{\#}, N)$ cannot select J_{vac}: those invariants are symmetric in the eigenvalues, so their minimum on the $N = 1$ cone is the symmetric point $(1, 1, 1)$, never the golden multiset (a provable no-go on the diagonal slice — the diagonal-slice shadow of the σ-even no-go, Prop. 2.1). The grading K is *not* an invariant; it is the preferred (σ-odd) direction that breaks the eigenvalue-permutation symmetry, exactly the defect of §3. Two *constraints* on the invariants, by contrast, *do* pin the multiset; that is the independent Route A of §8.

8 The overdetermined ϕ-vacuum (invariant routes)

Independently of the dynamics, the golden multiset is *overdetermined* by conditions the framework already carries. We give two routes to $J_{\mathrm{vac}} = \mathrm{diag}(\phi, 1, \phi^{-1})$.

Route A — the invariant selector $\{N = 1,\ \mathrm{Tr} = \mathrm{Tr}^{\#},\ \det G = 7\}$

Restrict to the inversion family $J(r) = \mathrm{diag}(r, 1, 1/r)$, so $N = 1$ automatically and $\mathrm{Tr} = \mathrm{Tr}^{\#}$ (palindromic spectrum). Let $G(J)$ be the Freudenthal/Gram form whose determinant on this family is $\det G = s^2 - 9$ with $s = r^2 + 1 + r^{-2}$.

Novelty label: new specialization proved here.

Lemma 8.1 ($\det G = 7 \Leftrightarrow r = \phi$)**.** *On the inversion family* $\mathrm{diag}(r, 1, 1/r)$ *with* $r > 0$*,*

$$\det G = 7 \iff s = 4 \iff r + \tfrac{1}{r} = \sqrt{5} \iff r = \phi.$$

Proof. $\det G = s^2 - 9 = 7$ gives $s^2 = 16$, hence $s = 4$ (positivity). Then $r^2 + r^{-2} = s - 1 = 3$, so $(r + r^{-1})^2 = r^2 + 2 + r^{-2} = 5$, i.e. $r + r^{-1} = \sqrt{5}$. The positive root of $r^2 - \sqrt{5}\, r + 1 = 0$ is $r = \frac{\sqrt{5}+1}{2} = \phi$. Direct check: for $r = \phi$, $\phi^2 = \phi + 1$ and $\phi^{-2} = 2 - \phi$, so $s = (\phi + 1) + 1 + (2 - \phi) = 4$ and $\det G(J_{\mathrm{vac}}) = 16 - 9 = 7$ exactly. □

The content of Lemma 8.1 is that the banked integer $\det G = 7$ (the same "7" in $\sin^2\theta_{23} = 7/16$ and $|V_{cb}| = 1/(9\sqrt{7})$; Appendix B) *algebraically encodes* $\sqrt{5}$, hence ϕ, with no ϕ placed by hand. The selector is built from E_6-invariants, so it lives on the gauge-invariant orbit; the diagonal $J_{\rm vac}$ is its gauge representative.

Route B — the grading-coupled potential

Take the diagonal-slice potential extracted from (1),

$$V(J) \;=\; -\ln \mathrm{Det} J \;+\; \tfrac{1}{2}\,\mathrm{Tr} J^2 \;+\; \gamma\,\mathrm{Tr}(K\cdot J), \qquad K = \mathrm{diag}(-1,0,1). \tag{2}$$

Novelty label: new specialization proved here.

Proposition 8.2 (unit coupling selects the golden vacuum)**.** *On $J = \mathrm{diag}(a,b,c)$, stationarity of* (2) *forces $b = 1$ and a common golden equation on the outer eigenvalues; at unit coupling $\gamma = 1$ the unique positive-definite minimum is $J_{\rm vac} = \mathrm{diag}(\phi, 1, \phi^{-1})$, with $\mathrm{Det} J_{\rm vac} = 1$ automatically and ϕ the positive root of $c^2 - c - 1 = 0$. The Hessian* restricted to the diagonal slice *is positive-definite; on the full algebra the invariant-potential Hessian is the saddle of §14, Q1, resolved on the gauged bed by Theorem 1.1.*

The two routes are independent: Route A uses the cubic/Freudenthal invariants and imposes inversion; Route B uses the grading direction K and unit coupling. Neither inserts ϕ; both reduce "ϕ" to a single named structural condition ($\det G = 7$, or $\gamma = 1 \Leftrightarrow$ AX1, §3). This is the precise, fenced sense in which the golden vacuum is *overdetermined*.

The 7/3 amplitude bridge — characterized as open (S116). The integer $\det G = 7$ is the dimension count $\dim \mathrm{Im}\,\mathbb{O}_s = 7$, and $7/3 = \dim \mathrm{Im}\,\mathbb{O}_s / \mathrm{rank}\, J_3$ is a proved algebraic invariant. Its promotion to a heavy-quark S-matrix residue was scouted to a wall: no literature theorem maps a Jordan Gram determinant to an LSZ residue/OPE/crossing ratio; the special-geometry Yukawa $Y_{ijk} = \partial_i\partial_j\partial_k N$ contracted at $J_{\rm vac}$ either yields a traceless ladder with mean 8/3 (not 7/3) or, after canonical normalization, the degenerate spectrum 1:1:1; and an explicit canonical Peirce-trilinear LSZ computation returns mass-residue ratio 1 and inclusive colour ratio 3. The decisive obstruction is Lemma 8.3: "7" is a *dimension*, whereas a physical residue channel-sum is weighted by the split-signature metric, under which $\mathrm{Im}\,\mathbb{O}_s$ has signature $(3,4)$ and signed trace -1. The metric-weighted-residue route is therefore *closed-negative* (Lemma 8.3, sharpened at Rev27; kernel `s902`): no unitary residue over the $\mathrm{Im}\,\mathbb{O}_s$ channels can return 7. What stays open is only whether a *non-residue*, index-theoretic exceptional-field-theory realization exists; the 7 survives as the finite-carrier *index* (Appendix K), not as a signed amplitude.

[LIB2-311]

Novelty label: new specialization proved here.

Lemma 8.3 (the 7 is a dimension, not a metric trace)**.** *On the split octonions $\mathbb{O}_s$ with trace form of signature $(4,4)$, the imaginary subspace $\mathrm{Im}\,\mathbb{O}_s$ (the metric-orthogonal complement of the identity) has* dim $= 7$ *and signature $(3,4)$, hence signed trace -1. Consequently $\dim \mathrm{Im}\,\mathbb{O}_s = 7$ and $\mathrm{Tr}\, g|_{\mathrm{Im}\,\mathbb{O}_s} = -1$ are distinct objects; under the equal-unit-weight channel prescription, an S-matrix residue weighted by g returns the latter (an arbitrary coupling-weighted contraction $\sum \eta_i |a_i|^2$*

does not — it reaches 7 *at amplitude* $\sqrt{7}$*,* `s1171` *T5; Rev32.9, A1652). The channel coefficients* $\tau(e_0, e_a, e_a) = (1, 1, 1, -1, -1, -1, -1)$ *realize this explicitly: positive Hilbert count* 7*, signed split trace* -1*.*

Sharpening (Rev27, kernel `s902`; `run1==run2`, hash `547d9265`). *The signed residue over* any *sub-collection of g-orthonormal channels equals (#positive − #negative) and therefore lies in the closed range* $[-4, +3]$ *(the negative and positive indices of the* $(3, 4)$ *form); the full-space value is* -1*. [LIB2-107] Since* $7 \notin [-4, +3]$*, i.e.* $|\text{residue}| \leq 4 < 7$*, the integer* 7 *is* unreachable *as a signed (unitary) residue and is realized only as the unweighted multiplicity. [LIB2-107] Hence the metric-weighted LSZ promotion of* $7/3 = \dim \operatorname{Im} \mathbb{O}_s / \operatorname{rank} J_3$ *is a structural* no-go*; the* 7*-as-*index *reading (the finite-carrier theorem, Appendix K) is the unique survivor. This closes the specific residue route on* $\operatorname{Im} \mathbb{O}_s$*; it does not assert that no other construction could realize* $7/3$ *physically. [LIB2-107, LIB2-311]*

(Verified in-house on the suite's `omul/oconj` *and an independent Cayley–Dickson implementation; S116, A825–A830.)*

OPE / colour-Casimir addenda (S116, observation register — not derivations). Three further recastings were tested and all return 7/3 only under an imposed selection, leaving the mass-boundary readout open: *(i) OPE coefficient.* Recast as a Wick/OPE coefficient $C_{00} = \sum_{i,j} \tau(e_0, e_i, e_j)^2$ over $\operatorname{Im} \mathbb{O}_s$, one finds $C_{00} = 7$ and $C_{00}/\operatorname{rank} J_3 = 7/3$; but $C_{AB} = \sum \tau\tau$ is a Gram matrix, positive-semidefinite *by construction*, so the 7 is again $\dim \operatorname{Im} \mathbb{O}_s$ recovered by a sum of squares. *(ii) Colour face.* Numerically $7/3 = 1 + C_F(SU(3)_{\text{fund}})$ since $\sum_a T^a T^a = C_F I = \frac{4}{3} I$, and $1 + C_F(SU(N)) = 7/3$ holds *(algebraically, up to permutation)* only at $N = 3$ ($3N^2 - 8N - 3 = 0$, positive root $N = 3$). This gives the invariant a colour-current face, but the physical colour effect on m_b/m_τ is renormalization-group logarithmic ($\gamma_m \propto C_F$, exponentiated), *not* an additive tree factor $1 + C_F$. *(iii) Same-line fence.* The 7/3 readout survives only the *same-line* scalar-mass+Casimir insertion; the literal *cross-line* colour-adjoint exchange gives $-\frac{1}{3}$, the sequential/additive variants give 49/9 and 11/3, and the canonical $U(3)$ current gives 3/2. Since the suite's physical Yukawa is the *cross-block* Peirce trilinear, the 7/3-surviving same-line readout is structurally mismatched with the framework's own Yukawa placement. Net: the colour face is suggestive and uniquely picks $N = 3$, but does not promote 7/3 to a derived mass coefficient. (S116, A831–A833.)

The closure projector has a canonical form; the generation assignment reduces to two physical guards (S116–S117, A841–A843). The selecting operator can be written $O = I + \Pi_{12}\, C_2(SU(3))$ with $C_F = \frac{4}{3}$, where the closure-current projector acquires an *exact* algebraic form $\Pi_{12} = 4\, L_{c_1} L_{c_2}$ — the unique off-diagonal Peirce block invisible to the c_3 idempotent. In-house on `s105b`: $\operatorname{rank} \Pi_{12} = 8$, idempotency $\|\Pi_{12}^2 - \Pi_{12}\| = 0$ (exact), c_3-invisibility, and orthogonality $\Pi_{12} \perp \Pi_{13}, \Pi_{23}$. Then O returns $7/3$ on closure-current quarks and 1 on direct-carrier and colour-singlet anchors. *What this does not do* is derive *which* fermions occupy the c_3-invisible block — the fermion→Peirce-block generation assignment. A structural defect rule (A843) selects exactly b, c but leaves *two load-bearing physical guards* that are inserted, not derived (the colour-current guard and the non-confinement-anchor guard). Two no-go results (A841) show the residual assignment must be *generation-topological*: the full one-generation gauge tuple is identical for top and charm, so no gauge/Casimir-only function gives $7/3$ on b, c but 1 on t; the cross-block Peirce signature is common to top, bottom and tau. This converges with the beat-order finding (A834) $n_b - n_\tau = \log_\varphi(7/3)$. Hence the heavy-quark $7/3$ relation, the projector Π_{12}, and the quark beat-order selector are *the same open object* — a piece of the generation-count frontier (the A1 datum of §5).

The readout side: a finite-operator form for the $7/3$ edge (S117, A844–A845). Three candidate readouts are positively excluded, leaving a single finite-operator survivor (all reproduced in-house). *(i)* A universal field-level same-line self-energy weight is impossible (a brute scan over $\{0,1\}^3$ returns zero solutions). *(ii)* Ordinary kinetic Z-normalization gives the *inverse*: $Z_L = Z_R = 7/3 \Rightarrow m_{\text{phys}} = \frac{3}{7} m_0$. *(iii)* The ordinary commutator/gauge-orbit Casimir vanishes. The *survivor* is the normalized double-anticommutator Casimir $C_+(D) = \frac{1}{4}\sum_a \{G_a, \{G_a, D\}\} = C_F D = \frac{4}{3} D$, whence $D + P_{\text{defect}} C_+(D) = \frac{7}{3} D$ on the closure edges and D on anchors (a post-EWSB audit forces the electromagnetic coefficient to zero). This upgrades the $7/3$ edge from a hand-written scalar to a canonical finite self-adjoint operator, but does *not* prove the physical mass boundaries *read* C_+ rather than the cross-block Yukawa singular value. $P3$ remains settled-open; no observable moves.

The tested finite endpoint/self-adjoint/source layer is exhausted: the 7/3 edge is a cotangent source response, and the residual is non-finite (S117–S118, A846–A863; 27 runs, none breached). A sustained probe pushed the readout problem to its terminus on the finite layer it tests. *(1) Operator form.* The survivor C_+ is the Hilbert gradient of the positive functional $S_+[D] = \frac{1}{8}\sum_a \mathrm{Tr}(\{G_a, D\}^\dagger\{G_a, D\})$ (A846) and equals the double Jordan product $\sum_a G_a \circ (G_a \circ D) = C_F D$ (A847), matching the Peirce closure law exactly. *(2) A sharp negative bounding the class.* The 7/3 is *not* a positive-parent-action minimizer: minimizing S_{rel} gives $c_{\min} = 3/(4\alpha+3)$, so $\alpha = 1$ returns the *inverse* 3/7; the factor survives only as an additive gradient *insertion* $D_{\mathrm{eff}} = D + \nabla S_{\mathrm{rel}}[D] = \frac{7}{3}D$ (A856–A857); integrate-out and kinetic-Z routes likewise return 3/7 (A859/A862). *(3) The physical object.* Restated symplectically, the 7/3 edge is the response of a *cotangent Yukawa source* $B \in J_{12}^\vee$ with $B = \nabla S_{\mathrm{edge}} = \frac{7}{3}D$ and $\mathrm{rank\,span}\{B_{y,z}\} = 8/8$ (A862–A863) — the same defect object §3 derives the unit alignment of. A decisive no-go (A851) confirms the residual is *generation-level* (top and charm collide). *Conclusion.* The tested finite endpoint/self-adjoint/source layer is exhausted — form, readout, uniqueness, and physical object pinned, minimizer/integrate-out classes excluded. The remaining content is explicitly *non-finite*: the full $E_{7(7)}/J_3(\mathbb{O}_s)$ fermion action with the source term $S_{\mathrm{edge}}[D] - \langle B, D\rangle$ plus the FDR-1 generation assignment (A1). Across 27 runs (A843–A863) none breached and no public observable moved; the 7/3 label stays STRUCTURAL/CONDITIONAL, never a derived mass prediction.

9 The geometric bounce

The kinetic metric $G_{MN} = -\partial_M\partial_N \ln N$ is the canonical Koecher–Vinberg metric on the symmetric cone $\{N > 0\}$. Along the radial ray $J(\rho) = e^\rho J_{\mathrm{vac}}$ the cone-distance element behaves as $\mathrm{d}\ell \sim \sqrt{2}\,\mathrm{d}\left[-\frac{1}{2}\ln(1-s)\right]$ toward the boundary, so the metric length from the Det = 1 vacuum to the Det = 0 locus diverges:

$$\ell(J_{\mathrm{vac}} \to \{\mathrm{Det} = 0\}) \;=\; \int -\sqrt{2}\,\mathrm{d}\ln(1-s) \;\longrightarrow\; \infty. \tag{3}$$

Novelty label: new specialization proved here.

Proposition 9.1 (classical bounce)**.** *The* Det = 0 *singular boundary lies at infinite KV cone-distance from the* Det = 1 *vacuum. A free (geodesic) trajectory therefore never reaches the singular locus: the cone is metrically complete toward* Det = 0*. Singularity resolution is classical and kinematic, located at the boundary, not at the vacuum (the* Det = 1 *interior point).*

Equivalently, the regulated null wall $W(N) = N^{-2/3} + 2\,N^{1/3} - 3$ ($W(1) = 0$, $W'(1) = 0$, $W \to \infty$ as $N \to 0$) has its minimum at the vacuum surface $N = 1$ and diverges at $N = 0$: $V_N(r) = \Lambda^4(e^{-2r} + 2e^r - 3)$ has minimum at $r = 0$, $V_N'' = 6\Lambda^4 > 0$, so any incoming radial trajectory *bounces*. This relocates the retired "Det = 0 as the vacuum" reading: the vacuum is the Det = 1 interior, Det = 0 is the wall it bounces off.

Fence. Proposition 9.1 is classical singularity-resolution. The exponents and scale Λ in $W(N)$ are not derived. No quantum area operator, holonomy spectrum, or Hamiltonian in physical seconds follows (the LQG-quantization reading is dead-for-now: $SO(4,4)$ is non-compact, no compact area operator follows).

10 The $E_{7(7)}$ clock connection and the quantum beat

The time layer of Appendix H realised the past↔future moment as a genuine $E_{7(7)}$ element T with $T^2 = -1$. The probe supplies the connection 1-form: an $E_{7(7)}$-valued clock connection $A^{\mathrm{clk}}_\mu = Q_\mu + a_\mu K$, with Q valued in the level-zero algebra $\mathfrak{e}_{6(6)} \oplus \mathfrak{so}(1,1)$ of the D_{56} grading and a_μ the clock 1-form along K. Its holonomy is

$$U_\gamma \;=\; \exp\Big(K \oint a\Big), \qquad \oint a = \tfrac{\pi}{2} \;\Longrightarrow\; U_\gamma = \exp(\tfrac{\pi}{2}K) = T, \quad T^2 = -1. \tag{4}$$

This is the "quantum beat" of the title. The half-turn $\oint a = \pi/2$ is one beat of the relational (Leibnizian) clock; $T^2 = -1$ is the Kramers/spinor double cover, so it takes *two* beats to return ($T^4 = +1$). Read physically, T reverses the shell (end) arrow H and *preserves* the clock's $(2,1)$ nappe, and it is not a symmetry of the gauging; the dynamics' end-flip O reverses both (`s1252`, A1770). So the "wormhole↔black-hole time-orientation flip" at the Det = 0 wall of §9 is a flip of the end arrow only; the holonomy reading at the wall is unaffected.

Fence. $T \in E_{7(7)}$, $T^2 = -1$ is certified (Appendix H, seed-independent). The clock connection $A^{\mathrm{clk}} = Q + aK$ is a STRUCTURAL construction; the parameter β in $\beta \oint a = \pi/2$ is a clock-period *convention* (β_K), not the old $\beta_d = 11/(6\pi)$ derivation, which is dead as a derivation ($K \notin \mathfrak{g}_2$). The reading of the turnover as a four-tick "past→past, future→future" return — T accruing a -1 phase on reflection at the Det = 0 wall ($T^2 = -1$, $T^4 = +1$, a Kramers/spinor double cover) — is validated as sound *classical holonomy* (the D905 review, S175): a genuine structural clarification of how the clock turns over at the bounce, but, being classical geodesic scattering rather than a Coleman event, it generates no dimensionless coupling (it is the fifth leg of the α fence, Appendix N).

11 The moment–annulus synthesis and the dual collapse

Speculative / interpretive layer (S118) — fenced; no public observable moves.

Moment ≡ annulus ≡ $\mathbb{Z}_2$-twisted cylinder (A858/A860). The dial moment multiset on the **56**, $\{-3{:}1,\ -1{:}27,\ +1{:}27,\ +3{:}1\}$, is exactly the annulus shell content $\mathbf{1}\oplus\mathbf{27}\oplus\mathbf{27'}\oplus\mathbf{1}$, and $60 = 30 + 30$ splits as the two $27/27'$ shells. The change of basis between the linear (charge) and geometric (mass) ladders is the *exact* Fibonacci identity $\varphi^n = F_n\varphi + F_{n-1}$. The half-turn satisfies $T^2 = -I$ on the full **56** (verified to 1.3×10^{-15}; spin-$\frac{1}{2}$/Kramers). The synthesis is *kinematic* and substrate-grounded; the rest-mass depth selector stays *dynamical* — no parameter-free container rule over $\{16, 27, 60, 3, 30\}$ reproduces the lepton depths $\{0, 11, 17\}$ (A860; the isolated coincidence $27 - 16 = 11 = n_\mu$ has no τ parallel), so the selector is the A1/dynamics wall again, not packing combinatorics. (Fenced: Möbius–LQG quantization walled at D804; the dial circle is the *internal* moment circle, not a spatial KK circle.)

The Koide half-depth link is a kinematic observation, explicitly fenced (S118, sharpened Rev18 per A865b). The Möbius double cover ($\times 2$) reproduces the $\sqrt{m}$ half-depth structure of the Koide relation as a *change-of-basis observation* on the grading circle, *not* a derived fermion propagator. The geometric origin of the factor 2 is the $T^2 = -I$ Kramers cover; we do *not* claim it is bound to a physical Koide self-energy. Absent an explicit propagator derivation this link is held at SPECULATIVE/INTERPRETIVE tier and contributes no scorecard datum — the standing fence against reading the $\times 2$ as numerology.

The golden saddle collapses moment-graded, dual about the present (A861). The invariant-potential $J_3(\mathbb{O}_s)$ Hessian (the statics of §7, *not* the gauged vacuum of §1) is stationary, Det = 1, signature $(15_+, 12_-)$. The A861 coordinate Hessian has twelve negative-curvature directions, coinciding with the twelve negative-norm directions of the declared tangent kinetic metric $G = -\partial\partial \log N$, while the corresponding $G^{-1}H$ eigenvalues are positive. They are the three off-diagonal octonion blocks $\times 4$, moment-graded under $K = \mathrm{diag}(-1, 0, 1)$: past $(1, 2)$ curvature -2φ, present $(1, 3)$ -4, future $(2, 3)$ $-2\varphi^2$. The collapse is graded past/present/future and *dual* about the grade-0 present (a fenced arrow-of-time seed). At rung one the absolute future/past coordinate-curvature ratio is ϕ, whereas the past/future generalized squared-frequency ratio in that kinetic convention is ϕ, giving a frequency ratio $\sqrt{\phi}$. A861's probe potential is not Eq. (1)'s, and the saddle is linearly oscillatory in that metric (`s1261`, A1774). This is STATICS only. *The bang/crunch curvature difference is not the cosmological constant*: the future−past difference is the dimensionless $O(1)$ number -2 (ratio φ), whereas $\Lambda \sim 1.4 \times 10^{-123}$ in Planck units — off by ~ 123 orders, a dimensionless Hessian versus a dimensionful energy density. The asymmetry is a real derived structural result; its identification with Λ is an honest NEGATIVE.

12 The weak-angle crossing (B4): a GeV-order, scheme-dominated note

A separate threshold probe (B4) tested whether the running weak mixing angle crosses $\phi^{-3} = 0.2360680$ at a fixed hadronic scale. The naive one-loop route fails (A870). With *proper multi-threshold* running the light quarks carry the rise, so the crossing migrates to the hadronic/IR scale: μ_{cross} is pushed sub-GeV (~ 0.15 GeV, band $[0.09, 0.55]$), and the once-quoted 1.8 GeV lies *outside* the band (kernel `s124h`, 4/4; A940; an intermediate single-threshold estimate gave ~ 1.04 GeV, A937). The honest statement is therefore: *a ϕ^{-3} crossing exists, at sub-GeV/GeV order, scheme- and threshold-dominated — not* a pinned $\mu^\star = 1.8$ GeV prediction. The sub-GeV region is non-perturbative (a hadronic R-ratio input is required); no observable moves.

13 The knot-neck geometry: figure-8 lepton neck and trefoil baryon neck (S233–S235)

The figure-8 lepton neck

The wormhole–black-hole neck is a Möbius $\mathbb{Z}_2$-twisted bundle: σ_C reverses the fibre and the first Stiefel–Whitney class $w_1 \neq 0$. Its base of events is the figure-8 knot complement — hyperbolic,

with volume 2.0298832… — so Mostow rigidity fixes *zero geometric moduli*; the holonomy is the spin cover $\mathrm{SL}(2,\mathbb{C}) \to SO(3,1)$ of $\mathrm{PSL}(2,\mathbb{C})$, with dilatation φ^2 (hence $\ln = 2\ln\varphi$, the banked τ)[1]. The throat glues smoothly by an affine map; the seam ring is an *intrinsic* curvature singularity ($R = \varepsilon/2\xi^2$, profile-independent) admitting no finite Israel thin shell. The one-loop gravitational determinant is analytic (Reidemeister) torsion: for the figure-8 monodromy $\det(I - tA)$ is the Alexander polynomial $t^2 - 3t + 1$, and the knot determinant $|\Delta(-1)| = 5$ = pentagon = $\mathbb{Z}/5$. The Euclidean no-boundary cap has action equal to the hyperbolic volume (dominant saddle); Perron–Frobenius φ^2 dominance sets the arrow. *Honest residues:* the single external scale (E4 by design — a conformal, Mostow-rigid geometry cannot manufacture a dimensionful constant) and the exact quantum-gravity one-loop measure.

The trefoil baryon neck

The baryon neck is the trefoil analogue: a Faddeev pole $R_B = \varphi$, knot determinant 3 = triangle, and — unlike the figure-8 — *two* shape moduli (not Mostow-rigid). Static shape selection is a **curvature-robust honest negative**: in the curved KV geometry ($K = -\frac{1}{4}$) the equilateral configuration is a curved-length *maximum*, with reduced shape Hessian in the circulant form $g\left(\begin{smallmatrix} -2 & 1 \\ 1 & -2 \end{smallmatrix}\right)$ (eigenvalue ratio exactly 3, both modes tachyonic); curvature softens the instability by $\approx 8\%$ but cannot flip its sign. Static-length functionals are therefore exhausted, and the remaining wall is the *quantum* source/propagator problem. Structurally, the spectrum closes in golden identities: $E_0 = \varphi^2$, $E_1 = \varphi^3$ (from $2\varphi^2 - 1 = \varphi^3$), $R_1 = \varphi$, and

$$R_2(\mu_3) = 2 - \frac{\mu_3}{2\varphi^2}, \qquad \rho^2 = \tfrac{1}{2}, \tag{5}$$

with ρ^2 derived given the identification contact-space ≡ two-cylinder twist space. This yields a *twist-sector map* (structural candidate, conditional): $\tau = +1 \Rightarrow \mu_3 = 2\varphi^{-2} \Rightarrow R_2 = 2 - \varphi^{-4} = 1.854$ ($N(1710)$-like); $\tau = -1 \Rightarrow \mu_3 = 0 \Rightarrow R_2 = 2$ ($N(1880)$-like/null). A house coverage theorem underwrites it: the cross-cubic trilinear has 64 nonzero unit channels lying on the 7 Fano quaternion lines, each single associative slice covering exactly 25%, the full vertex requiring the union = $\mathbb{O}_s$. The golden magnitude is pinned to a single exact target, $\kappa_{\mathrm{wall}} = \kappa_{\mathrm{golden}} = 9\varphi^{-2}/4$ (derived given E_{th}), with a measure-parity reading (democratic wall measure → null branch; quadratic/spectral → golden). Everything here is a spectroscopy *discriminator*, explicitly not a fit; $\sqrt{\sigma}$/tube-tension stays fenced and no MeV appears.

14 Open leads and kill gates (pre-registered)

The dynamics layer is deliberately incomplete. The decisive next computations are pre-registered; none is promoted here.

Q1. **Invariant-potential full-$J_3(\mathbb{O}_s)$ Hessian — statics, superseded dynamically.** On the *ungauged invariant potential* the full 27×27 Hessian at J_{vac} has signature $(15_+, 12_-, 0_0)$: the 12 negative modes are the split $(4,4)$ octonion directions, per-block curvature $\pm 2(x_i + 1) = \pm(2\phi, 4, 2\phi^2)$. This is G_2-protected — the unique G_2-invariant block quadratic *is* the indefinite

[1] An exact identity, recorded as a reading: $\tau = 2\ln\varphi = \sqrt{5}\,L(1,\chi_5) = \sqrt{5}\,\mathrm{Res}_{s=1}\zeta_{\mathbb{Q}(\sqrt{5})}(s)$ (the class-number formula for $\mathbb{Q}(\sqrt{5})$: class number 1, regulator $\ln\varphi$; `s1315`). No physical claim attaches to it.

split norm, so no G_2/F_4-covariant scalar term lifts it; only a G_2-breaking compact uplift reaches positive-definiteness, at $\eta_c = 2\phi^2$ (kernels `s112a`, `s113a`, reproducible). *Dynamically this is superseded*: on the gauged $E_{7(7)}/SU(8)$ bed at $\omega = \pi/4$ the golden point is tachyon-free Minkowski (Theorem 1.1); the slice saddle is the statics of the invariant toy, not the dynamical verdict.

Q2. **The one-loop stationary-point classification on the flat branch (the open stabilization target).** The C_{iab} build is done (Appendix Q, Gate H: inertia $(9, 0, 1)$); what is open is whether the one-loop potential has a certified stationary point on the six-dimensional flat branch, or a controlled stabilising contribution along D_{56} (§4).

Q3. **Is $\gamma = 1$ forced beyond AX1?** The unit magnitude is equivalent to AX1 $\rho^2 - \rho - 1 = 0$ (§3); whether AX1 itself is forced by the octonion/Peirce self-similarity (the σ fixed point $s = 1 + 1/s$) is the standing deeper question. If only AX1 is needed, ϕ is selected modulo the framework's single golden axiom.

Q4. **The generation↔Morse-cell assignment (A1).** The count $\chi(\mathbb{O}P^2) = 3$ is algebra-derived (§5); the assignment of the three families to the cells $\{0, 8, 16\}$ needs the golden mass-ordering (dynamics) and is folded as honest-open.

Q5. $\det G = 7$ **provenance.** Derive $\det G = 7$ from the E_7/annulus/missing-**16** geometry, or it remains the deepest named INPUT.

Rev25 update (the S216–S220 state of the deferred C_{iab} target). The stabilization item above was subsequently executed in the dynamics lane, and this appendix's account is updated to the S218–S220 state rather than its original one. (i) An intermediate AMBER verdict (S215) was *overturned* by a convention fix (S216) — it is void, not a standing wound. (ii) The named wall — A-tensor projector precision (S217) — was *killed* at S218 by the exact rebuild (kernel `s638`: GAP = 0.0; Gate C 1.1×10^{-15}). (iii) Gate H then ran the Coleman–Weinberg inertia readout: result $(9, 0, 1)$ — against the pre-registered blind prediction $(10, 0, 0)$ (A1329), scored per protocol as a **recorded loss**. The candidate mechanism therefore carries STRUCTURAL CANDIDATE wording (S220), neither promoted nor buried; the receipts, including the loss, are collected in Appendix Q. The dynamics tier itself stays held for Rev26; nothing in this appendix's fences moves.

Frame fence on every Coleman–Weinberg Hessian number (S217 kernels `s618/s619/s621/s623`; landed S272). Three separate live-frame recomputations bear on how CW quantities in this lane may be quoted, and they do not all point the same way.
(i) The sign is frame-robust. Recomputing $\mathrm{Str}(M^4 \log M^2)$ in the live self-consistent frame (coset scalar, actual DWS E-frame vectors, live $A_1^2 = 2.0106$) gives -45.66 (natural, $c_S = 12$, $c_V = 1$) and -45.89 (M^2/M^4 co-calibrated, $c_S = 12.011$, $c_V = 0.9971$), against -43.89 and -45.61 from the earlier stale-input frame. The CW log is **negative in all four live variants** and the magnitude moves by $\lesssim 2$ units. Unlike the sub-percent supertrace residual — which *does* flip sign stale↔live — the CW log is dominated by the heavy DN-scaled fermion sector, so the 0.2–0.8% input shift never flips it.
(ii) The finite-difference route to $H^{(1)}$ is branch-limited, not merely noisy. The ten flats are exactly massless at the vacuum, so a displacement h lifts nearby modes across the 10^{-4} "massive" threshold and the sector counts $[36, 48, 24, 8]$ do not stay fixed; shrinking h avoids the crossing but the outer Hessian amplifies the inner finite-difference noise as $1/h^2$. **There is no single h that is both branch-stable and low-noise.** At flat 0 the nominal diagonal $H_{aa} = +48.8$ carries `branch_ok = False` and a first-order Ward residual $d_a \,\mathrm{Str}M^2 = -1.78$, so that value is contaminated by a branch discontinuity and is **not quotable**. This is the pre-registered risk ("same DWS branch counts at every sample"), realized.
(iii) ⋆ The binding fence: two published flat bases are not the same subspace. The principal cosines between the live flat basis and the earlier one are $[\,1.000,\ 0.868,\ 0.693,\ 0.669,\ 0.626,\ 0.457,\ 0.424,\ 0.365,\ 0.143,\ 0.057\,]$ — agreement only in the first direction, and effectively orthogonal by the last. The kernel records `same_subspace = False`. **A CW Hessian computed on the earlier basis is therefore an object on a different subspace, not a competing estimate of the same number, and the two must never be compared or averaged.** On the live three-flat subset the fermion CW Hessian inertia is $(3, 0, 0)$ with eigenvalues $(-0.1927, -0.1727, -0.1421)$; the partial Ward gates are nonzero by construction (the scalar and vector sectors are absent), so **this is a scoped diagnostic and nothing here is promoted**. α and colour remain fenced, the scale remains the unit anchor U, and the engine is untouched.

Dead / quarantined (do not revive without a new hook). (a) $\beta_d = 11/(6\pi)$ as a *derivation* — dead (clock-period convention only). (b) Area spectrum / LQG quantization — dead-for-now. (c) An RG flow from Det = 0 (UV) to Det = 1 (IR) "explaining" the $t/b/c$ tensions — quarantined (conflicts with the firewall-locked A713 result). (d) Triality / the 3-axis as the fermion beat-depth selector — closed-negative (D805). (e) Selecting ϕ by minimizing a *single* invariant — gives the symmetric $(1, 1, 1)$ (the σ-even no-go, Prop. 2.1); only the three-constraint Route A and the σ-odd defect work. (f) Coleman–Weinberg as a *unique* golden *selector* — closed-negative in the tested σ-equivariant layers (Prop. 2.1); do not re-open it as the selector. (g) any "$9 + 9$" or "8 flats" count of the golden point's moduli — withdrawn (Rev33.2; §4); the certified one-loop inertia is Appendix Q's $(9, 0, 1)$.

Status

Derived selection layer, Definition A. **Banked:** the derived dynamical vacuum — golden = tachyon-free Minkowski critical point of dyonic $\omega = \pi/4$ $SO^*(8)$ (Theorem 1.1, A887); the σ-even/σ-

odd selection dichotomy — CW unable to uniquely select golden in the tested σ-equivariant layers (Prop. 2.1), the σ-odd unit defect the sole selector with $F = 1 \Leftrightarrow$ AX1 (Lemma 2.2); the BPS / source-completion theorem — direction, $+\frac{1}{2}\|B\|^2$ completion, and radial $m^2 = 7/4$ derived, unit magnitude $\equiv$ AX1 (Theorem 3.1); the generation count as a Morse index $\chi(\mathbb{O}P^2) = 3$ (Prop. 5.1); plus the retained probe layers (overdetermined invariant routes, geometric bounce, $E_{7(7)}$ clock $T^2 = -1$, moment–annulus synthesis).

Honest end-state. Golden vacuum *selection* is a *derived* σ-odd-defect mechanism *modulo the AX1 unit*, with Coleman–Weinberg structurally unable to uniquely select golden in the tested σ-equivariant layers. Full *stabilization* is *not* closed: the golden point is a tree-level Morse–Bott minimum on a six-dimensional flat branch that reaches an unstable origin, and at one loop (inertia $(9, 0, 1)$, Appendix Q) it is neither stationary nor below the origin; no source-free vacuum is certified. "Selected — yes; fully stabilized — not yet." This is the strongest honest form of the answer to the no-dynamics killer.

Fences. All on the certified substrate but Definition A: the unit magnitude is AX1, not derived from nothing; the residual one-loop lift is Appendix Q's Gate H, not a parity-block readout; the no-scale dilaton is a separate input; the tested finite endpoint/self-adjoint/source layer is exhausted for the 7/3 edge but its residual is non-finite; B4 is a sub-GeV/GeV-order scheme-dominated crossing, not a 1.8 GeV claim; the bounce is classical; the mass scale is INPUT. No public observable moves.

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Appendix J — The Code Hamiltonian from the Parent's Transversal Weyl Symmetry: the $C_3 \times C_3$ Penalty Form and the Reynolds-Projector Unification of the One-Generation Yukawa Sector (Rev33.2)

Derived at the symmetry level (s239/s240). NOT from a written SGTOE action — no Lagrangian yet imposes P_X, P_Z; that identification is the lone residual. No public observable moves.

Tom O'Sieg

August 2026

1 Scope and standing assumptions

This appendix records two results obtained at the *symmetry level* on top of the banked Peirce-monad qutrit code of A1039–A1041. We work in the parent Hilbert space

$$\mathcal{H} \;=\; \mathbb{C}^3_A \otimes \mathbb{C}^3_B \otimes \mathbb{C}^3_C \;\cong\; \mathbb{C}^{27},$$

the three tensor factors being the three Peirce monads, each modelled on the compact qutrit slice $H_3(\mathbb{C}_u) \hookrightarrow J_3(\mathbb{O}_s)$ identified in A1040. Let X, Z be the qutrit Weyl–Heisenberg clock and shift, satisfying the single-leg cocycle relation $ZX = \omega XZ$ with $\omega = e^{2\pi i/3}$. Define the *transversal* (three-leg) operators

$$S_X \;=\; X \otimes X \otimes X, \qquad S_Z \;=\; Z \otimes Z \otimes Z,$$

and the cycle averages

$$P_X \;=\; \tfrac{1}{3}\Big(I + S_X + S_X^2\Big), \qquad P_Z \;=\; \tfrac{1}{3}\Big(I + S_Z + S_Z^2\Big), \qquad P_{\text{code}} \;=\; P_X\, P_Z.$$

A1039/A1040 established (and s239 re-verified from the banked matrices) that $[P_X, P_Z] = 0$, $\operatorname{rank} P_{\text{code}} = 3$, and that the rank-3 image $\operatorname{im} P_{\text{code}}$ is the Peirce-qutrit code. The code Hamiltonian was, at that stage, a *stated* commuting-stabilizer choice,

$$H_{\text{code}} \;=\; (I - P_X) + (I - P_Z),$$

with $\ker H_{\text{code}} = \operatorname{im} P_{\text{code}}$ and a spectral gap $\Delta = 1$. The framework gates of A1039/A1040 (`all_SGTOE_embedding_gates_pass` and `all_full_SGTOE_derivation_gates_pass`) were correctly left **False** on exactly the point "*the SGTOE parent action does not yet impose P_X, P_Z.*"

The two results below are honestly partial. **Result 1** (s239) shows that the frustration-free penalty *form* of H_{code} is forced (not a free choice) by a symmetry the parent *already has* — the transversal Weyl–Heisenberg $C_3 \times C_3$ — with the symmetric point $a = b = 1$ a normalization (no proven $X \leftrightarrow Z$ swap is exhibited here that would force $a = b$); and that the cubic (three-body) arity is forced by

the qutrit cocycle and coincides with the arity of the exceptional Jordan cubic. **Result 2** (s240) shows that P_{code} and the A1048 hypercharge projector $\mathcal{P}_{Y=0}$ are the *same* operator class — the Reynolds (group-average) projector. That class result does not by itself produce the democratic one-generation Yukawa ray (Rev32.9, A1652 F01b: the "this forces the democratic ray" gloss printed here through Rev32.8 is deleted; see the multiplicity-one hypothesis of Proposition 3.2). Neither result touches a public observable. Both carry forward all standing fences: the compact unit u is unselected (the S^2 weak-plane family of A1041), $N_{\text{gen}} = 3$ is an input, and the single actual outcome is RED.

2 Result 1 (s239): the forced frustration-free penalty form of the transversal Weyl $C_3 \times C_3$

2.1 Three-body arity is forced by the qutrit cocycle

On a single qutrit, $ZX = \omega XZ$ (verified residual 6×10^{-16}). On n transversal legs the cocycle multiplies:

$$(Z^{\otimes n})(X^{\otimes n}) \;=\; \omega^{\,n}\,(X^{\otimes n})(Z^{\otimes n}).$$

Hence the transversal clock and shift *commute* iff $\omega^{\,n} = 1$ iff $3 \mid n$.

Novelty label: new specialization proved here.

Lemma 2.1 (Cocycle-forced cubic arity)**.** *$S_X^{(n)} = X^{\otimes n}$ and $S_Z^{(n)} = Z^{\otimes n}$ commute if and only if $3 \mid n$; $n = 3$ is the minimal positive leg number that abelianises the transversal clock and shift.*

The gate values are unambiguous: at $n = 1$ and $n = 2$ the operators do *not* commute (at $n = 2$ the commutator norm is $\|[S_X^{(2)}, S_Z^{(2)}]\| = 5.20$, the ω^2 phase), while at $n = 3$ they commute to $\|[S_X, S_Z]\| = 2.8 \times 10^{-15}$. An independent cross-check script reproduces the 5.20 (non-commuting) versus 2.8×10^{-15} (commuting) values. Therefore the three-monad interaction is *not* a modelling choice: it is the minimal leg count at which a commuting-stabilizer code of this transversal form can exist at all, and 3 is exactly the arity of the exceptional Jordan cubic trilinear. This discharges the prior open item B5 ("three monads = legs of one trivalent cubic vertex").

2.2 The code is the Weyl-singlet sector

The group $\langle S_X, S_Z \rangle$ is an abelian $C_3 \times C_3$ acting on $\mathbb{C}^{27}$. Classifying $\mathbb{C}^{27}$ by the nine (j, k) isotypes via the cycle-average projectors

$$P_j^{(X)} \;=\; \tfrac{1}{3}\sum_{m=0}^{2} \omega^{-jm} S_X^m, \qquad P_k^{(Z)} \;=\; \tfrac{1}{3}\sum_{m=0}^{2} \omega^{-km} S_Z^m,$$

each of the nine joint isotypes $P_j^{(X)} P_k^{(Z)}$ has dimension exactly 3 ($27 = 9 \times 3$, the regular-representation structure), and the trivial–trivial isotype coincides with the code:

$$P_0^{(X)}\, P_0^{(Z)} \;=\; P_{\text{code}} \qquad (\|P_{(0,0)} - P_{\text{code}}\| = 1.9 \times 10^{-15}).$$

Novelty label: new specialization proved here.

Proposition 2.2 (Code = Weyl singlet)**.** *P_{code} is the (trivial, trivial)-isotype projector of the transversal Weyl–Heisenberg $C_3 \times C_3$; equivalently the group-average projector onto the Weyl-singlet sector. The monad code is precisely the gauge-singlet subspace of the parent's transversal Weyl symmetry.*

2.3 The penalty form is forced (relative weight a normalization)

Within the abelian $*$-algebra $\mathcal{A} = \langle S_X, S_Z \rangle$ (the $C_3 \times C_3$ group algebra), consider the most general Hermitian penalty built from the two cycle averages with the code as zero-energy ground:

$$H(a,b) \;=\; a\,(I - P_X) + b\,(I - P_Z), \qquad a, b > 0.$$

Novelty label: new specialization proved here.

Theorem 2.3 (Forced frustration-free Weyl-invariant penalty form)**.** *For all weights $a, b > 0$:*

1. *the ground space of $H(a,b)$ equals $\operatorname{im} P_{\text{code}}$ (rank 3), independent of a, b;*
2. *the spectral gap is $\Delta = \min(a,b)$;*
3. *each term is positive semidefinite and annihilates the code $\big((I-P_X)P_{\text{code}} = (I-P_Z)P_{\text{code}} = 0\big)$, so $H(a,b)$ is frustration-free;*
4. *both C_3 factors are required: dropping either term leaves a rank-9 ground space, not the rank-3 code.*

Consequently the functional form *of the penalty — both terms $(I - P_X)$ and $(I - P_Z)$ required, frustration-free, with ground space exactly $\operatorname{im} P_{\text{code}}$ — is forced by the transversal Weyl $C_3 \times C_3$ symmetry. The symmetric representative $H_{\text{code}} = (I - P_X) + (I - P_Z)$ ($a = b = 1$) is the natural normalization of this family.*

Remark 2.4 (What is and is not fixed)**.** Clauses (i)–(iv) establish a *two-parameter* family $H(a,b)$, $a, b > 0$, all sharing the code ground space; they do *not* single out $a = b = 1$. Selecting the symmetric point as *unique* would require an additional $X \leftrightarrow Z$ duality that actually *swaps* P_X and P_Z, together with an overall normalization. The involutions exhibited in §2.4 (Weyl inversion R, golden reversal J) *fix* P_X and P_Z individually and do not supply that swap. We therefore claim only that the penalty *form* is symmetry-forced and that $a = b = 1$ is its normalized symmetric point; the relative weight $a : b$ is a normalization, not a derived number. Exhibiting the swap duality (and thereby promoting "form forced" to "H_{code} unique") is left open.

The scan over $(a,b) \in \{(1,1), (0.4,1.7), (2.3,0.6), (1,3)\}$ returns rank-3 code ground in every case with $\Delta = \min(a,b)$; the single-term Hamiltonians $(I - P_X)$ and $(I - P_Z)$ each have a rank-9 ground, confirming clause (iv). Thus the Weyl symmetry *forces* the functional form, and H_{code} is its normalized symmetric point. The overall positive scale (the gap units) remains a normalization, not a derived number.

2.4 The primal/dual involution is a dynamical symmetry of H_{code}

The s236 primal/dual involution motif (T2) was previously a *structural* choice. Two distinct involutions of the qutrit parent are exact symmetries of the *derived* H_{code}:

- **Weyl inversion** $R : |i\rangle \mapsto |-i \bmod 3\rangle$, with $RXR = X^{-1}$, $RZR = Z^{-1}$. Each cycle average is invariant under $g \mapsto g^{-1}$ (since $\{I, S, S^2\} = \{I, S^{-1}, S^{-2}\}$), so R fixes P_X, P_Z. With $U_\sigma = R \otimes R \otimes R$ one finds $[U_\sigma, H_{\text{code}}] = 2.1 \times 10^{-15}$ (the ghost-sign / round-trip leg).
- **Golden reversal** $J : |i\rangle \mapsto |2 - i\rangle$, with $JD_\varphi J = D_\varphi^{-1}$ (the dual-VEV reciprocity $D_\varphi \leftrightarrow D_\varphi^{-1}$ on the golden vacuum $D_\varphi = \text{diag}(\varphi, 1, \varphi^{-1})$), and $[U_J, H_{\text{code}}] = 2.8 \times 10^{-15}$ (the dual-VEV leg).

Novelty label: new specialization proved here.

Proposition 2.5 (Structural → dynamical)**.** *The s236 primal/dual involution motif (ghost sign and dual VEV $D_\varphi \leftrightarrow D_\varphi^{-1}$) is realised as an exact symmetry of the derived H_{code}: $[U_\sigma, H_{\text{code}}] \approx 2 \times 10^{-15}$ and $[U_J, H_{\text{code}}] \approx 3 \times 10^{-15}$. It is thereby upgraded from a structural choice to a dynamical symmetry, discharging the B4-pointer rung at the symmetry level.*

3 Result 2 (s240): the Reynolds-projector unification of the one-generation Yukawa sector

3.1 P_{code} and $\mathcal{P}_{Y=0}$ are one operator class

A *Reynolds projector* is the canonical projector onto the trivial-isotype (singlet) subspace of a finite or compact group G:

$$\mathcal{P}_G \;=\; \frac{1}{|G|} \sum_{g \in G} \rho(g) \quad \text{(finite } G\text{)}, \qquad \mathcal{P}_G \;=\; \int_G \rho(g)\, dg \quad \text{(compact } G\text{)}.$$

It satisfies the defining identities $\mathcal{P}^2 = \mathcal{P}$, $\mathcal{P}^\dagger = \mathcal{P}$, and the group-absorption relation $\mathcal{P}\,\rho(g) = \rho(g)\,\mathcal{P} = \mathcal{P}$.

The code projector is exactly the $C_3 \times C_3$ group average:

$$P_{\text{code}} \;=\; \frac{1}{9} \sum_{j,k=0}^{2} S_X^j S_Z^k \qquad \text{(residual } 1.9 \times 10^{-15}\text{)},$$

idempotent (5.8×10^{-16}), Hermitian (0.0), and group-absorbing ($\|P_{\text{code}} S - P_{\text{code}}\| = 1.2 \times 10^{-15}$).

The A1048 hypercharge projector $\mathcal{P}_{Y=0} = \text{indicator}(Y_{\text{total}} = 0)$ is the $U(1)_Y$ group average onto $\ker(Y_{\text{total}})$. Reconstructed from the banked Y_{16} grading and $Y_{\text{Higgs}} = \{\pm\frac{1}{2}\}$, it equals the shipped hypercharge mask exactly (err 0.0), is idempotent, satisfies $\mathcal{P}_{Y=0}\, T_{\text{raw}} = T_{\text{proj}}$ (err 0.0), and removes precisely the forbidden mirror half $Y_{\text{total}} \in \{-1, +1\}$.

Novelty label: standard theorem used.

Theorem 3.1 (Reynolds unification)**.** *P_{code} (transversal Weyl $C_3 \times C_3$) and $\mathcal{P}_{Y=0}$ ($U(1)_Y$) both satisfy the defining Reynolds identities $\mathcal{P}^2 = \mathcal{P}$, $\mathcal{P}^\dagger = \mathcal{P}$, $\mathcal{P}\rho(g) = \mathcal{P}$ (group absorption). They are the same operator* class *— each is the Reynolds projector onto the singlet sector of its own group — but not the same operator: Reynolds projectors of different groups have different images, so a conclusion established for one is not thereby established for the other (Rev32.9, A1652 F01b). The loaded one-generation Yukawa sector is therefore*

$$T_{\text{loaded}} \;=\; (\textit{Weyl-singlet Reynolds}) \circ (\textit{hypercharge-singlet Reynolds})\, T_{\text{raw}},$$

i.e. the raw exceptional cubic averaged over the parent symmetry and restricted to its singlet.

The shipped norms anchor the half-leak: $\|T_{\text{raw}}\|^2 = 64$, $\|T_{\text{proj}}\|^2 = 32$ (leak ratio exactly $\frac{1}{2}$), with the removed $\|\cdot\|^2 = 32$ carried entirely by the forbidden $Y = \pm 1$ components.

3.2 Democracy is Reynolds-forced; splitting requires a compensator

A Reynolds projector *selects* the singlet sector; it does not reweight within it (item 3 below is the load-bearing statement). Whether the selected sector can then be split depends on its multiplicity (Proposition 3.2). The banked data make this concrete:

1. the invariant coefficients are democratic, $(1, 1, 1, 1)$, with spread 2.2×10^{-16};
2. democracy is genuine, not a unit-normalization artifact: the channel basis is orthogonal (off-diagonal Gram 0.0) but has *unequal* norms (Gram-diagonal spread 8.0, from distinct channel multiplicities), yet the cubic coefficients are all exactly 1;
3. re-projecting the democratic sum with $\mathcal{P}_{Y=0}$ keeps $(1, 1, 1, 1)$ (spread 2.2×10^{-16}): the Reynolds projector *selects*, it does not reweight;
4. a deterministic non-invariant golden spurion (weighting channel f by φ^{k_f}, $k = [3, 0, 1, 2]$) lifts the ray to coefficient spread 3.236, breaking democracy.

Novelty label: standard theorem used.

Proposition 3.2 (Compensator required)**.** At trivial multiplicity one — *when the singlet sector of the averaged symmetry G has multiplicity one on the four-channel span — no operation invariant under G can split the democratic* $(1, 1, 1, 1)$ *ray of the one-generation Yukawa sector (Schur). Without that hypothesis the statement is false:* $\text{diag}(2, 1, 1, 1)$ *is an invariant operator that splits the ray at multiplicity four (`s1172` T2).* Exhibition *(Rev32.9, A1652 F01b): the multiplicity-one mechanism is exhibited by `s243` on the model group S_4 permuting the four channels ($\mathbb{C}^4 = \mathbf{1} \oplus \mathbf{3}$; its gate G4 is the $m = 2$ counter-model). The suite has* not *exhibited a registered group acting on the four physical channels with trivial multiplicity one: $U(1)_Y$ fixes all four channels, and $|C_3 \times C_3| = 9$ admits no orbit of size* 4*. The hypothesis is therefore a premise of this proposition, not a derived fact. A compensator* outside *the invariance is mathematically required to lift the ray; an explicit golden spurion is shown to do so (spread* 3.236*), naming golden J_{vac} as a viable candidate structure.*

This is the in-house half of the D885 question: it converts "*is a compensator needed?*" into "*a compensator is mathematically required*," and identifies the golden J_{vac} as the candidate. It does *not* derive the four physical Yukawa coefficients — the spurion shows breaking is *possible* and supplies a candidate, not the physical hierarchy.

4 Result 3 (s241–s246): the written-action layer and the off-diagonal no-go theorems

This section folds the S160–S161 written-action upgrades (in-house kernels `s241-s246`), which sharpen Results 1–2 from the symmetry level toward an explicit one-generation action and close several off-diagonal routes at theorem grade. Nothing here moves a public observable.

(s241) Manifest kinetic term. The code Hamiltonian is exhibited as a manifest sum of squares,

$$H_{\text{code}} = (I - P_X) + (I - P_Z),$$

a frustration-free, manifestly positive-semidefinite kinetic form whose zero modes are exactly the joint ± 1 eigenspace of the primal/dual projectors P_X, P_Z. This closes the W2 written-Lagrangian residual at the level of the kinetic term.

(s242) Tiered one-generation Lagrangian and the quarantine theorem. Assembling the kinetic term with the cubic-invariant Yukawa of Result 2 gives a tiered one-generation Lagrangian in which the core democratic symmetry is realised exactly.

Novelty label: new specialization proved here.

Theorem 4.1 (Quarantine, `s242`)**.** *In the assembled one-generation action the within-generation hierarchy is confined entirely to the input tier: the democratic core is exact and symmetry-protected, and every hierarchy-generating quantity is an external input, not a derived coefficient.*

This is the constructive companion to Appendix K's negative: the action *isolates* the undetermined coefficients rather than hiding them.

(s244–s246) Off-diagonal no-go at theorem grade. Three further kernels close the off-diagonal selector routes.

Novelty label: new specialization proved here.

Theorem 4.2 (Joint frame$\times\eta$ Klein-four no-go, `s244`)**.** *No joint action of the frame rotation and the η-grading (a Klein four-group of candidate off-diagonal selectors) generates the within-generation splitting; the multiplicity-one fence forbids it and an explicit $m = 2$ counter-model saturates the bound.*

The nucleation analysis (`s245`) shows the "perfect-imperfection" defect $O = \frac{1}{2}\text{diag}(1, 0, -1)$ is a *seed, not a source*: it can mark where a splitting would nucleate but cannot supply its magnitude. Kernel `s246` locks the frame orientation σ_{frame} to the loaded vacuum J_{vac}, removing it as a free off-diagonal handle. Together these upgrade the Reynolds/Schur no-go (Appendix K, `s243`) to theorem grade.

5 Residual debt and firewall status

Remark 5.1 (What is derived, what is not)**.** Result 1 derives H_{code} *at the symmetry level*: it is the unique frustration-free penalty enforcing the parent's transversal Weyl $C_3 \times C_3$, with cubic arity forced by the qutrit cocycle, and with the primal/dual involution realised as a dynamical symmetry. Result 2 unifies the one-generation mass mechanism as a single Reynolds projection and proves democracy is forced. There is still *no* written microscopic SGTOE Lagrangian exhibiting P_X, P_Z as explicit operator terms.

Residual fence (the lone open piece). The remaining debt is to exhibit the transversal Weyl $C_3 \times C_3$ as a symmetry of the *written* SGTOE parent Lagrangian. This is conditional on the standing A1039/A1040 identification of the qutrit clock/shift with the Peirce-triality structure of the Albert cubic $J_3(\mathbb{O}_s)$. Everything in this appendix is derived at the symmetry level; the written-Lagrangian identification of that Weyl symmetry is the single outstanding item. Carried fences, unchanged: the compact unit u is unselected (the S^2 weak-plane family of A1041, an unbroken Sp(1) no-go), the overall positive scale of H_{code} is a normalization, $N_{\text{gen}} = 3$ is an input, and the four physical Yukawa coefficients are not derived (compensator required, not supplied). Debt *narrowed*, not closed.

Firewall. No public observable moves. The single actual outcome remains **RED / untouched**: no collapse claim is made. The engine and presentation workbooks are untouched (the kernels read banked cold-storage `npz` only; RNG-free, run1=run2 byte-identical). Nothing in this appendix is promoted to a prediction.

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Appendix K — The Within-Generation Yukawa Coefficients are Irreducibly External to the Banked Parent Action: a Scoped Honest Negative from Five Independent Angles (Rev33.2)

Derived: 4 SM Yukawa tensors + democratic normalization. Coefficients = irreducibly external (scoped to the banked framework). No public observable moves.

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August 2026

1 Purpose and scope

This appendix records a *conditionally-closed honest negative*. It is the sharpest statement the SuperGrokTOE program has reached on what the exceptional-Jordan parent action can and cannot fix about the Standard-Model (SM) Yukawa sector. The result has two halves, and keeping them separated is the whole point of the appendix.

1. **What is derived (positive).** A $\mathrm{Der}(\mathbb{O}_s)$-invariant trilinear on the loaded Albert cubic (uniqueness of the invariant not claimed; see Theorem 2.1) projects onto exactly the four SM Yukawa *tensors* $\{Q\,u^c H,\ L\,\nu^c H,\ Q\,d^c H^c,\ L\,e^c H^c\}$, with a single *democratic* common normalization $(1,1,1,1)$. [LIB2-111] This is a UV/matching statement; it is real and it is banked.
2. **What is not derived (negative, scoped).** The four distinct physical coefficients $\{y_u, y_d, y_e, y_\nu\}$ — i.e. the entire within-generation mass hierarchy — do *not* follow from the banked parent action. [LIB2-118] Six independent analytic and numerical angles agree on this. A symmetry-breaking compensator lying *outside* the banked invariance structure would be required, and none of the banked enhanced symmetries supplies one.

The negative is **scoped**, not universal. It is a no-go over the *banked* exceptional-Jordan / loaded-gauge parent action together with perturbative running and the banked enhanced-symmetry set. Non-perturbative and composite strong-coupling dynamics are *not* banked, hence are *not* excluded in general. The honest published claim is therefore exactly: *the four SM Yukawa tensors plus their democratic normalization, full stop.* No public observable moves; no dimensionful anchor is dropped; $N_{\mathrm{gen}} = 3$ remains an input.

2 What the framework does derive

2.1 The unique exceptional-cubic coupling

Work over the split octonions $\mathbb{O}_s$ (signature $(4,4)$; imaginary part $\mathrm{Im}(\mathbb{O}_s)$ of signature $(3,4)$) and the Albert algebra $J_3(\mathbb{O}_s)$. The derivation algebra $\mathrm{Der}(\mathbb{O}_s)$ is the split form $\mathfrak{g}_{2(2)}$, of dimension 14, and the G_2-invariant symmetric form on $\mathrm{Im}(\mathbb{O}_s)$ is one-dimensional (proportional to the polarization $\eta_{3,4}$).

Novelty label: standard theorem used.

Theorem 2.1 (An invariant trilinear)**.** *The cubic Yukawa coupling*

$$T(c,b,a) \;=\; \mathrm{Re}\big(c\cdot(b\cdot a)\big)$$

is $\mathrm{Der}(\mathbb{O}_s)$*-invariant and nonzero. [LIB2-110]* Uniqueness caveat: *that T is the* only *such invariant trilinear is* not *established here — it would require computing the invariant multiplicity* $\dim \mathrm{Hom}_{G_{2(2)}}(V_1\otimes V_2\otimes V_3,\mathbb{R})$ *with the exact representations and scalar components specified. We use only invariance (verified below), not uniqueness; the democracy conclusion does not depend on uniqueness. [LIB2-110]*

The invariance is verified to machine precision: the maximal Leibniz residual obeys

$$\max\Big|\sum T(D\cdot,\cdot,\cdot)\Big| \;=\; 4.2\times10^{-15}, \qquad \max|T| \;=\; 12.8,$$

so T is genuinely nonzero and genuinely invariant. The conjugated form $\tau(x,y,z)=\mathrm{Re}\big[(\bar{x}y)\bar{z}\big]$ used by an independent component-level check is the same coupling up to conjugation, and is likewise $\mathrm{Der}(\mathbb{O}_s)$-invariant.

2.2 G_2 transitivity forces democracy

Novelty label: standard theorem used.

Lemma 2.2 (Transitivity on a fixed-norm imaginary orbit, split form)**.** $\mathrm{Im}(\mathbb{O}_s)$ *carries an* indefinite $(3,4)$ *form, so it is* not *the compact S^6; the compact identity $G_2/SU(3)\cong S^6$ may not be imported directly. The relevant group is the* split *real form $G_{2(2)}$, which acts on* $\mathrm{Im}(\mathbb{O}_s)$ *preserving the norm, with distinct orbit classes for positive-, negative-, and null-norm vectors. On a fixed positive-norm pseudo-sphere the* Der*-orbit tangent has dimension* 6*, and $G_{2(2)}$ acts transitively on that single norm-class orbit. The four gauge-allowed channels (the $Y=0$ components of the census in Angle (b), §3.2) lie within one such orbit class, on which any $G_{2(2)}$-invariant trilinear is constant. (Equality of a local tangent dimension with* 6 *is consistent with, but does not by itself prove, global transitivity; the latter is the standard split-G_2 orbit statement, used here per-orbit-class.)*

Novelty label: new specialization proved here.

Proposition 2.3 (Democratic readout)**.** *Evaluated across the seven G_2-rotated copies of a single channel, the cubic coupling T is constant (spread 3.6×10^{-12}; all values $=1.044419$). Consequently channels that differ only by their hypercharge assignment receive* identical *cubic weight.*

Theorem 2.1 plus Lemma 2.2 give Proposition 2.3: an invariant trilinear, evaluated on an orbit on which the symmetry acts transitively, cannot distinguish points of that orbit. *Premise, stated and not derived (Rev32.10, A1717-F02):* Proposition 2.3 exhibits transitivity on the orbit of a *single* channel vector (its seven G_2-rotated copies); passing from that vector orbit to the four channel *triples* requires that the four normalized ordered triples lie on one common $G_{2(2)}$ orbit under a single group action. No registered-frame component certificate identifying the four triples under one action is exhibited here, so common triple-orbit membership is a premise of this subsection. Under that premise the four gauge-allowed channels enter with the common democratic ray

$$(c_u,\, c_\nu,\, c_d,\, c_e) \;=\; (1,\, 1,\, 1,\, 1).$$

This is the G_2 tight-frame value. It is a UV normalization statement, not a low-energy prediction. [LIB2-113]

Net of §2. The exceptional cubic derives *the four SM Yukawa tensors* $\{Q\,u^cH,\, L\,\nu^cH,\, Q\,d^cH^c,\, L\,e^cH^c\}$ *and one democratic normalization* $(1,1,1,1)$ — and *nothing finer.* Everything that remains (the four coefficient ratios = the fermion mass hierarchy) is external to the invariant cubic.

3 Six independent angles that the within-generation hierarchy is external

The following six lines were obtained by distinct methods — abstract group theory, explicit component census, Reynolds-projector algebra, an enhanced-symmetry candidate sweep, one-loop renormalization-group (RG) running, and the parent-action cubic-cone kinetic metric. They converge on a single conclusion.

3.1 Angle (a): the 7–3 fence (s238)

Distinct coefficients $\{y_u, y_d, y_e, y_\nu\}$ would have to be supplied by the loaded $U(1)_Y$, whose Casimirs are distinct,

$$q_d = 0.494, \qquad q_u = 0.538, \qquad q_\nu = 0,$$

but $U(1)_Y \not\subset \mathrm{Der}(\mathbb{O}_s) = G_2$. A G_2-invariant cubic cannot carry an object that lives outside G_2. The cubic therefore yields only the democratic value; any splitting requires G_2-breaking loading. This is precisely the standing k=1 LSZ / 7–3 running fence.

3.2 Angle (b): exact half-norm mirror leakage (A1048)

At the explicit component level, the raw octonionic trilinear τ carries exactly half of its squared norm as $Y = \pm 1$ "forbidden mirror" leakage:

$$|\tau_{\mathrm{raw}}|^2 = 64.0, \qquad |\tau_{\mathrm{proj}}|^2 = 32.0, \qquad \text{leakage fraction} = \tfrac{1}{2}\ \text{(exact)}.$$

The loaded $U(1)_Y$ Reynolds projector $\mathcal{P}_{Y=0}$ removes this leak exactly — the projected tensor equals the mask-times-raw tensor ($\|\mathrm{diff}\| = 0$) and equals the four SM tensors ($\|\mathrm{diff}\| = 0$). [LIB2-111, LIB2-112] The component census confirms every forbidden component sits at total hypercharge ± 1

(HQd^c, HLe^c at $+1$; $H^cQu^c, H^cL\nu^c$ at -1) and every allowed component at $Y = 0$. Thus the raw J_3 cubic is *not* the physical Yukawa tensor without the $U(1)_Y$ projection; the projection that makes it physical is precisely the loaded $U(1)_Y$ Reynolds average. That average removes the nonzero-total-hypercharge components and acts as the identity on the four allowed tensors, so it *retains* their common coefficients: this is preservation, not selection — the averaging by itself neither singles out equality nor excludes an unequal quartet on the same span (Angle (c); `s1171` T1).

3.3 Angle (c): democracy is Reynolds-preserved (s239/s240; cross-ref. Appendix J)

The monad-code penalty H_{code} (Appendix J) is the symmetry-forced transversal-Weyl-singlet penalty form, and its projector P_{code} ($C_3 \times C_3$) lies in the *same Reynolds-projector class* as $\mathcal{P}_{Y=0}$ ($U(1)_Y$). Both are group averages of *different* groups, so they are one class, not one operator (Appendix J). The Reynolds projector *selects* the singlet sector and does not reweight within it; $\mathcal{P}_{Y=0}$ is the identity on the span of the four displayed tensors, which all carry total hypercharge 0 (`s1171` T1). [LIB2-113] The democratic ray is protected *at trivial multiplicity one* (Appendix J, Proposition on the compensator; exhibition by `s243` on a model group), and that hypothesis is a premise here, not derived: no registered group acts transitively on the four channels ($U(1)_Y$ fixes them; $|C_3 \times C_3| = 9$ admits no orbit of size 4). (Rev32.9, A1652 F01b/S330.7: through Rev32.8 this paragraph derived the ray from "a group average over a symmetry that acts transitively on the channels", which is unavailable for both projectors.) Under that premise democracy is not an accident of normalization. [LIB2-113] To split $(1, 1, 1, 1)$ one needs a compensator lying *outside* the invariance being averaged over; within the banked structure such a compensator is *required*, not optional.

3.4 Angle (d): the candidate-compensator sweep fails (A1049/A1050)

A sweep of the banked enhanced-symmetry candidates was performed; each fails to produce the required complete within-generation hierarchy while respecting the projected channel census (Rev32.9, A1652: the dual-VEV bullet below does split up-type from down-type, so "fails to split democracy" was too broad):

- $F_4 \supset G_2$, $E_6 \supset F_4$, $B-L$, J_{vac}**:** each either preserves the joint $G_2 \times U(1)_Y$ invariance (and so stays democratic) or reintroduces mixing with the already-projected-out mirror sectors (violating the A1048 tensor census). [LIB2-114] $U(1)_Y$ *relabels* the channels; it does not *split* them.
- **Dual-VEV** ($\tan\beta \neq 1$)**:** splits up-type from down-type by $O(10\text{–}100)$ but *structurally cannot cross the quark↔lepton axis* — it cannot separate m_t from m_u, nor m_b from m_τ. [LIB2-114]
- **Perturbative Z-factors:** $Z_i \approx 1 - \frac{g^2}{16\pi^2} C_2(R_i) \ln(\Lambda/\mu)$ are bounded well below 10^3; reaching 10^{-5} would require non-physical exponentiation over trans-Planckian scales. The observed intra-generation split is $\sim 10^5$.
- J_{vac} **spurion:** reaching $y_e/y_t \sim 3 \times 10^{-6}$ needs depth $\approx J_{\text{vac}}^{24}$ with $\varphi^{-24} \approx 9.6 \times 10^{-6}$ — a dimension-$\sim$28 insertion. This is numerology, not a leading-order mechanism. (Verified in-house: $\varphi^{-24} = 9.645 \times 10^{-6}$; exact depth for 3×10^{-6} is $n \approx 26.4$.) J_{vac} governs the $O(1)$ inter-generation beat-depth ratios well, but categorically fails the 10^5 intra-generation split.

3.5 Angle (e): residue map rank-4; running alone fails (A1051)

Write the field-residue map in log variables,

$$y_f \;=\; \frac{y_\star}{\sqrt{Z_{L_f}\, Z_{R_f}\, Z_{H_f}}},$$

with channel supports $\{Qu^cH\}, \{Qd^cH^c\}, \{Le^cH^c\}, \{L\nu^cH\}$, written with a single Higgs H and its conjugate H^c (Paper 1 §6 names H_u, H_d). **The count below treats Z_H and Z_{H^c} as formally independent**: eight residues, rank 4, kernel dimension 4. Identifying $Z_{H^c} = Z_H$ merges the two Higgs columns and leaves seven residues at rank 4 and kernel dimension 3, so moving from one conjugate pair to two independent doublets takes the kernel dimension from 3 to 4, not from 4 to 3 (`s1190`; `s1171` T4; Rev32.9, A1652). Rev32.10 and earlier printed that direction reversed and attached the eight-residue count to the single-Higgs label. The corresponding 4×8 incidence matrix has *rank* 4 (each channel carries a unique pivot Z-factor). [LIB2-115]

Novelty label: new specialization proved here.

Theorem 3.1 (Residue underdetermination)**.** *Because the residue map is rank* 4 *over eight independent Z-factors, the eight residues can realize* any *positive Yukawa quartet. [LIB2-115] The four coefficients are therefore* underdetermined *unless the parent action itself derives the residues and thresholds. If the residues stay Peirce-edge uniform, all four Yukawas remain equal (the golden edge product cancels:* $\varphi \cdot 1 \cdot \varphi^{-1} = 1$*, residual* $\sim 10^{-16}$*).*

Running alone cannot rescue the hierarchy. One-loop SM-plus-Dirac-ν running from the democratic unit ray lands at M_Z on

$$(y_u, y_d, y_e, y_\nu) \;=\; (1.0207,\ 0.9958,\ 0.3959,\ 0.3581),$$

whose ratios to the reference quartet are $(1.02,\ 41.8,\ 38.8,\ 1.2 \times 10^{12})$: the down/charged-lepton channels overshoot by $\sim 40\times$ and the Dirac-ν channel by $\sim 10^{12}\times$. Running produces $O(1)$ spread, not the observed $\sim 10^5$ intra-generation hierarchy. [LIB2-116] The Gram structure $\mathrm{diag}(12, 4, 12, 4)$ (color multiplicity) with unit coefficients confirms democracy is genuine, not a unit-normalization artifact.

3.6 Angle (f): the parent-action kinetic metric has relative rank zero (A1052)

The deepest angle works directly with the parent action's kinetic geometry rather than the cubic vertex. Take the cubic-cone metric $G_{AB}(J) = \partial_A \partial_B[-\log N(J)]$ at $J_{\mathrm{vac}} = \mathrm{diag}(\varphi, 1, \varphi^{-1})$ ($N = 1$; analytic metric matches the finite-difference Hessian to 1.7×10^{-6}; raw signature $(15, 12)$, $\det = 1$ tangent signature $(14, 12)$, reflection-positive metric strictly positive with minimal eigenvalue φ^{-2}). On the three off-diagonal Peirce edges the reflected positive metrics are component-uniform scalars,

$$G^+_{12} = 2\varphi^{-1} I_8, \qquad G^+_{13} = 2 I_8, \qquad G^+_{23} = 2\varphi I_8.$$

Every loaded Yukawa vertex draws one field from each edge ($J_{12}(H) \times J_{13}(\Psi_L) \times J_{23}(\Psi^c_R)$), so its canonical normalization is the *common* factor

$$\left[G^+_{12} G^+_{13} G^+_{23}\right]^{-1/2} = \left[8 \det J_{\mathrm{vac}}\right]^{-1/2} = \tfrac{1}{\sqrt{8}},$$

the golden weights cancelling in the product ($\varphi \cdot 1 \cdot \varphi^{-1} = 1$). After $y_t = 1$ this is again the ray $(1, 1, 1, 1)$.

Novelty label: new specialization proved here.

Proposition 3.2 (Parent relative rank zero)**.** *The parent edge-residue map is rank* 1 *with* relative rank 0*: every channel sees the identical edge product, so the edge-uniform parent geometry cannot resolve Q from L inside J_{13} nor u^c, d^c, e^c, ν^c inside J_{23}. [LIB2-117] More generally, any two-derivative operator built by functional calculus from $\{N, J_{\rm vac}, L_{J_{\rm vac}}, U_{J_{\rm vac}}, \Theta\}$ is scalar on each eight-dimensional Peirce edge. The full physical residue map (Theorem 3.1) has rank* 4*, relative rank* 3*; hence* three independent *loaded symmetry-breaking kinetic or threshold directions are needed to* span *a generic three-dimensional space of relative quartets. They are not required merely to split the four Yukawas: a more restrictive nonlinear law selects a particular quartet with fewer. $y(t) = (1, e^t, e^{2t}, e^{3t})$ has four distinct entries for every $t \neq 0$ and relative-log image of rank* 1 *(`s1190`), so one parameter suffices.*

This is the quantified form of Angle (c): "a compensator is required" becomes "the parent geometry has relative rank 0, so three independent external directions are needed to span the generic space." It is independently SHA-verified (in-house re-run, run1=run2; the physical-value gate is correctly negative), and its signature $(15, 12)$ coincides with the Θ Jordan-automorphism signature of Appendix L.

Convergence. Six independent angles — (a) the $G_2/U(1)_Y$ fence, (b) exact half-norm mirror leakage, (c) Reynolds-preserved democracy, (d) a failed candidate-compensator sweep, (e) a rank-4 underdetermined residue map plus failed running, and (f) a parent-action kinetic metric of relative rank zero — all return the *same* conclusion: the within-generation Yukawa hierarchy is irreducibly external to the banked parent action.

4 The scope fence

Scope fence (read before citing this result).

This is a no-go *over the banked exceptional-Jordan parent action only*: the $G_2 \times U(1)_Y$ invariants, perturbative one-loop running, and the banked enhanced-symmetry set $\{F_4, E_6, B-L, J_{\rm vac}\}$. [LIB2-118]

Non-perturbative and composite strong-coupling dynamics are NOT banked. They are therefore *not excluded* by anything above. [LIB2-114, LIB2-118] This is *not* a universal no-go: it does not claim that no conceivable parent could fix the coefficients.

The honest published claim is exactly: the four SM Yukawa *tensors* $\{Qu^cH,\ L\nu^cH,\ Qd^cH^c,\ Le^cH^c\}$ plus their *democratic* $(1, 1, 1, 1)$ normalization — full stop. The four physical coefficient ratios (the fermion mass hierarchy) are recorded as *irreducibly external* to the banked parent, with a stated, falsifiable mechanism for what would have to be added.

Remark 4.1 (What would close it)**.** A genuine close would require the parent action to *derive* the eight Z-residues and thresholds (Theorem 3.1), *or* to supply a G_2-breaking compensator outside the banked invariance that survives the A1048 mirror-census — and to do so without re-introducing a dimension-$\sim$28 spurion (Angle d). Until then the coefficient ratios remain an honest external input.

Remark 4.2 (Ledger status)**.** This appendix moves the Yukawa-coefficient / 7–3 fence from "open blocker" to **conditionally closed — honest negative** (PI-approved this cycle), under the scope fence above. X_{actual} remains RED; $N_{\text{gen}} = 3$ remains an input; the engine and the rest of the paper suite are untouched; *no public observable moves.* Nothing is promoted on the strength of this negative.

Novelty label: new specialization proved here.

Theorem 4.3 (The 7/3 finite-carrier / trace-normalization theorem; S229)**.** *The heavy-quark trace ratio* $7/3 = \mathrm{Tr}_\Theta(\mathbf{1} \oplus \mathbf{3} \oplus \bar{\mathbf{3}})/\mathrm{Tr}(\mathbf{3})$ *is* forced*, not a chosen Markov normalization. On the reflected module* $\mathbf{1} \oplus \mathbf{3} \oplus \bar{\mathbf{3}}$ *the most general trace weight is* $\tau_{a,b} = a\,\mathrm{Tr}_{\mathbf{3}} + a\,\mathrm{Tr}_{\bar{\mathbf{3}}} + b\,\mathrm{Tr}_{\mathbf{1}}$*, giving readout* $r = 2 + b/(3a)$*. The Frobenius identity* $B([s, u_i], v_i) = \sqrt{2}\,a = B(s, [u_i, v_i]) = \sqrt{2}\,b$ *on the exact split-octonion (Zorn) product forces* $a = b$*; the twelve nonzero closure defects are all* $\propto (a - b)$*, so the constraint variety is exactly* $\{a = b\}$ *and* $r = 7/3$ *is unique. [LIB2-053]*

Tiering (strict). This closes the last trace-weight freedom on the 7/3 road; it is *not* an LSZ residue, a low-energy mass theorem, or a physical export theorem. [LIB2-053] It is distinct from — and does not soften — the within-generation coefficient negative that is the subject of this appendix: the *trace floor* of 7/3 is now a theorem, while the physical export arrow (the loaded heavy-quark carrier of Appendix R's "one carrier, three loadings" picture) remains the standing wall. Verified two independent ways in-house (verbatim byte-identical rerun; scratch re-implementation of the Zorn product); no public observable moves.

Remark 4.4 (Theorem-grade upgrade, `s243`; off-diagonal closure cross-reference)**.** The Reynolds/Schur underdetermination (Theorem 3.1) was subsequently upgraded to theorem grade by kernel `s243`: the residue map is multiplicity-one fenced, and an explicit $m = 2$ counter-model shows the bound is saturated rather than an artifact of the search. [LIB2-113] (That upgrade is a statement about the residue map of Theorem 3.1 and the model group of `s243`; the Registrar entry on Reynolds preservation, LIB2-113, asserts only that $\mathcal{P}_{Y=0}$ is the identity on the four-channel span, and is not a certificate of this upgrade.) The complementary off-diagonal routes (a joint frame$\times\eta$ Klein-four action; the perfect-imperfection nucleation defect; and the frame-orientation handle) are closed at theorem grade in Appendix J's written-action section (kernels `s244-s246`). The negative of this appendix is therefore not a single missed route but a closed family.

5 Summary

The exceptional-Jordan parent action fixes the SM Yukawa *structure* completely — the four tensors and one democratic normalization — and fixes the within-generation *coefficients* not at all. The latter is not a failed search but a proved structural fact within the banked framework: G_2-invariance plus G_2-transitivity force democracy (Proposition 2.3); the splitting lives in the loaded $U(1)_Y \not\subset G_2$; the residue map is rank-4 underdetermined (Theorem 3.1); no banked enhanced symmetry supplies a compensator; the parent kinetic metric has relative rank zero (Proposition 3.2); and running alone misses by orders of magnitude. Six independent angles agree. The result is a clean, falsifiable stopping point, scoped honestly to the banked parent and claiming no more.

Leibniz Quantum Beats Newton

Appendix L — Quantum-Mechanical Foundations on the Loaded Subimage: Born from Jordan–Gleason, the Tsirelson Bound from the Associative Envelope, and the Θ Jordan Automorphism (Rev33.2)

Born ✓ & Bell ✓ conditional on the channel-count axiom; single-outcome X_{actual} remains RED. No public observable moves.

Tom O'Sieg

August 2026

Abstract

This appendix collects the quantum-mechanical foundations of the SuperGrokTOE framework as they stand on the *loaded subimage* — the dimensionally-reduced, mass-bearing 4D effective category onto which Θ-compactification projects the otherwise scale-free exceptional parent. Three results are assembled. (i) The **Born rule** $\omega(P) = \mathrm{Tr}(\rho \circ P)$ is the *unique* non-contextual probability measure on the compact-positive order-unit space C^+_{phys}, by the Bunce–Wright (Jordan–Gleason) theorem, precisely when the J_3 trace form is positive-definite $(27, 0)$; the master axiom $\mathrm{Tr}(\eta\Theta) = 7$ is exactly that condition, with the raw split form being indefinite $(15, 12)$. (ii) The map $\Theta = \mathrm{sign}(\eta)$ is a *Jordan automorphism* of $J_3(\mathbb{O}_s)$ to machine precision, closing the Jordan-product/automorphism closure gap left open by the Born derivation. (iii) The **Tsirelson bound** $\|\mathcal{B}\| \le 2\sqrt{2}$ follows from the associative envelope of the loaded $\mathbb{C}$-subimage via the identity $\mathcal{B}^2 = 4I - [A_1, A_2] \otimes [B_1, B_2]$, the exceptional parent being PR-box-*compatible* (under a chosen maximal tensor product, not by derivation); Euclidean JBW structure *alone* is shown to be insufficient. **Everything here is conditional on the still-open channel-count axiom and on the H1a/H1b selectors, and — emphatically — the single-outcome problem X_{actual} remains RED.** This appendix derives the *statistical/relational* structure of quantum mechanics (Born weights, Bell correlations, the quantum bound), not the selection of a single actual outcome. The measurement problem is not solved here.

Contents

1 Setting: the loaded subimage and the firewall **1**

2 The Born rule from Jordan–Gleason **2**

2.1 The named theorem and its single discriminator 2

2.2 The master axiom: $\mathrm{Tr}(\eta\Theta) = 7$ as the positive-definiteness switch 2

2.3 Composite (bipartite) Born 4

3 The Θ Jordan automorphism: closing the closure gap **4**

4 The Tsirelson bound from the associative envelope **4**

4.1 Bell lives on the associative subimage 4

4.2 The Tsirelson identity and the commutator bound 5

4.3 Phase-3: the honest negative and the associativity firewall 5

4.4 Physical narrative 6

5 The firewall, stated as conditions **6**

6 Rev28: the X_{actual} row splits **7**

7 A positivity obstruction: the bare flow is not unitary in disguise (Rev30, A1482) **8**

1 Setting: the loaded subimage and the firewall

The framework's kinematic arena is the rank-3 exceptional Jordan (Albert) algebra $J_3(\mathbb{O}_s)$, equipped with the split-signature metric η on the imaginary split-octonions $\mathrm{Im}(\mathbb{O}_s)$. The *flat* (scale-free) parent is non-associative and indefinite; it admits neither a mass unit nor a bipartite composite. The physically realized sector is the *loaded* (massed, dimensionally-reduced) image obtained by the Θ-compactification $\Theta = \mathrm{sign}(\eta)$, written C^{+}_{phys}. Two distinct quantum structures live on this loaded image: the *Born rule* (the probability measure) and the *Bell/Tsirelson* structure (the bipartite correlation bound). The central claim of this appendix is that one mechanism — the Θ-loading, i.e. the channel-count axiom $\mathrm{Tr}(\eta\Theta) = 7$ — underlies both.

Before any derivation, we state the firewall once, prominently, because every result below is fenced by it.

> **★ FIREWALL — the measurement problem is NOT solved here.**
>
> All results in this appendix derive the *statistical / relational* structure of quantum mechanics: the Born *weights* $\omega(P) = \mathrm{Tr}(\rho \circ P)$ and the Bell *correlation bound* $\|\mathcal{B}\| \le 2\sqrt{2}$. They do *not* address the selection of a single actual outcome. The single-outcome quantity X_{actual} — which state of affairs is realized in an individual measurement — remains **RED (unsolved)**. Nothing here is a hidden-variable theory, a collapse dynamics, or a derivation of definite outcomes. **Born ✓ and Bell ✓ are CONDITIONAL** (see Sec. 5); X_{actual} is **not ✓**.

2 The Born rule from Jordan–Gleason

2.1 The named theorem and its single discriminator

The decisive input is the Jordan generalization of Gleason's theorem.

Novelty label: standard theorem used.

Theorem 2.1 (Bunce–Wright / Jordan–Gleason, 1992)**.** *Let M be a Euclidean (formally real) JBW algebra of rank ≥ 3 with no type-*I_2 *(spin-factor) summand. Then every state on the projection*

lattice of M is of the form $\omega(P) = \mathrm{Tr}(\rho \circ P)$ for a unique density ρ. In particular the Born rule is the unique *non-contextual measure.*

For $J_3(\mathbb{O}_s)$ the rank is 3 (past the I_2 obstruction), so the only remaining hypothesis is that the algebra be *formally real*, equivalently that the Jordan trace form be *positive-definite*. The entire Born question thus reduces to a single discriminator: the *signature of the J_3 trace form.*

Novelty label: standard theorem used.

Proposition 2.2 (Trace-form signature discriminator; s215)**.** *The compact algebra $J_3(\mathbb{O})$ has trace-form signature $(27, 0)$ (Euclidean / formally real). The raw split algebra $J_3(\mathbb{O}_s)$ has signature $(15, 12)$ (indefinite, not formally real), decomposing as $(3, 0)_{\text{diag}} \oplus 3 \times (4, 4)_{\text{off-diag}}$. [P-002]*

The numerical content (s215, 5/5 gates, run1=run2): the octonion norm signatures are compact $\mathbb{O} = (8, 0)$ and split $\mathbb{O}_s = (4, 4)$; the J_3 trace forms are $(27, 0)$ and $(15, 12)$ respectively; rank-3/no-I_2 holds; and the trace form is a frame function, $\sum_i \mathrm{Tr}(\rho \circ e_i) = \mathrm{Tr}(\rho)$, verified over 2000 $J_3(\mathbb{C})$ frames with $\max |\Sigma - 1| = 1.7 \times 10^{-15}$.

2.2 The master axiom: $\mathrm{Tr}(\eta\Theta) = 7$ as the positive-definiteness switch

The compactification $\Theta = \mathrm{sign}(\eta)$ is designed to flip the sign of the negative-norm generators in the trace evaluation. Define the Θ-invariant subalgebra and its order-unit (OS) inner product

$$J_{\text{phys}} = \{A \in J_3(\mathbb{O}_s) : \Theta A \Theta^\dagger = A\}, \qquad \langle A, B\rangle_\Theta = \mathrm{Tr}\big(\Theta(A \circ B)\big). \tag{1}$$

The positivity argument (A1024): on J_{phys} the insertion of Θ into the trace renders $\langle A, A\rangle_\Theta > 0$ for every nonzero A ("sum of squares of real components"), so $A^2 + B^2 = 0 \Rightarrow A = B = 0$ — the formally-real criterion. Bunce–Wright then applies directly.

The bridge from the $\mathrm{Im}(\mathbb{O}_s)$ metric to the J_3 trace form is the canonical **3-block decomposition**, which is the load-bearing geometric step.

Novelty label: new specialization proved here.

Theorem 2.3 (Master axiom; canonical 3-block proof; A1024)**.** *The $(15, 12)$ signature of $J_3(\mathbb{O}_s)$ traces to the $(3, 4)$ signature of $\mathrm{Im}(\mathbb{O}_s)$ via the three off-diagonal octonionic blocks: each contributes 4 negative dimensions, $3 \times 4 = 12$ negative dimensions in total, the diagonal 3×3 Hermitian block contributing $(3, 0)$. Writing $\eta_{\text{phys}} = \eta\Theta$ on $\mathrm{Im}(\mathbb{O}_s)$, one has the biconditional chain*

$$\boxed{\mathrm{Tr}(\eta\Theta) = 7 \iff \eta\Theta = I_7 \iff J_3 \text{ trace form is } (27, 0) \iff C^+_{\text{phys}} \text{ formally real} \iff \text{Born unique.}} \tag{2}$$

Proof sketch (arithmetic of the 3-block decomposition). The diagonal block contributes $(3, 0)$. In the raw split case each of the three off-diagonal octonionic blocks contributes $(4, 4)$, for a total (3+12, 12) = $(15, 12)$. The Θ-flip acts as $+1$ on the four compact directions and -1 on the four split directions of each off-diagonal block, converting each $(4, 4)$ to $(8, 0)$; the diagonal $(3, 0)$ is unchanged. The total becomes $(3{+}24, 0) = (27, 0)$. Rendering η positive-definite on $\mathrm{Im}(\mathbb{O}_s)$ is exactly $\eta\Theta = I_7$, i.e. $\mathrm{Tr}(\eta\Theta) = 7$ (the maximal trace of the 7×7 block, attained only when every eigenvalue of $\eta\Theta$ is $+1$). Each link is reversible, giving the biconditional. □

Remark 2.4 (Formal reality is a property of the product, not only the metric)**.** The link "$(27, 0)$ trace form $\Leftrightarrow$ C^+_{phys} formally real" is stated above at the level of the *trace form* (signature). A positive Θ-twisted bilinear form is *necessary* for formal reality but is *not by itself sufficient*: formal reality ($\sum x_i^2 = 0 \Rightarrow x_i = 0$) is a property of the Jordan *product*, and the $+1$ fixed space of the $(15, 12)$ involution is not *automatically* a 27-dimensional compact Albert algebra closed under that product. A complete proof requires exhibiting the explicit Euclidean fixed-point subalgebra (or an appropriate isotope) and verifying its rank, cone, and projection lattice. We therefore read the boxed chain as established at the trace-form level and *conditional* (on that subalgebra construction) at the product level — consistent with the overall conditional status of the channel-count axiom $\text{Tr}(\eta\Theta) = 7$.

Remark 2.5 (One axiom, three closures; s221)**.** Via the s221 sharpening, the *single* axiom $\text{Tr}(\eta\Theta) = 7$ simultaneously closes (a) Born uniqueness (Bunce–Wright), (b) the channel-count normalization $7/3$, and (c) gauge-emergence. It is the master geometric switch of the framework: Born-unconditional $\Longleftrightarrow$ gauge-emergence.

Remark 2.6 (The projector P_+ is load-bearing; A1034)**.** The honest form of the count is $\text{Tr}(\eta\Theta P_+) = 7$, with P_+ the loaded positive-frequency/physical projector: it is *not* an unrestricted global trace. Without P_+ the restricted count is 5 and the global J_3 trace is 27; only the loaded projection isolates the 7. This is exactly why the axiom remains *open* (it is the LSZ-level statement that $P_+(R_{\text{pole}})$ exhausts the 7-dimensional physical block), and why neither $A1007$, s226, nor $\kappa = 1$ alone forces it — explicit counterexamples R_7, R_8, R_9 survive. The biconditional above should be read with P_+ understood throughout.

2.3 Composite (bipartite) Born

For Bell scenarios one needs Born on the bipartite system. Under the loaded frame J_{phys} is a Type I complex JBW factor (an associative complex matrix algebra $\mathcal{B}(\mathbb{C}^n)$). The tensor product of two formally-real associative JBW algebras is itself formally real, and the induced product inner product factorizes,

$$\langle A_1 \otimes B_1,\, A_2 \otimes B_2\rangle_\Theta = \langle A_1, A_2\rangle_\Theta \, \langle B_1, B_2\rangle_\Theta > 0, \tag{3}$$

so Bunce–Wright extends to the composite, giving the bipartite Born rule $\omega(P_A \otimes P_B) = \text{Tr}(\rho_{AB}(P_A \otimes P_B))$. The BGW non-associativity and the indefinite $(15, 12)$ signature are both stripped by the loaded projection.

> **Conditional, not unconditional.** The Type I identification rests on $\text{Im}(\mathcal{F}_{\Theta,\text{load}}) \subseteq \{\text{associator} = 0\}$, which is asserted (leading conjecture, A1022 Q1) rather than derived from the algebra action. The composite Born is therefore banked *conditional* on that projection lemma and on the master axiom $\text{Tr}(\eta\Theta) = 7$.

3 The Θ Jordan automorphism: closing the closure gap

The Born derivation of Sec. 2 left one analytic step open: the subalgebra $J_{\text{phys}} = \{A : \Theta A \Theta^\dagger = A\}$ is closed under the Jordan product only if Θ is an *automorphism* of that product, $\Theta(A \circ B) = (\Theta A) \circ (\Theta B)$. This is now established numerically to machine precision (s226).

Novelty label: new specialization proved here.

Theorem 3.1 (Θ is a Jordan automorphism of $J_3(\mathbb{O}_s)$; s226)**.** *The map* $\Theta = \mathrm{sign}(\eta)$ *satisfies*

$$\Theta(A \circ B) = (\Theta A) \circ (\Theta B) \qquad \text{for all } A, B \in J_3(\mathbb{O}_s), \tag{4}$$

*verified over a complete basis (*729 *basis pairs) with maximum error* 3.1×10^{-16} *(worst pair* $(i, j) = (3, 10)$*), and over* 500 *random pairs with maximum error* 6.2×10^{-15}*. Moreover* $\Theta^2 = I$ *to* 4.4×10^{-16} *and* Θ *is symmetric (deviation* 0.0*), with signature* $(15, 12)$*. All gates pass.*

Remark 3.2 (What this closes)**.** Theorem 3.1 closes the Jordan-product/automorphism closure gap that A1023/A1024 flagged as the remaining analytic step of the Born derivation. With Θ a genuine Jordan automorphism, J_{phys} is a bona fide Jordan subalgebra, the OS inner product $\langle \cdot, \cdot \rangle_\Theta$ is well-defined on it, and the positivity argument of Sec. 2 is self-consistent. Note this is a *numerical* verification at machine precision, not yet a closed-form algebraic identity; the signature it reports, $(15, 12)$, is that of the raw split form (it is the *trace* $\mathrm{Tr}(\eta\Theta) = 7$ that selects the loaded $(27, 0)$ image).

4 The Tsirelson bound from the associative envelope

4.1 Bell lives on the associative subimage

The Bell scenario is well-posed only where a bipartite tensor product exists, i.e. where the algebra is associative. The discriminator is the split-octonion associator $[x, y, z] = (xy)z - x(yz)$ (s214, 5/5 gates, run1=run2):

- On the *flat* parent $\mathbb{O}_s$ the associator is large, $\max\|[x, y, z]\| = 59.4$ over 2000 random triples — non-associative, no associative embedding, BGW (Bondi–Gardner–Williams) obstruction real, no composite, Bell ill-posed.
- On the *loaded* $\mathbb{C}$-subimage $\mathrm{span}\{1, u\}$ with $u \circ u = -\frac{1}{2}\mathbf{1}$ the associator vanishes (5.1×10^{-15} over 500 general elements) — associative, the tensor product exists, standard quantum Bell applies.
- On the associative subimage the CHSH value is $S = 2\sqrt{2}$ (saturated), the classical LHV maximum is 2, the PR-box (4) is excluded, no-signalling holds ($\max|\Delta| = 0$), and determinism holds.

The split-octonion associator *is* the concrete BGW obstruction: nonzero on the exceptional parent, zero on the $\mathbb{C}$-subimage. The same compactification that generates the 4D mass unit (radion/dilaton $\to$ KK scale) is the loading that projects onto the associative frame.

4.2 The Tsirelson identity and the commutator bound

Let A_1, A_2 (Alice) and B_1, B_2 (Bob) be dichotomic observables, $A_i^2 = I$, $B_j^2 = I$, and let $\mathcal{B} = A_1 \otimes (B_1 + B_2) + A_2 \otimes (B_1 - B_2)$ be the CHSH operator. The load-bearing algebraic identity is (s225 gate G1, residual 8.9×10^{-16}):

Novelty label: standard theorem used.

Theorem 4.1 (Tsirelson identity and bound; s225 / A1022)**.** *On an associative envelope,*

$$\mathcal{B}^2 = 4I - [A_1, A_2] \otimes [B_1, B_2]. \tag{5}$$

From $A^2 = I$ one has the commutator bound $\|[A, A']\| \le 2$ (analytic via $[\sigma_x, \sigma_y] = 2i\sigma_z$, $\|2i\sigma_z\| = 2$; numerically $\max = 1.9999992$ *over* 5000 *random involutions). Hence*

$$\|\mathcal{B}^2\| \le 4 + \|[A_1, A_2]\|\,\|[B_1, B_2]\| \le 4 + 2 \cdot 2 = 8, \qquad \textit{so} \qquad \|\mathcal{B}\| \le 2\sqrt{2}, \tag{6}$$

*tight at $2\sqrt{2}$ (attained on Φ^+ with optimal observables, $S = 2.82843 = 2\sqrt{2}$). The PR-box (*CHSH $=$ *4) is excluded, since $\|\mathcal{B}\| = 4$ would require $\|[A, A']\| = 4$, violating the $A^2 = I$ commutator bound of* 2*. The classical bound is ≤ 2 (exhaustive).*

The commutator bound is the key: a Cauchy–Schwarz estimate gives only the trivial 4 (the PR-box value); the commutator identity is the correct and tight tool.

4.3 Phase-3: the honest negative and the associativity firewall

A sharper question (Phase-3) is whether $2\sqrt{2}$ can be forced from the C^+_{phys} axioms — Euclidean JBW plus reflection positivity — without *presupposing* Hilbert-space operators. The honest answer is negative (A1033).

Novelty label: new specialization proved here.

Theorem 4.2 (Phase-3 honest negative; A1033)**.** *Euclidean JBW structure together with the OS inner product is* insufficient *to force $\|\mathcal{B}\| \le 2\sqrt{2}$. The obstruction is precise: without an associative envelope the product $(A_1 \otimes B_1) \circ (A_2 \otimes B_2)$ is undefined, so the identity $\mathcal{B}^2 = 4I - [A_1, A_2] \otimes [B_1, B_2]$ cannot be formed. The maximal tensor product $C_{\max}$ (compatible with no-signalling) contains extreme points saturating* CHSH $= 4$*. OS positive-definiteness secures the Born rule but does not constrain the state-space geometry to the quantum boundary.*

Novelty label: standard theorem used.

Theorem 4.3 (Associativity firewall; the "if" direction; A1033)**.** *On an associative Type I complex JBW factor, $\|\mathcal{B}\| \le 2\sqrt{2}$* is *a theorem: the tensor product is associative, so $(A_1 \otimes B_1)(A_2 \otimes B_2) = (A_1 A_2) \otimes (B_1 B_2)$ is well-defined; the $\mathcal{B}^2$ identity follows algebraically; and by Gelfand–Naimark the associative JBW algebra is the self-adjoint part of a C^*-algebra, whose C^*-norm yields $\|[A, A']\| \le 2$ and hence $\|\mathcal{B}\| \le 2\sqrt{2}$. This requires* no *import of Hilbert operators: associativity generates the C^*-envelope. The biconditional "$\|\mathcal{B}\| \le 2\sqrt{2}$ iff Type I complex JBW factor" has its "if" direction proven; the "only if" is supported by Proposition 4.4 but not fully proven.*

Novelty label: new specialization proved here.

Proposition 4.4 (Parent is PR-box-*compatible* under $C_{\max}$; A1033)**.** *On the full exceptional $J_3(\mathbb{O}_s)$ (Albert algebra) the bipartite composite is* not *canonically defined: since Albert algebras embed in no associative algebra (BGW), there is no associative tensor product, and Bell is genuinely ill-posed on the parent itself (consistent with the BGW obstruction above: no composite, Bell ill-posed). One must therefore* choose *a composite.* If *one adopts the maximal tensor product $C_{\max}$ (the most permissive no-signalling composite — a choice not fixed by the algebra), then $C_{\max}$ has flat edges hosting extreme states ω_{PR} with $\langle \omega_{\text{PR}}, \mathcal{B} \rangle = 4$.* Caveat (do not overclaim): *this makes the parent PR-box-*compatible under $C_{\max}$*; it is* not *a derivation that the parent "is" a PR-box theory with* CHSH $= 4$*, because no canonical maximal composite follows from the mere absence of an associative embedding. "Bell undefined on the parent" and "*CHSH $= 4$ *on a chosen $C_{\max}$" are compatible only with the composite named explicitly.*

4.4 Physical narrative

The exceptional parent (scale-free, 5D/11D) is PR-box-*compatible* (under the adopted maximal tensor product $C_{\max}$): it can reach CHSH up to the algebraic bound 4 only once $C_{\max}$ is chosen as the composite. The descent to the quantum boundary $2\sqrt{2}$ is the kinematic consequence of KK-mass loading (D874): the overall-scale modulus (radion/dilaton, the sign-S decompactification sector) supplies the mass unit, KK momentum phases require a complex structure, and the reduction projects the physical S-matrix onto the associative $\mathbb{C}$-subimage. The supergravity backbone is the standard de Wit–Nicolai reduction $E_{6(6)}/USp(8)$ (42 scalars, 5D) $\to E_{7(7)}/SU(8)$ (70 scalars, 4D) via a circle. In one sentence: *the exceptional parent is PR-box-compatible under $C_{\max}$; KK-mass loading selects the associative $\mathbb{C}$-subimage; the associative envelope is the quantum boundary $2\sqrt{2}$.*

5 The firewall, stated as conditions

We close by making explicit exactly what is conditional and what remains red.

Conditions on which Born ✓ and Bell ✓ rest:

1. **Channel-count axiom** $\mathrm{Tr}(\eta\Theta) = 7$ (the LSZ-open piece). This is the master switch (Thm. 2.3): it makes the J_3 trace form $(27, 0)$, hence C^+_{phys} formally real, hence Bunce–Wright applies. It is *not* yet derived from first principles (the OS-norm shortcut was refuted in s201; the LSZ route is open).
2. **The H1a + H1b selectors** (s224 / A1025). Gauge-emergence — equivalent to Born-unconditional (s221) — decomposes into H1a (B−L removal needs an equivariant bosonic defect-lift Morse selector, a new portal) and H1b ($\mathrm{SU}(2)_L$ selection needs separate structure; the hypercharge direction remains a loaded input). Both are open, so Born stays *conditional*.
3. **The associative-projection lemma** $\mathrm{Im}(\mathcal{F}_{\Theta,\mathrm{load}}) \subseteq \{\text{associator} = 0\}$, needed for the Type I identification of the Bell composite (A1022 Q1), is a leading conjecture, not a derived map.

Given these, Born $= \mathrm{Tr}(\rho \circ P)$ is unique (Jordan–Gleason) and $\|\mathcal{B}\| \leq 2\sqrt{2}$ is a theorem on the associative subimage.

★ FIREWALL (restated) — X_{actual} is RED. The measurement problem is NOT solved.

This appendix derives the *statistical and relational* structure of quantum mechanics: the Born *weights* and the Bell *bound*. It does *not* derive the selection of a single actual outcome. The single-outcome quantity X_{actual} remains **unsolved (RED)**: nothing above tells you *which* eigenvalue is realized in an individual run. Born and Bell are ✓ **conditional** on the conditions in the gray box; X_{actual} is **not** ✓ under any condition stated here. No public observable moves.

Result	Status	Conditional on
Born uniqueness (Jordan–Gleason)	✓ conditional	$\mathrm{Tr}(\eta\Theta) = 7$; H1a+H1b
Θ Jordan automorphism	✓ (machine precision)	— (verified)
Tsirelson bound $2\sqrt{2}$	✓ conditional	associative envelope; assoc. lemma
PR-box excluded on subimage	✓	associative envelope
Phase-3 from JBW alone	× (honest negative)	insufficient without associativity
Single outcome X_{actual}	**RED (unsolved)**	not addressed

6 Rev28: the X_{actual} row splits

The firewall stated in this appendix — between the conditional Born/Tsirelson structure derived here and the single-outcome ("actuality") problem — is retained, and it is now theorem-backed from two directions. What changes at Rev28 is that the blanket form of the red row narrows.

Novelty label: new specialization proved here.

Proposition 6.1 (The actuality no-go, generalized)**.** *A probability law does not imply one realized outcome: the symmetric two-outcome state is swap-invariant while no character is, and the argument generalizes to any nontrivial transitive G-set by a fixed-point argument. Demanding a* deterministic natural section *of the actualization morphism is a category error for a stochastic law — probability theory returns a random element, not a deterministic function of its own law.*

Novelty label: new specialization proved here.

Proposition 6.2 (The record type is derived)**.** *For the record process of Appendix T: finite-stage characters are exactly point evaluations, compatibility gives* $\operatorname{Spec} C(\Omega_T) \cong \Omega_T$*, and history, character and compatible boundary record are* one object*. The morphism $1 \rightsquigarrow \Omega_T$ exists in the Markov-kernel category. The law is atomless ($p^* = 5\varphi^2/16$), so every point history has measure zero, and a realized history carries* 1.168 *bits per step: it is* extensive record data*, not one selector bit. The asymptotic-boundary rescue (tail/Poisson/Martin) is refuted for this chain — bounded harmonics are constants.*

X_{actual} **— the split (Rev28).** $X_{\text{record-law/type}}$: DERIVED. $X_{\text{realized-value}}$: NOT DETERMINED BY THE LAW — sample/record data, by theorem. The firewall of this appendix **stays in force for the specific realized outcome**; what narrows is the blanket claim that no actuality object exists. Remaining: one realized record (data), the conditional conserved-$\mathbb{Z}_2$ sector identification, and AX-MAP-HISTORY as a candidate only (named, not adopted — it is an atypical, measure-zero orbit, and adopting it changes the question).

7 A positivity obstruction: the bare flow is not unitary in disguise (Rev30, A1482)

A recurring hope in this programme has been that a hyperbolic-looking flow might be recast as unitary dynamics by a better choice of polarization. For the bare quartic flow that hope is closed, and the argument is short enough to state completely.

The lemma. If A commutes with a compatible complex structure J and $g = \Omega J > 0$, then A is g-skew-adjoint and its spectrum is purely imaginary. *The bare-I_4 linearization has* 54 *real hyperbolic modes.* Therefore **no positive compatible polarization reinterprets the bare I_4 flow as unitary dynamics.** The conclusion is absolute in the sense that matters here: positivity cannot be bought by choosing a basis — the flow itself must be reduced.

Provenance, stated because it is mixed. The lemma was checked in house *numerically*, against a randomly generated positive g ($\max|\mathrm{Re}\,\lambda| < 10^{-8}$, with $A^{\mathsf{T}}g + gA = 0$ confirmed), rather than proved symbolically here; and the count of 54 real modes is imported from the accompanying kernel rather than re-derived. **The real form is part of the statement**: signatures and mode counts in this family are consequences of the split declaration recorded in Appendix X, and quoting the 54 without it would repeat exactly the defect this era spent its rounds diagnosing. One fact in the same computation *is* real-form invariant and is worth recording beside the negative — the 26 flat directions are 26 in both real forms, in both the full and the tangent Hessians. Nothing here promotes an observable, moves a tier, or touches the engine.

Leibniz Quantum Beats Newton

Appendix M — The Order-Five Orientation and the Categorical 60-Clock: $M = 2h(E_8)$, the Parity Grading, and the Galois-Rigid Localization of the Rest-Mass Occupation (Rev33.2; entered Rev21; occupation selector promoted at Rev25)

Clock modulus and parity sector: positive/structural, conditional on the vacuum-to-projector bridge. Occupation $O = \{0, 3, 11, 17\}$: DERIVED-CONDITIONAL as of Rev25 (three-layer selector derivation, §7; live public falsifier $\{31, 39\}$). No public observable moves.

Tom O'Sieg

August 2026

Abstract

This appendix folds the order-five / clock arc (in-house kernels `s260-s283`, sessions S165–S171, together with the externally-delivered, in-house-verified build-grade assessments A1085/A1086) into the lab notebook. The rest-mass *occupation* (bound as in Paper 0: occupation *of golden mass-ladder rungs*, **not** of atomic orbitals) — the set of integer beat-depths $O = \{0, 3, 11, 17\}$ at which the charged leptons and the up quark sit on the golden ladder — is the framework's deepest open selector. **The S165–S171 arc recorded here did not derive** O (current status: DERIVED-CONDITIONAL since Rev25 — the update at the end of this abstract; one status, stated Rev32.9). What it records is sharply delimited: (i) the rung *parity* is an *exact universal* $\mathbb{Z}_2$ grading (a theorem, M-independent); (ii) the clock *modulus* is derived target-independently *two* ways, both giving $M = 60 = 2h(E_8)$ — a Chinese-remainder carrier $\mathbb{Z}_{60} = \mathbb{Z}_3 \times \mathbb{Z}_4 \times \mathbb{Z}_5$ and, independently, the binary-icosahedral McKay/E_8-Coxeter construction; and (iii) the occupation is localized *exactly* as a Galois-rigid cyclotomic-frame datum: $\mathrm{Stab}_{(\mathbb{Z}/60)^\times}(O) = \{1\}$, so every Galois-covariant construction is provably blind to O, and the only target-independent source of the missing data is an anomaly-forced full-frame selector (fenced). Every positive statement here is *conditional* on the still-open vacuum-to-projector bridge of the golden Peirce construction, and on the banked input $T^2 = -I$. [LIB2-334] **Rev25 update:** the occupation selector has since been *derived in all three layers* (mod-2 Möbius parity, mod-5 torsion walls, mod-7 Fano/Artin adjacency; S224–S225, A1349/A1350-verified), and the occupation is promoted from Input/Open to DERIVED-CONDITIONAL (§7), with the hash-stamped period-two continuation $\{31, 39\}$ standing as the live public kill condition.

Contents

1 Setting and posture **2**

2 The clock as a non-invertible-symmetry-reduced system; the parity theorem **2**

3 Categorification: $\mathrm{Rep}(D_{60})$ and the spectral obstruction **2**

4 The clock modulus: $M = 60 = 2h(E_8)$, two independent ways **5**

4.1 Chinese-remainder carrier 5

4.2 McKay / E_8-Coxeter carrier 5

5 The order-five orientation (conditional) **5**

6 Galois-rigid localization of the occupation **6**

7 The occupation selector: Derived-Conditional (Rev25) **6**

8 Status ledger **8**

1 Setting and posture

On the loaded subimage the charged-lepton and light-quark rest masses sit, to within a few percent, at integer powers of the golden ratio $\varphi = (1+\sqrt{5})/2$ relative to the electron:

$$n_f \;=\; \log_\varphi \frac{m_f}{m_e}, \qquad n_e = 0, \quad n_u \approx 2.99, \quad n_\mu \approx 11.07, \quad n_\tau \approx 16.94,$$

so that the nearest integers form the *occupation set* $O = \{0, 3, 11, 17\}$ (kernel `s263`). The depth n_f read off from a measured mass is *descriptive*; the integer *selector* that would say *which* rungs are occupied is the open object.

What this appendix claims and does not claim. *Claimed (structural, target-independent):* the parity grading (§2), the clock modulus $M = 60 = 2h(E_8)$ and its odd-character spectrum (§4), and the exact Galois localization of O (§6). *Conditional:* the order-five orientation (§5) rests on the golden-Peirce vacuum-to-projector bridge, which is itself Open; and the central sign $T^2 = -I$ is a banked input, not derived here. *Rev25 status of the values:* the occupation $O = \{0, 3, 11, 17\}$ is Derived-Conditional — the selector derivation now exists in all three layers (§7) — conditional on the same standing fences as the rest of this appendix (the vacuum-to-projector bridge; the banked $T^2 = -I$), and falsifiable through the live $\{31, 39\}$ continuation.

2 The clock as a non-invertible-symmetry-reduced system; the parity theorem

The golden clockwork organizes as a $\mathbb{Z}_M/\mathbb{Z}_2$ non-invertible-symmetry-reduced (NISR) system (kernels `s265-s267`). Of the 28 inter-rung pairs, a conjugate-pair lemma splits them 15:13 into opposite-parity (all-order protected) and same-parity (soft) classes (`s266`); the electron (rung 0, even) is all-order protected from the odd rungs $\mu(11), \tau(17)$.

Novelty label: new specialization proved here.

Theorem 2.1 (Exact universal parity grading, `s267`)**.** *The rung parity is an exact $\mathbb{Z}_2$ grading of the clock, independent of the modulus M. The pointed (invertible) subcategory is $C_{\mathrm{pt}} = \{0, 30\} \cong \mathbb{Z}_2$.*

This promotes the `s266` lemma to a theorem. It is a statement about *structure* (which rungs can mix to all orders), not about occupation. Kernel `s269` provides the matching honest negative: parity is *falsified* as a flavor-mixing (CKM) rule — roughly two-thirds of the empirical mixing weight, including $|V_{cs}| \approx 0.973$ and $|V_{tb}| \approx 0.999$, sits in parity-forbidden entries. The grading governs selector *stability*, not mixing.

3 Categorification: $\mathrm{Rep}(D_{60})$ and the spectral obstruction

The clock is categorified as $C_{60} = \mathrm{Rep}(D_{60})$, the $\mathbb{Z}_2$-gauged NISR (kernel `s275`): it is group-theoretical with F-symbols equal to the D_{60} $6j$-symbols, has 33 simple objects with $\sum_\lambda (\mathrm{FPdim}_\lambda)^2 = 120 = |D_{60}|$, and its module categories lie on the di Francesco–Zuber ADE ladder. The spectral functor $r : \mathrm{Irr}(\mathcal{M}) \to \mathbb{Z}_{60}/\pm$ images *no* module category onto O (kernel `s276`); the obstruction is that $11, 17$ are coprime to 60 and the Galois group $(\mathbb{Z}/60)^\times$ places $\{11, 17\}$ in a single eight-element orbit. The twist $\theta(j, m) = \zeta_{60}^{jm}$ is the Green's-function pairing of the two signature windings (`s277`), and the modular S, T data are Galois-*covariant*. These are *structured negatives* — a covariant construction provably blind to O — not no-go theorems.

Remark 3.1 (Three categorical occupation no-goes and the weak-gauge annihilation, `s270`–`s273`)**.** A hostile audit added three further categorical attacks, all confirming the occupation is not bulk-forced. *Condensation* (`s270`): $O = \{0, 3, 11, 17\}$ is not fusion-closed and carries non-bosonic components, so it cannot be a condensable algebra in the chiral folded ring. *Boundary* (`s271`): enumerating all 12 Lagrangian algebras (gapped boundaries) of the achiral double $D(\mathbb{Z}_{60})$ shows O is no boundary subgroup or rung projection. *Dynamics* (`s272`): any symmetry-breaking potential built from the bulk J_{vac} geometry lacks the per-rung tuning needed to target four coprime integers. Separately, the weak sector is not native to the diagonal frame (`s273`): the Eilenberg–Moore category of the Reynolds monad P_{code} isolates the Weyl-singlet diagonal $(+,+)$ frame but *annihilates* the off-diagonal $(-,-)$ Peirce spaces where the non-commuting $SU(2)_L$ generators must reside, so $SU(2)_L$ is not extracted from the invariant frame. All four are structured negatives that reinforce the input status of the gauge group (Appendix N) and of the occupation; none touches the *continuous-geometry* mixing derivation of Paper 3, whose angles do not invoke the parity grading (the cross-cycle adjudication is recorded in Paper 3, Definition-A reconciliation, and the session notes).

Remark 3.2 (No canonical order on $\mathrm{Irr}(\mathrm{Rep}(D_{60}))$: a two-lane blind nonexistence, `s999`–`s1000`)**.** The freeze-first arc put the ordering question to two lanes *blind* — neither saw the other's return, and both were adjudicated against a house table sealed beforehand (D1183/A1490, Gemini; D1184/A1491, Grok). Both proved **nonexistence, each inside an explicitly declared class**. The Grok class is stated with three clauses: maps $\mathrm{Irr}(\mathcal{X}) \to \mathbb{R}$ built *solely* from the fusion, F, braiding and annular data, *free of continuous and discrete choices* beyond D_{60} itself, and *invariant* under $\mathrm{Aut}_\otimes(\mathcal{X})$; the Gemini class is "canonical categorical invariants, no external imports".

The load-bearing argument is the same in both, and it is an invariance argument, not a counting one: canonical $\Rightarrow$ $\mathrm{Aut}_\otimes$-invariant $\Rightarrow$ constant on orbits $\Rightarrow$ non-injective wherever a nontrivial orbit exists. The house supplied the witness the lanes asserted but did not exhibit (`s999`): $\varphi_7 : r \mapsto r^7,\ s \mapsto s$ is an automorphism of D_{60} — homomorphism and bijection checked exhaustively on all 120 elements —

inducing a tensor autoequivalence that permutes the 29 two-dimensional simples by $j \mapsto \pm 7j \bmod 60$, with a genuine 2-cycle $V_5 \leftrightarrow V_{25}$.

One universal lemma of the Grok return is false as written, and the house corrects it here rather than repeating it. The return asserts "the orbits are larger than singletons." Computing the full induced Aut-action on the 29 two-dimensional labels over all sixteen units k (`s1000`) gives orbit sizes $[1, 1, 1, 2, 2, 2, 4, 4, 4, 8]$: *$V_{10}$, V_{15} and V_{20} are fixed by every automorphism.* The proof needs only the existence of *one* nontrivial orbit, so the verdict is unaffected — and the resulting bound is in fact stronger than the lane claimed: with linear-character orbits $[1, 1, 2]$, an $\mathrm{Aut}_{\otimes}$-invariant map takes at most **13** values on 33 simples, so non-injectivity is forced with room to spare.

Scope, stated because the scope is the result. This is a nonexistence *within the declared classes above*; it is not a statement that no order of any kind exists. The subsidiary leg — that FPdim, degenerate on $\{1, 2\}$ across 33 simples, is the only canonical real-valued invariant — is ASSERTED, not proved, in both returns, and is recorded as such; the invariance leg carries the result alone. Two further steps of the Grok return (that no other modular-data invariant is injective without additional choices, and that the relevant centre eigenvalues are circle-valued or highly degenerate) are likewise asserted. Both returns were text-only, with no executable shipped; every number quoted here is house-computed. **Adjudicator disclosure:** the same house scorer adjudicated both lanes in one session with the Gemini return already in context, so blindness holds *between the lanes* while house-side independence is procedural rather than epistemic. Finally, the companion rigidity result of the same era — that $\mathbf{1} \subset Y \otimes Y^*$ in any rigid fusion category makes reachability *indiscrete* — is **not** folded here: it was corroborated by neither lane, and remains a single-lane-plus-house result.

What it costs the occupation programme. Together with the structured negatives above, this closes the categorical route to an occupation ordering: fusion supplies composition, braiding supplies exchange, and Fock occupation *consumes* a labelling rather than manufacturing one. None of the three supplies an energy order. The occupation's input status (Appendix N) is reinforced, and nothing here promotes or moves any observable. *Interpretive corollary, printed beside the fence only (Rev32.3, PI wording S300): the $\otimes$-layers are order-free by theorem; apparent motion is re-presentation along exchange isomorphisms; dynamics is priced extra structure.*

Remark 3.3 (What *does* carry an order, and what does not (Rev30, A1484/A1485))**.** The categorical no-go above is one half of the ordering question. The other half is positive, and it cost the house a sealed prediction to establish, so it is recorded exactly.

An intrinsic partial order does exist. The weight-lattice dominance order on the weights of the **7** of $G_2 = \mathrm{Aut}(\mathbb{O}_s)$ is a genuine intrinsic partial order: 24 comparable ordered pairs and 9 *incomparable* pairs, so it is genuinely partial and not total, and it is built from root data alone — no n, no ℓ, no $n + \ell$, no Z and no energy enters its construction.

Read this correctly, because the sealed text and the physics point different ways. The house had sealed the claim that *no filling-order carrier exists in the framework*, and against the refutation condition as drafted — which asked for a total *or partial* order — **that claim lost**. It is recorded as a loss. But the object above is **not a filling order**, and nothing here says the framework contains one: a dominance order on weights is not a rule for traversing states. The loss is a drafting loss, and reading it as "the framework supplies a filling order" inverts it. **Scope:** the check used the **7** of G_2 — one representation of one group — and is not a survey of the corpus. The corpus has still not been swept for ordering carriers by anyone; that remains an open task.

Fock occupation is not a source of order either. Nothing in the construction of the Fock space or of its number operators manufactures the labelling of the index set: that labelling is an external input. **Fock occupation does not supply an ordering — it consumes one.** Taken with the categorical result above, the position is that fusion supplies composition, braiding supplies exchange, and occupation consumes a labelling; *none of the three supplies an energy order*, and the missing object is a Hamiltonian or transfer law rather than a categorical invariant.

A naming debt, acknowledged. Calling this set an "occupation" invites a false parity with physical subshell filling, and that is a debt against the whole corpus rather than a one-off. The binding fence for the term is given at its first use (Paper 0) and again in §7 of this appendix: occupation *of golden mass-ladder rungs*, not of atomic orbitals nor of states by particles. Nothing in this remark promotes an observable, moves a tier, or touches the engine.

4 The clock modulus: $M = 60 = 2h(E_8)$, two independent ways

4.1 Chinese-remainder carrier

The central sign $T^2 = -I$ forces a $\mathbb{Z}_4$ factor, the qutrit a $\mathbb{Z}_3$, and the order-five orientation a $\mathbb{Z}_5$; on the carrier $\mathbb{Z}_{60} = \mathbb{Z}_3 \times \mathbb{Z}_4 \times \mathbb{Z}_5$ the frozen, target-independent grading is $U_{60} = U_3^2 \otimes U_4^{-1} \otimes U_5^{-2}$, primitive because

$$\frac{2}{3} - \frac{1}{4} - \frac{2}{5} = \frac{40 - 15 - 24}{60} = \frac{1}{60}$$

is a unit mod 60, so U_{60} is a bijection onto $\mathbb{Z}_{60}$ (kernel `s281`).

4.2 McKay / E_8-Coxeter carrier

Independently (assessment A1086, in-house-verified), the binary icosahedral group $2A_5 \subset SU(2)$ has, under tensoring by its defining spinor, the affine $\widetilde{E}_8$ McKay graph; deleting the tensor-unit node gives finite E_8, whose Coxeter transformation has order $h(E_8) = 30$ with characteristic polynomial Φ_{30}. Its Clifford (Pin) lift Q_{E_8} satisfies $Q_{E_8}^{30} = I$; paired with the banked operator T ($T^2 = -I$) on the common carrier $\Sigma_{E_8} \otimes L_T$,

$$U_{60} = Q_{E_8} \otimes T, \qquad U_{60}^{30} = -I, \qquad U_{60}^{60} = I,$$

with minimal polynomial $x^{30} + 1$ and characteristic polynomial $(x^{30} + 1)(x^2 + 1) = \Phi_4^2 \Phi_{12} \Phi_{20} \Phi_{60}$. The spectrum is exactly the 30 odd 60th roots of unity (labels $15, 45$ doubled). Hence

$$\boxed{M = 60 = 2h(E_8)}$$

on the McKay–Clifford carrier. The central product $(2A_5 \times C_4)/\langle z = t^2 \rangle$ (order 240) contains *no* element of order 60, so the clock genuinely requires the McKay/Coxeter layer and not the central product alone.

Remark 4.1. The two derivations are independent and consistent: the CRT route fuses the 3- and 5-structure as separate factors, whereas the Coxeter route fuses them inside a single E_8 Coxeter element, with T supplying the doubling $30 \to 60$. This is the strongest internal convergence signal of the arc. The result is target-independent for the modulus and the odd-parity sector only; it does not touch occupation, and the even label 0 is absent from the spin clock (the tensor unit is a candidate origin, not a derived one).

5 The order-five orientation (conditional)

The native clock symmetries are all affine, contained in $\mathrm{AGL}(1,60)$ (order 960, solvable); the Galois-breaking order-five element is the icosahedral A_5 (simple, $|A_5| = 60$, with *no* element of order 60), whose binary cover is $2I = SL(2,5)$ — the E_8 McKay node (kernel `s280`). The order-five orientation is therefore an irreducible input relative to the affine clock.

It is *realized* on the exceptional Jordan algebra (assessment A1085, in-house-verified two ways): golden Peirce reflections H_0, H_1 on $J_3(\mathbb{O}_s)$ with $v_0 = (\varphi, 1, \varphi^{-1})$ give $U_5 = H_0H_1$ of exact order 5 and trace φ, with $\langle P, H_0\rangle$ closing to an icosahedral group whose determinant-+1 subgroup is A_5; the centralizer of U_5 in A_5 is exactly C_5 (the C_3–C_5 wall). The canonical lift $2I = SL(2,5)$ has the exact order census $\{1{:}1, 2{:}1, 3{:}20, 4{:}30, 5{:}24, 6{:}20, 10{:}24\}$, a unique involution $-I = T^2$, and every order-four element squaring to $-I$ (kernel `s281`).

Fences (carried verbatim). The order-five realization is *conditional* on the vacuum-to-projector bridge $J_{\mathrm{vac}} \mapsto v_0 \mapsto p_0$ of the golden-Peirce construction, which is a naturalness claim, *not* proven canonical. [LIB2-029] The central sign $T^2 = -I$ (equivalently the $2I$ centre, the Dic_{30} relation) is a banked *input*: "the central sign is forced" holds *given* one is in $2I$; choosing $2I$ is the input.

6 Galois-rigid localization of the occupation

The decisive structural result narrows the occupation to a single, fully rigid cyclotomic frame.

Novelty label: new specialization proved here.

Theorem 6.1 (Maximal Galois irregularity, `s283`)**.** $\mathrm{Stab}_{(\mathbb{Z}/60)^\times}(O) = \{1\}$ *exactly. Consequently O requires the full cyclotomic frame $\mathbb{Q}(\zeta_{60})$: it is fixed only after three further $\mathbb{Z}_2$ bits beyond the golden field are pinned (the ζ_4 spinor, ζ_3 qutrit, and ζ_5-square icosahedral signs).*

A blind golden/icosahedral index can be Galois-equivariant only under $H = \{u \equiv \pm 1 \bmod 5\} \cong \mathbb{Z}_2^3$ (order 8, the $\sqrt{5}$-fixer); but the H-orbits of 11 and 17 are disjoint, so O is not H-invariant and is unreachable by any blind golden index (kernel `s282`; this sharpens the earlier full-Galois statement of `s279`). The Galois closure of $\{3, 11, 17\}$ has $24, 20, 10$ labels over $\mathbb{Q}, \mathbb{Q}(\sqrt{5}), \mathbb{Q}(i, \sqrt{5})$ respectively; no affine map $\ell \mapsto a\ell + b$ cycles $3 \to 11 \to 17$; and the targets are simple eigenvalues, so degeneracy supplies no rule (assessment A1086). Every Galois-covariant level is thus sealed.

Remark 6.2 (Erratum to `s282`)**.** Kernel `s283` corrects the `s282` wording: the residual gap is *three* $\mathbb{Z}_2$ bits, not a "single $\sqrt{5}$ sign" (the $\sqrt{5}$ sign is one of the three).

7 The occupation selector: Derived-Conditional (Rev25)

Superseded note (Rev25). Through Rev24 this section carried the red "missing theorem" box: *construct a target-independent selector whose support is exactly* $O = \{0, 3, 11, 17\}$ — with the occupation values held as strictly Input/Open. [LIB2-029] That demand has now been met at derivation (not theorem-chain- complete) grade by the S224–S225 arc, and the occupation is promoted to DERIVED-CONDITIONAL. [LIB2-029] The original no-go results of this appendix stand unchanged — they proved a *Galois-covariant* construction cannot do it, and the selector that succeeded is indeed not Galois-covariant: it fixes the frame by anomaly-forced/monodromy data, exactly the type §6 said was the only one available.

The three-layer derivation (in-house kernels `s696`-class, `s704`; hostile-lane verified A1349/A1350):

1. **Mod 2 — Möbius parity.** The rung parity is the exact universal $\mathbb{Z}_2$ grading of Theorem 2.1, realized as the Möbius (orientation) parity of the winding substrate (`s696`-class theorem).
2. **Mod 5 — torsion walls.** The order-five orientation's torsion structure walls off the $\mathbb{Z}_5$ residues, fixing the ζ_5-sector frame data.
3. **Mod 7 — Fano/Artin adjacency, now derived.** The open spinor boundary is the support of one primitive Artin generator; $\sigma_1 \mapsto T_{\text{Artin}} = \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$ is *the* canonical $B_3 \to PSL(2, \mathbb{Z})$ map (not a choice); mod 7, T_{Artin} has order 7 (it is not the central-sign operator $T = T_{\text{clock}}$, $T_{\text{clock}}^2 = -I$, used elsewhere in this appendix — two operators, named apart at Rev32.9, A1637) and acts as $z \mapsto z{+}1$ on $\mathbb{F}_7$, forcing the boundary pair $\{0, 1\}$ to cyclic distance 1. The Fano line completes as $\{0, 1, 3\} = \{0, s(0), s^3(0)\}$ — the boundary pair plus the cube point — and the vector leg literally *is* T^3, converging with the banked T_S^3 monodromy. Exclusions are typed, not assumed: $d = 2$ is Möbius-parity even (no open half-twist); $d = 3$ is the vector leg (a spinor endpoint there is a typing collapse). Verified by enumeration (A1350, kernel `s711`).

The kill condition (live). The derived selector's period-two continuation is $\{31, 39\}$, hash-stamped (`7fd5775e...`) *before* verification rounds and confirmed by verbatim re-run (A1349). [LIB2-029] It is the standing **public falsifier**: an established occupied rung outside the selector's support — or a demonstrated failure of $\{31, 39\}$ where the selector's continuation is testable — kills the derivation and reverts the occupation to Input status. **Conditional on:** the vacuum-to-projector bridge (§5 fences, carried verbatim) and the banked $T^2 = -I$. [LIB2-029] Nothing else in the suite consumes the promotion; no public observable moves.

Historical record (pre-Rev25): two fenced routes were held pointing at the then-open gap from opposite sides (opt-in; the anomaly-forced type of the first is the type the successful selector in fact has):

- the $G_2(2)$ extremal-CFT anomaly — $G_2(2) = \text{Aut}(\mathbb{O}_s)$ and $J_3(\mathbb{O}_s)$ is the carrier of the order-five orientation, so the gauge axis and the order-five axis are the same substrate; a central-charge-fixing CFT fixes the modular T-data ($\theta = \zeta_{60}^{jm}$ has trivial natural covariance, hence is of the right type); and

- a noncentral physical index — a Peirce/generation-coupled rank-three index with one zero-mode line in each of the three native blocks $(3, 11, 17)$, under a tight blind contract (freeze before unblinding; rank derived, not stipulated; the same projector must govern the physical mass operator).

Remark 7.1 (the oriented-Schur torsion $\kappa = \sqrt{3}$ and the all-orders firewall (`s292`–`s294`, S175; opt-in))**.** The noncentral-index route above has a concrete, verified intermediate result that nonetheless leaves the occupation open. The orientation-odd entangling torsion the hunt needs is canonical: eliminating the terminal complement of the paired-orientation parent vertex by the *oriented Schur complement* forces $K_{\text{eff}} = i\, g_R g_J\, \beta\, (RJ - JR) = \kappa\, K_{--}$ with spec $= \{-\sqrt{3}, 0, +\sqrt{3}\}$, i.e. $\kappa = \sqrt{3}$ — nonzero generically whenever both terminal bridges are present (the democratic-singlet gap cancels; coefficient theorem). But it lives *detached* from the current commutative vertex algebra ($\tau_z \otimes K_{\text{eff}} \perp \mathbb{C}[\tau_x] \otimes \mathbb{C}[D_\varphi]$; cross-bridges $\sim 10^{-15}$), so installing it is *new physics*. The accompanying *action-match firewall* is an all-orders no-go gate with a sharp binary falsifier: if the physical R- and J-bridges land in orthogonal terminal sectors the interference vanishes ($\kappa = 0$) and the mechanism is dead. The frozen-index test — are the three odd sectors of D_{ent} exactly $\{3, 11, 17\}$? — remains untested. This remark promotes nothing by itself; the occupation's Rev25 status (Derived-Conditional, §7) rests on the three-layer selector derivation, not on this route (banked A1090).

Remark 7.2 (independent corroboration: the adjacent-integer support floor (S245, `s901`/D1121))**.** The nonzero occupation $\{3, 11, 17\}$ is re-derived, independently of the three-layer selector, by a clean minimax argument. Reading the continuous golden depths off the physical ratios gives (2.996, 11.080, 16.945); rounding each to its nearest integer and scoring a candidate support by its worst-case depth mismatch, $\{3, 11, 17\}$ is the *global minimax* — objective floor 0.0795 (set by the muon offset $n_\mu - 11$), with the nearest competitor $(3, 12, 17)$ far above at 0.588. This is corroboration, not a new promotion: it strengthens the Derived-Conditional support of §7 from a second, convention-light direction. It is fully consistent with the exact additive-χ completion of the mass *ratios* on this support being [closed-negative] at the ratio tier (Appendix O, and the closed-selector ledger of Appendix S): the integer support skeleton is robust; only the exact numerical completion on it fails. Nothing here moves the $\{31, 39\}$ falsifier or any public observable.

8 Status ledger

Result	Status	Conditional on
Parity $\mathbb{Z}_2$ universal grading (`s267`)	✓ theorem	—
$C_{\text{pt}} = \{0, 30\} = \mathbb{Z}_2$	✓	—
$\text{Rep}(D_{60})$, 33 simples, $\sum \text{FPdim}^2 = 120$	✓	—
$M = 60 = 2h(E_8)$ (two ways)	✓ target-indep.	modulus + odd sector only
Order-five orientation on $J_3(\mathbb{O}_s)$	✓ conditional	vacuum→projector bridge
$T^2 = -I$ central sign	input (banked)	choice of $2I$
$\text{Stab}_{(\mathbb{Z}/60)^\times}(O) = \{1\}$	✓	—
Parity as CKM mixing rule (`s269`)	× (honest negative)	falsified
Condensation/boundary/dynamics no-go (`s270`–`s272`)	× structured negative	O not bulk-forced
$SU(2)_L$ from P_{code} monad (`s273`)	× (annihilated)	off-diagonal Peirce killed
Occupation $O = \{0, 3, 11, 17\}$	**Derived-Conditional** (Rev25)	3-layer selector (§7); $\{31, 39\}$ live falsifier
Adjacent-integer support floor $\{3, 11, 17\}$ (`s901`)	✓ corroboration (S245)	minimax floor 0.0795; independent of 3-layer selector
$G_2(2)$ / noncentral-index selector	**FENCED (opt-in)**	not run
Oriented-Schur torsion $\kappa = \sqrt{3}$ (`s292`–`s294`)	**FENCED (opt-in)**	detached vertex; firewall pending

The clock modulus and parity are structural results on the categorical carrier; the occupation is Derived-Conditional as of Rev25 (three-layer selector, $\{31, 39\}$ live falsifier), with the fences of §5 carried verbatim. The engine and all public observables are unchanged by this appendix.

Leibniz Quantum Beats Newton

Appendix N — Selector Fences: Why the Fine-Structure Constant and the Compact Colour Form Are Inputs, Not Outputs (Rev33.2)

Two honest negatives. α stays fenced (now five-legged); the compact colour real-form selector is named as input. No public observable moves.

Tom O'Sieg

August 2026

Abstract

This appendix folds two settled-negative arcs into the lab notebook. Part N.A (kernels `s255-s262`, sessions S165–S166) records the attempt to *un-fence* the fine-structure constant α and its fivefold refutation: the Wyler bounded-domain route is numerically falsified; the golden field makes any near-137 target numerically dense, so closeness carries no information; "perfect-imperfection" (noble/KAM) stabilization is real mathematics that selects nothing; and an energy/dissipative principle locks to rationals, with the golden value unpinned only below the Aubry threshold (it pins through K_c; `s261`); and the dynamical-scale-via-vacuum-decay route is closed (D905): on its normalized radial potential the $\omega = \pi/4$ golden vacuum has no lower point ($V_N \geq 0$), so no nontrivial decay bounce exists in that radial problem (the trivial vacuum has zero action relative to itself), and any bounce would in any case need a second input scale (Leg 5). The conclusion is structural: a fixed geometric number for a *running* coupling is a category error absent a derived privileged scale, and the rest-mass integer selector is a discrete topological (winding) datum, not a continuous offset (under the Leg 4 deformation-class premise, Part N.A). Part N.B (kernels `s253`, `s255`, `s257`, sessions S164–S165) names the real-form / colour selector: the compact $\mathfrak{su}(3)$ colour algebra cannot embed in the available compact part (maximal compact dimension $6 < 8$), so the choice of real form is an input. Both results are honest negatives; α remains an input modulus and nothing is promoted.

Contents

1 Part N.A — The fine-structure constant is fenced **1**

1.1 Leg 1 — the Wyler route is falsified (`s256`) 1

1.2 Leg 2 — golden numerology density (`s258`, `s259`) 2

1.3 Leg 3 — perfect-imperfection is robustness, not selection (`s260`, `s261`) 2

1.4 Leg 4 — the selector is topological (`s262`) 2

1.5 Leg 5 — the dynamical-scale route is closed (`s175`, D905/A1088/A1089) 2

2 Part N.B — The compact colour real-form is an input **3**

3 Part N.C — The electroweak framing is loaded: the permanence theorem (S231–S232) 5

4 Part N.D — The dual ruling: what was killed, and what stayed open (Rev30, A1500) 5

5 Status ledger 7

6 Part N.E — Two fences added at Rev32.6 (A1566, A1567, A1568) 7

1 Part N.A — The fine-structure constant is fenced

The framework declares α (the $U(1)_{\mathrm{EM}}$ fibre scale) an input. [LIB2-236, P-008] Session S165 tested whether it could instead be *derived*. It cannot. Five independent legs establish the fence.

1.1 Leg 1 — the Wyler route is falsified (`s256`)

The Wyler bounded-domain normalization, computed exactly, gives $\alpha^{-1} = 137.0360824$ against the CODATA value 137.035999177 — a discrepancy of thousands of standard deviations. The associated multi-tuple template family is excluded en masse. The bounded-domain programme is retired as a derivation of α. [LIB2-236]

1.2 Leg 2 — golden numerology density (`s258`, `s259`)

Scale-free ratios built from φ populate the neighbourhood of 137 densely; under a controlled density discipline (a Beatty/Wythoff admissibility count), a near-137 hit is *expected by chance* and therefore carries no selective information. [LIB2-092] The counts are the kernels': a pool of 21 golden and substrate invariants, monomials up to complexity 6, 503 distinct values near 137 (`s259`; `s258`: 30 up to complexity 5 without the golden ray); the 1% band is reached at complexity 3 and the 1.5 ppm band is not reached at all (Rev32.9, A1647 U2). The "it is close" observation is a numerology-class artifact, not evidence. [LIB2-092]

1.3 Leg 3 — perfect-imperfection is robustness, not selection (`s260`, `s261`)

The golden value is the most-irrational (noble) number and hence the KAM-most-stable torus, with critical coupling $K_c \approx 0.9716$ in the standard-map sense (`s260`). But in a Frenkel–Kontorova / Peierls–Nabarro setting an energy/dissipative principle *mode-locks to rationals*; the golden value slides freely *below* the Aubry threshold $K_c = 0.971635$ and pins at and through it (`s261`: `golden_unpinned_below_Kc`, `golden_pins_through_Kc`; Rev32.9, A1647 F03 — through Rev32.8 this said "the unpinned, freely sliding extreme" without the regime). [LIB2-093] Noble stabilization is a robustness fact, not a mechanism that fixes a coupling. [LIB2-093]

1.4 Leg 4 — the selector is topological (`s262`)

The depth ladder

$$R_d = \frac{1}{\varphi^{2d} - 1}$$

is a resolvent / Green's function. *The winding datum, named (Rev32.10, A1711-F05):* the registered winding map is the occupation map of Appendix M, $n : \{\text{ladder configurations}\} \to \mathbb{Z}$, which integer rung of R_d is occupied; it is the selector in Paper 0's bound sense and nothing else is meant by "winding" here. Its invariance is a premise about the deformation class, not a consequence of discreteness alone: the admissible class is continuous one-parameter shifts of the rung positions that preserve the ordering of the rung energies, and n is locally constant on that class. Where two rung energies cross the minimising label switches (the two-level control $E_0(a) = a^2$, $E_1(a) = (1-a)^2$ switches at $a = \frac{1}{2}$), so a crossing is outside the admissible class by definition, and the statement below is conditional on that exclusion. A continuous spurion (the Reynolds offset) — the deformation class here is continuous one-parameter shifts of the rung positions — can shift positions on the ladder but cannot leap its discrete integer steps; therefore the integer *selector* (the occupation of Appendix M, in Paper 0's bound sense: which integer rungs are occupied; Rev32.9, A1647 U1) is a discrete topological (winding) datum, not a continuous correction. [LIB2-094]

1.5 Leg 5 — the dynamical-scale route is closed (`s175`, D905/A1088/A1089)

A final route would *generate* a privileged scale dynamically — a Coleman vacuum-decay rate $\sim e^{-S_{\text{bounce}}}$ off the golden vacuum — and use it to pin α. It fails on three counts. *(i)* In the radial problem the golden vacuum is *absolutely stable*: the normalized radial potential $V_N(r) = \Lambda^4(e^{-2r} + 2e^r - 3)$ has a unique critical point at $r = 0$ with $V_N(0) = 0$, $V_N''(0) = 6\Lambda^4 > 0$ and $V_N(r) > 0$ for all $r \neq 0$, so there is nowhere lower to tunnel to and the Euclidean equation of motion has only the trivial solution: no nontrivial decay bounce exists in this radial problem (positive canonical kinetic term; a regular finite-action solution approaching $r = 0$; vanishing boundary term), and the trivial vacuum has zero action relative to itself (Rev32.9, A1647 F04: through Rev32.8 this read "$S_{\text{bounce}} = 0$ exactly", which is the trivial vacuum's own action, not a tunnelling exponent). [LIB2-095] Count (i) is a statement about the radial model only. On the full scalar manifold the golden point is a tree-level Morse–Bott minimum joined at zero cost, along the $V \equiv 0$ golden ray, to the $SO(6,2)$ origin, which is not a minimum and from which the potential runs away (Appendix I; `s1361`, `s1249`); whether a decay path off the golden point exists there is open, so count (i) is not decisive on the full manifold. The route stays closed on count (ii), which holds for any bounce, and on count (iii). The "bounce" of Appendix I is *classical* geodesic reflection off the Det = 0 wall (a metric-completeness fact), not a Coleman tunneling event; with no decay rate there is no dynamical scale. [LIB2-095] *(ii)* Even granting metastability, the $O(4)$ bounce action $S_E \propto (M_{\text{kin}}/\Lambda_{\text{pot}})^4$ is Λ-dependent and requires a second input scale — never geometry alone. *(iii)* The bounce lives on the *radial cone modulus*, with no banked term linking it to the $U(1)_{\text{EM}}$ fibre kinetic coefficient α; a scale there would set a dimensionful quantity, not the dimensionless coupling. [LIB2-095] Convergent double refutation (Grok A1088 + Gemini A1089), leg *(i)* verified in-house (V_N minimum = 0 at $r = 0$, no $V_N < 0$). The companion four-tick reading — T accruing a -1 on reflection at the Det = 0 wall ($T^2 = -1$, $T^4 = +1$) — is validated by both reviewers as sound *classical holonomy* of the clock turnover, but it too generates no coupling.

Ledger. α is an input modulus, fenced on five independent legs (Wyler-falsified; golden-dense; robustness-not-selection; running-coupling category error; dynamical-scale route closed). [P-008] It *runs*, so a fixed geometric value is ill-posed without a derived privileged scale — none exists in the banked framework. φ remains the single golden axiom (AX1); α is not derived from it.

2 Part N.B — The compact colour real-form is an input

The colour selector concerns which real form of the relevant Lie algebra carries the physical gauge content.

Novelty label: new specialization proved here.

Proposition 2.1 (Compact-colour obstruction, `s253`)**.** *The compact colour algebra* $\mathfrak{su}(3)$ *(dimension* 8*) cannot embed in the compact part available to the loaded subimage, whose maximal compact dimension is* $6 < 8$*. [LIB2-096, LIB2-297] Hence the physical colour content is not forced by a compactness criterion; the choice of real form is an input. [LIB2-096]*

The constructive side is recorded for completeness: the compact $\mathfrak{g}_2(-14)$ is built from the split $\mathfrak{g}_2(2)$ by an explicit Cartan involution (`s255`), and the conformal algebra $\mathfrak{so}(4,2) \cong \mathfrak{su}(2,2)$ together with $\mathfrak{g}_2(2)$ is located inside $\mathfrak{so}(4,4)$ (`s257`). These fix the *ambient* real-form geometry but do not remove the selector: the real form is chosen, not derived. [LIB2-297]

Ledger. The real-form / colour selector is an input, on the same footing as the gauge-group assignment of the main text. Proposition 2.1 makes the obstruction explicit (a dimension count), so the input is named rather than hidden. No public observable moves.

The colour input enters twice. Its compactness: colour is the compact unitary-class $\mathfrak{su}(3)$ centraliser in $\mathfrak{su}(8)$, never a gauging, and the split frame supplies only $\mathfrak{sl}(3,\mathbb{R})$ (`s1216`). Its orientation: the frame $\mathbb{Z}_2$ is $\psi = \det A$ of the real colour three-frame, a gauge-invariant label, not a vacuum (A1750). Both are faces of one object: compact colour's complex structure is $J_c = P \cdot J$, the split para-complex colour structure twisted by the slot-norm involution P — the same P that carries κ's T_3 weighting and, as an additive charge, the Majorana obstruction (`s1218`, `s1219`).

The carrier truncation is also chosen: a pre-registered null (`s709`, S225)

The selector above is stated at the level of the real form. There is a second, finer place the same question can be asked, and it was asked under a pre-registered test whose result is recorded here for the first time.

The loaded colour/Ω_Q provenance is carried as *conditionally* selected, the condition being that the 56-dial — clock-weight multiplicities $(1, 27, 27, 1)$ — may legitimately be truncated to one state per weight,

$$E_Q^{\text{loaded}} \;\cong\; E_{-3} \oplus E_{-1} \oplus E_{+1} \oplus E_{+3}.$$

`s709` tested whether that truncation is *extracted* or merely *chosen*, using the three measured translator velocities R_k $(k = 3, 4, 5)$ acting on the weight-graded 56. Its four gates and their tiers were frozen before the run.

Gate	Result	Reading
C1 nonzero	PASS	Every a_k, b_k has norm $> 10^{-10}$ (norms 0.0283, 0.1216, 0.1409): the translators do connect the end states.
C2 cross-translator agreement	**null**	$\mathrm{sv}_2/\mathrm{sv}_1 = 0.468$ against pre-registered tiers FORCED $< 10^{-10}$ and STRONG $< 10^{-2}$; pairwise $\lvert\cos\rvert$ of 0.73, 0.62, 0.52. The three translators pick *different* lines in the 27.
C3 ladder closure	FORCED	Top cos with e_{+3} equal to 1.0 at $k = 3, 4, 5$.
C4 up/down consistency	NULL	$\lvert\cos(b_k, v_{2,k})\rvert = 0.042$, 0.00045, 0.0022 at $k = 3, 4, 5$: the down-ladder does not meet the up-ladder (row added Rev32.9, A1652; the frozen fourth gate was not printed through Rev32.8).

Verdict: `S709_CARRIER_NULL_TRANSLATORS_INSUFFICIENT`.

What this adds to the fence. C2 was the gate that mattered: had the three translators picked the same line, one-state-per-weight would have been *extracted* rather than chosen, and the conditional under the loaded colour provenance would have discharged itself. It returned NULL, and not marginally — 0.468 against a 10^{-2} threshold. **So the truncation is a choice, exactly as the real form above is a choice**, and the conditional selection remains conditional. [LIB2-097] Note also what did *not* fail: C1 and C3 passed, so this is a specific negative about cross-translator agreement, not a general failure of the construction. [LIB2-097] No public observable moves either way.

3 Part N.C — The electroweak framing is loaded: the permanence theorem (S231–S232)

The electroweak scale registration $v_{\mathrm{EW}}/\sqrt{\sigma} = \varphi^{105/8}$ (Appendix O, the σ-bridge section) rests on a framed-depth package $\mathcal{D}_H = (P_H, N_{\mathrm{ann}}, q_H)$ — a Higgs line, a winding level, and a quadratic refinement. The natural hope, that this package is selected frame-free, is closed *negative* at theorem grade, on the same footing as the α fence of Part N.A:

Novelty label: new specialization proved here.

Proposition 3.1 (Electroweak permanence; S231/S232)**.** *No frame-free package $\mathcal{D}_H$ exists on the bare Freudenthal system $\mathfrak{F} = \mathbb{R}\oplus\mathbb{R}\oplus J_3(\mathbb{O}_s)\oplus J_3(\mathbb{O}_s)$ under its automorphism group $G_0 = E_{7(7)}$. [LIB2-098] Specifically: (i) LOAD-EW-LINE — the* **56** *is real-irreducible, so no rank-one invariant projector (Higgs line) exists (Appendix F); (ii) LOAD-EW-INDEX — the level $N_{\mathrm{ann}} = 13$ is homed ($= F_7 = a_8$) but has no frame-free basepoint to attach to; (iii) LOAD-EW-FRAME — the quadratic refinement is a torsor, not a function of the form; the two valid refinements ($\beta = 1$ and $\beta = 7$, $q(1) = 1/4$ vs $3/4$) have equal $\lvert\gamma\rvert = 1$ and are exchanged by the metaplectic lift-sign $q \mapsto -q$, so no intrinsic selector fixes $\beta = 1$. [LIB2-098]*

Ledger. The electroweak framing is a *loaded* input, on the same footing as the real-form/colour and α selectors. [LIB2-098] Consequently $v_{\mathrm{EW}}/\sqrt{\sigma} = \varphi^{105/8}$ is the unique *visible* framed-depth registration *relative to* the marked compact-positive carrier (Paper 4), not a frame-free scale theorem — the honest tier at which Appendix O foregrounds it. No public observable moves.

4 Part N.D — The dual ruling: what was killed, and what stayed open (Rev30, A1500)

The final round of the freeze-first arc returned a *dual* ruling, and its two halves are routinely misquoted in opposite directions. This section states all four statuses together, because any one of them quoted alone is a false claim.

The four statuses, and they must travel together. (1) Row 1 is KILLED — but only as a *sole parent datum*, and only inside the declared class. [LIB2-099] The class was committed, not gestured at: the frozen-V_{56} category $\mathrm{BeatFlip}_{56}^{\mathrm{fr}}$, named before the target artifacts were loaded. Inside it, the orphan doublet is the carrier-level compact-reduction *shadow* of σ_{sel} — an exact frozen-carrier shadow and nothing more — not the missing external boundary datum.

(2) GLOBAL Row 1 remains open, for genuinely external categories that do not factor through the orphan-image class. The kill does not reach them.

(3) The conditional Row-2 subgate PASSES.

(4) The ORIGINAL full Plan-v2 Row 2 remains open. The finite subgate is *not* the row and must never be quoted as if it were.

Why Row 1 fails in its class. The vertical K_{sel} action moves L_∂ with rank exactly ten, so the assignment $gK_{\mathrm{sel}} \mapsto gL_\partial$ is **not well-defined**: changing the representative changes the alleged boundary image in every direction of a ten-dimensional orbit. The orbit dimension reached is 43 where 53 is required, leaving a residual fibre of 10, and the stabilizer arrives at dimension 20 where the dynamical requirement is 10.

What the subgate did and did not establish. Four checks passed: an exact homogeneous reduction to the reduced action with 32×32 Hessian residual 0.0; the full ten-generator first-class constraint algebra, all 100 commutators and all 1000 Jacobi triples at residual 0.0; a gauge-reduced principal symbol $\operatorname{spec} \sigma_{\mathrm{phys}} = 30^{(62)}$ at $k^2 = 30$, with a wrong-sign control returning $-30^{(32)}$; and a positive finite transfer truncation (81×81, $\lambda_{\min} = 4.27 \times 10^{-5} > 0$) together with a four-time reflection Gram ($\lambda_{\min} = 4.55 \times 10^{-5} > 0$), against a time-reversed control at $\lambda_{\min} = -1.0000039$. **None of that is a continuum result.** No nontrivial stable renormalized four-dimensional continuum trajectory was constructed, which is precisely why status (4) is open.

Two things that must never be printed as results. *First,* the reduced-sector gaps 0.3235, 0.3290, 0.3334 at $a = 0.5, 0.4, 0.3$ are a **temporal-regulator sanity sequence, not a continuum trajectory**. Throughout, a is typed as a regulator and not as a measured scale; reading the sequence as an extrapolation would manufacture a continuum limit that was not constructed, and both the lane and the house say so explicitly.

Second — and this is the more instructive one — there was a control that ***succeeded****, and it was refused.* The object "orphan $+ L_\partial$" does land on orbit dimension 53 with the required stabilizer and residual fibre 0: numerically perfect. It is also **exactly the forbidden circular repackaging**, because it reaches the target by importing the very datum it was supposed to supply. A round reporting only that success would have read as a solved row. It was typed as circular and **not banked**. The gate that separates the two outcomes is the non-circularity pre-test, and it is the reason the dispatch was built the way it was.

The adjudication was reproduced across two hosts — 20/20 gates, 20/20 should-fire controls, semantic hash exact, with 3 of 398 result leaves differing and all three being environment stamps. **Nothing was promoted.** No observable, no tier and no comparator moved; α remains fenced (Part N.A) and the engine is unchanged.

5 Status ledger

Result	**Status**	**Note**
α from Wyler domain (`s256`)	× falsified	137.0361 vs 137.036, $\gg \sigma$
golden near-137 density (`s258/9`)	× numerology	closeness uninformative
noble/KAM stabilization (`s260/1`)	robustness only	not a selector
selector is topological (`s262`)	✓ structural	winding, not offset
dynamical scale via decay (`s175`/D905)	× closed	no nontrivial bounce; stable vacuum
α overall	**FENCED (input)**	five-legged; runs
compact $\mathfrak{su}(3)$ embedding (`s253`)	× obstructed	max compact dim $6 < 8$
real-form / colour selector	**input (named)**	chosen, not derived

Both arcs are honest negatives scoped to the banked framework. They sharpen the input ledger (Appendix X) by naming *why* α and the colour real form are inputs, and they tie the rest-mass selector to the topological occupation of Appendix M. Nothing is promoted; no public observable moves.

6 Part N.E — Two fences added at Rev32.6 (A1566, A1567, A1568)

The θ_{23} octant registration: fence (A1566/D1244; `s501` retyped, `s1156`). `s501` is a *structural candidate/interface*: the trefoil chirality supplies a possible odd $\mathbb{Z}_2$ carrier and the even sector is blind to the octant sign, but the chirality torsor $\{-1,+1\}$ and the octant torsor $\{7/16, 9/16\}$ admit exactly two $\mathbb{Z}_2$-equivariant identifications (`s1156`: $-1 \mapsto 7/16$ and its mirror), and no registered action term, operator, intertwiner or readout map excludes the countermap. [LIB2-100, LIB2-241, LIB2-308] The internal identity $\sin^2\theta_{\text{mismatch}} = 7/16$ is theorem-grade; its attachment to the physical lower octant is LOADED-CORRESPONDENCE (Paper 3). [LIB2-100] The Rev32.5 claims "selector built", "locked to ε_Q" and "free bits $2 \to 1$" are retired. [LIB2-241] *Reopen condition:* a registered loaded odd object Ω_{23} with a theory-fixed sign in a coupling such as $S_{\text{link}} = -\kappa\,\chi_{\text{odd}}\cos 2\theta_{23}$, plus a pre-registered mirror control proving the countermap impossible without using θ_{23}, δ, or observed charge-sign data.

[LIB2-308]

Complex-ratio ladder: fence (A1567/D1246; A1568/D1248, `s1159` v2). A1567 finds no registered phase-bearing ladder ratio: a census of 29 ladder/family rows against 39 phase/rotation rows across the shipped components registers no $q = \phi^{-1}e^{i\alpha}$; the one commuting contact $[Y, L_{\text{vac}}] = 0$ (Appendix I, `s105b`) does not tie the $U(1)$ angle to a rung. [LIB2-101] The missing object is typed: $Q_L = S_L\, e^{\alpha J_L}$ with $(J_L|_E)^2 = -I_E$ and $[J_L|_E, S_L|_E] = 0$ on the ladder two-plane E, with E and its orientation named by the registering datum (as the antecedent D1247 states the target "on the ladder 2-plane"), and *one* source law fixing both the real rung step and a nonzero α; installing J_L, choosing the angle, or selecting after comparison does not qualify. [LIB2-101, LIB2-309] The obstruction is now computed, not typed. For the two computed carriers, $C(L_{\text{vac}}) \subset \mathfrak{e}_{6(6)}$ has dimension/signature/centre $30,\ (18,12,0),\ 2$, while $C(H) \subset \mathfrak{e}_{7(7)}$ has $79,\ (43,36,0),\ 1$. [LIB2-102] Both have positive-dimensional compact subalgebras, but Y, L_K, H are noncompact and the Appendix-H K is not in $C(H)$. [LIB2-103] Appendix G separately supplies a commuting compact Z_2, but leaves its angle independent of D_{KK}. [LIB2-103] No one-law (J_L, α) is selected; any attachment remains LOADED-CORRESPONDENCE. [LIB2-102] *Loss condition (A1567), met at Rev32.5:* no theory-selected compact $U(1)$ in any ladder-scaling centralizer, or only ones whose angles stay free. *Reopen:* a registered compact datum commuting with the scaling, with its angle fixed without comparison. [LIB2-101, LIB2-309] The census is invariant under $SU(8) \to SU^*(8)$ (`s1160`; Appendix G).

What a phase-bearing defect must deliver: the CP-cycle criterion (Rev32.9; A1576 check 3, house re-run to 10^{-12} and again at S341). After canonical kinetic normalization, with $H_u = \mathrm{diag}(a_1, a_2, a_3)$,

$$\mathrm{Tr}[H_u, H_d]^3 = 6i\,(a_1 - a_2)(a_2 - a_3)(a_3 - a_1)\ \mathrm{Im}\big[(H_d)_{12}(H_d)_{23}(H_d)_{31}\big],$$

the Jarlskog commutator invariant. A single off-diagonal link, or a two-link tree, gives mixing but no three-family Dirac CP violation (the invariant vanishes); a non-zero value needs a closed three-link product with a non-removable phase. Hence $[Y_u, Y_d] \neq 0$ is necessary only: a candidate defect for the Ω_{23} charter must export a registered, gauge-compatible response into $\mathrm{Tr}\, C^3 \neq 0$, and a half-period or compact rotation frequency is not a CP phase until it is so exported. Two limits of the criterion (S367; house kernel `s1205`): the invariant is proportional to the Jarlskog $J \propto s_{23}c_{23}$, which is symmetric under $\theta_{23} \to 90^\circ - \theta_{23}$, so the criterion cannot decide the octant; and on today's assembled up/down pair it evaluates to a multiple of $\sin\delta_{\mathrm{CKM}}$ of the formula-installed phase — a read-back, not an export — so the test applies to a future registered defect, not to today's pair. What *does* see the octant is the CP-even sector (S370; house kernel `s1208`): the flip $\theta_{23} \to 90^\circ - \theta_{23}$ is the $\mu \leftrightarrow \tau$ relabelling composed with $\delta \to \delta + \pi$, so every weak-basis invariant of the assembled lepton pair up to degree six separates $\sin^2\theta_{23} = 7/16$ from $9/16$ in its real part — the lowest is $\mathrm{Re}\,\mathrm{Tr}(H_e H_\nu) = \sum m_\alpha^2 m_i^2 |U_{\alpha i}|^2$ — through the splitting $m_\mu^2 - m_\tau^2$ alone, while every CP-odd part is blind. A theory-side test of the octant would therefore be a CP-even criterion, never a CP-cycle; none is registered, and the attachment of 7/16 stays LOADED-CORRESPONDENCE.

Why the target must be local: a parity lemma (Rev32.9; A1576 check 2, A1652; house kernel `s1176`). A real linear J with $J^2 = -I$ and $[J, S] = 0$ exists only if every real eigenspace E of S is even-dimensional, since J preserves E and $(\det J|_E)^2 = \det(-I_E) = (-1)^{\dim E} \geq 0$. The registered scaling spectra are L_{vac} on the **27** with multiplicities $(1, 1, 1, 8, 8, 8)$ — three odd blocks — and H on the **56**, from $\mathrm{diag}(1, 1, 1, 1, 1, 1, -3, -3)/2$, with eigenvalues $\{-3, -1, +1, +3\}$ and multiplicities $(1, 27, 27, 1)$ — all four odd (`s1176`, 6/6 gates, both lemma controls). Hence *no* everywhere-defined J_L commutes with L_{vac} on the full **27** or with H on the full **56**: this is stronger than "the scaling centralizers contain compact subalgebras but do not select the phase law". The phase target must be an even-dimensional invariant subcarrier — at most 24 of 27 and 52 of 56; these are parity bounds and select nothing — and the H_ϕ spectrum $(12, 32, 12)$ is exempt. No tier moves.

Leibniz Quantum Beats Newton

Appendix O — The Charged-Lepton Rest Mass as a White/Black Self-Energy Residual: a Built-but-Open Operator Program (Rev23) — Closed as Ratios-Complete with One Registered Scale Input (Rev28; suite Rev33.2)

A lab-notebook record of the S179–S192 rest-mass arc, closed at Rev25 by the S225–S226 scale-theorem chain (§10): the program is ratios-complete*; the framework carries exactly one dimensionful input, counted by cohomology and registered in Appendix X. Definition A; α fenced; no public observable moves.*

Tom O'Sieg

August 2026

Posture (binding). This appendix is an *honest-open* record of an active research program, not a results chapter. It documents what the framework's charged-lepton rest-mass machinery has and has *not* achieved across sessions S179–S192. Every statement is tagged [DERIVED], [CONDITIONAL], [INPUT], [CLOSED-NEGATIVE], or [OPEN]. *No mass or phase value is promoted*; the charged-lepton spectrum is reached only as a two-parameter fit; the framework's published predictions are unaffected. The engine workbooks are unchanged across the whole arc (Machine `727b0bf01eb5`, Outputs `7f3bc6f92b74` — the hashes of that era, S179–S192; the shipped pair is named in Appendix X); no public observable depends on anything below.

1 Scope and the one-line summary

The framework fixes the charged-lepton mass *ratios* only through the Koide relation. What it derives is the *shape* relation — $Q = 2/3 \Leftrightarrow r = \sqrt{2}$, the circulant null condition (§3) — and not the *value*: the number 2/3 enters as an external selector (*Rev29*; WATCH box in §3). It does *not* derive the individual masses: those require two further numbers (the "gaps") that, on all present evidence, are irreducible *inputs* of Yukawa type (§5). Between S179 and S192 the supporting operator architecture was built out in full and verified deterministically in-house (§4), and every physical route to *deriving* the two gaps was found to terminate at a single upstream object — a valid gauge-preserving S_{125} carrier, the same object that gates the Standard Model colour charges (§6). An independent AI-coalition review converged 2/2 on the conservative reading (§7): the shape is the result, the gaps are the input.

2 The reframe: winding primary, mass a self-energy residual (S184)

The organizing hypothesis of the arc replaces "select the mass" with "select the integer, then correct it." A charged lepton occupies an integer *winding* N on the golden ladder, and its mass is written

$$m \;=\; m_e\,\varphi^{\,N+\delta}, \tag{1}$$

with N primary and the mass detail carried by a smooth residual δ. The integer N is fixed by a spinor tension-balance closure that is *winding-blind* ($\det(\varphi^{+N}, \varphi^{-N}) = 1$ for every N; Appendix H), which is precisely why every earlier attempt to dynamically *select* a preferred rung failed — there is no preferred rung to select. [CONDITIONAL] reinterpretation: consistent with all data and it dissolves the "phantom rung" impasse, but it is not a derivation of δ. The integer windings themselves stay [INPUT] (engine column H, `SELECTOR=OPEN`).

The residual δ is read as a *gauge self-energy* correction to the propagator (kernels `s346`/`s348`): the leptons carry a Koide/EM self-energy (the winding-only $Q_{\text{pure}} = 0.6728$ is refined to the physical $Q_{\text{real}} = 2/3$, with mean $|\delta| \approx 0.045$); the quarks carry an equal-and-opposite weak-isospin T_3 self-energy ($\approx \pm 12.4\%$, isospin-singlet part ≈ 0). This unifies two previously separate framework items under one reading but is shown only as self-energy *structure*, not computed from a propagator. [CONDITIONAL]. The construction *models* the mediator as the white-hole/black-hole (WH/BH) dyonic two-sheet object: the electron is *modelled as* a T-bridged $(4, 4)$ pair, BH$(+E)$/WH$(-E)$, and the rest mass *as* the particle–hole *half-gap* the fermionic Möbius twist protects from cancellation (no negative rest mass; `s308`). [CONDITIONAL]. *Rev29 (F12)*: "is" is withdrawn throughout this reading. No identification of a physical mediator with the WH/BH object is claimed or shown; what exists is an operator model with the stated algebraic properties, and every downstream statement inherits the modelling step, not an identification.

3 The Koide shape: derived; the $2/9$ phase: closed-negative

Novelty label: standard theorem used.

Proposition 3.1 (Koide as an exact circulant null condition, `s430`)**.** *For a Hermitian circulant amplitude $A = c_0 I + c_e K_{++} + c_o K_{--}$ on the $\mathbf{1} \oplus \mathbf{2}$ generation split, the Koide value $Q = 2/3$ holds* iff *the radial ratio is $r = \sqrt{2}$. The printed constant is $C = \sqrt{2}/\dim J_2(\mathbb{O}_s) = \sqrt{2}/10$. [DERIVED] as a shape/null condition — and the attribution is split: the biconditional $Q = 2/3 \Leftrightarrow r = \sqrt{2}$ for a circulant $\sqrt{m}$ amplitude is the known geometric form of Koide's relation (R. Foot, arXiv:hep-ph/9402242, 1994; the circulant form is also Brannen's, 2006),* not *new here. What this appendix derives is the circulant structure itself on the triality-organized $\mathbf{1} \oplus \mathbf{2}$ split, and the tracing of the constant $\sqrt{2}$ to $\dim J_2(\mathbb{O}_s)$ (`s430`). Hence the novelty label "standard theorem used" above, and the [DERIVED] tag on the shape (Rev33.0, PI call S362).*

This is the genuine, parameter-free content: the circulant (democratic) structure, the triality organization, and $Q = 2/3 \Leftrightarrow r = \sqrt{2}$. What it fixes is the *shape* of the lepton triple, not its two independent scales.

WATCH (W02, *Rev29*) — the shape is derived, the value is an external selector, and the two are not the same claim. Proposition 3.1 is a biconditional: *given* $Q = 2/3$ the radial ratio is forced to $r = \sqrt{2}$, and conversely. That is a statement about shape. It does *not* produce the number $2/3$ — nothing in the circulant null condition selects that value over any other admissible Q, and the value is supplied from outside the algebra. Against this, `Trackers/HONESTY_LEDGER.md:38` records the Koide selector $K = 2/3$ as **RED — INPUT, external selection**, and the electron closure is carried as KOIDE-CONSISTENT *under that external rule* throughout Appendices A/O/X. The tension is **recorded here, not resolved**: both statements are true of different objects, and no route from the algebra to the *value* $2/3$ has been closed anywhere in the corpus.

Remark 3.2 (the $\delta = 2/9$ "phase = weight" route is closed-negative as a derivation (S185))**.** The empirical Koide-Foot angle $\delta = 0.222230$ sits a fraction of a percent from $2/h^\vee(F_4) = 2/9 = 0.222\overline{2}$, which prompted an $F_4 = \mathrm{Der}(J_3(\mathbb{O}_s))$ "phase = weight" derivation candidate. It is [CLOSED-NEGATIVE] for three independent reasons: (i) the exact value $2/9$ is 442σ from the CODATA pole ratio m_μ/m_e (`s356`) — the "0.003%" framing hid the far tighter μ/e failure; (ii) $F_4 = \mathrm{Aut}(J_3)$ is *isospectral* (`s357`: it preserves the characteristic-polynomial multiset) and Jordan derivations have zero diagonal part, so no f_4 element rotates the diagonal primitive-idempotent plane — the Koide 3-cycle is a discrete Weyl/triality element, not a continuous Cartan rotation; (iii) $h^\vee = 9$ is not unique to F_4. Retained only: the Foot $= SO(2)$ geometry and the $2/9$ arithmetic. Not banked as a result.

4 The WH/BH operator architecture (A1169–A1175, S192)

Over the A1169–A1175 arc the operator content behind (1) and Prop. 3.1 was constructed explicitly and reproduced in-house (all kernels deterministic, `run1==run2`). The architecture is *complete and self-consistent*; this is a real, cumulative advance. It is also, by itself, *value-free*.

Remark 4.1 (the $\sqrt{m}$ amplitude-square rule is derived, `s434`)**.** From a quadratic heavy-mirror action $\left(\begin{smallmatrix} 0 & A \\ A^\dagger & \mu I \end{smallmatrix}\right)$ the low-energy Schur mass is $m_{\text{light}} = \frac{1}{2}(\sqrt{\mu^2 + 4a^2} - \mu) \to a^2/\mu$, so $AA^\dagger$ is a *derived* low-energy mass, not an ad-hoc fourth-order rule. [DERIVED]. A single scalar gap μ cannot fit the spectrum (it needs $\mu = 1.53$ for μ/e but 6.58 for τ/e), which is what forces the two-block structure below.

Object	Content	Status
Finite parent action (A1169)	nonfactorizing, Hermitian $\succeq 0$; Schur complement carries the oriented torsion K_{--}; $M = AA^\dagger$ gives $\sqrt{m}$ an origin	[DERIVED] (appended, *not* extracted from S_{125})
$\sqrt{m}$ mirror rule (A1171)	Rem. 4.1	[DERIVED]
Qutrit terminal (A1171)	minimal common terminal of both rank-3 bridges; operator-Schmidt rank 2; compressed parent 192-dim	[DERIVED] (minimal)
Gauge-centre species C_{16} (A1171)	$C_{16} = (-1)^{N_c} = e^{2\pi i T_3} = -\Gamma_{LR}$, the unique non-identity Hermitian involution in the SM gauge centre	[DERIVED]
Polar–Möbius quotient (A1172)	maps the two clock null lines to one particle/conjugate orbit per block on the full Fock $32 = 16 \oplus \overline{16}$	[DERIVED]
Vectorlike $16 \oplus \overline{16}$ block (A1172)	full-rank (32), symmetric, gauge-invariant to 5.8×10^{-16}; charged Dirac masses *require* the doubled rep	[DERIVED]
3|2 reciprocity (A1175)	assuming an orthostochastic symmetric $K = \frac{1}{5}\left(\begin{smallmatrix}1&4\\4&1\end{smallmatrix}\right)$ gives $\mu_p - \mu_d = 3/5$, $\mu_d = 14/15$, $\mu_p = 23/15$	[CONDITIONAL] on the assumed reciprocity; still inexact
Direct-mass static contract (A1170)	a quadratic Schur self-energy is a *mass* operator, not a $\sqrt{m}$ amplitude; the static near-match is Dyson-unstable	[CLOSED-NEGATIVE]
One-body $B-L$ conjugacy (A1175)	impossible; mismatch $= 2P^{\text{odd}}_{N=3}$ (rank 10) $\Rightarrow$ a degree-3 cocycle is required	[CLOSED-NEGATIVE]

5 The two-gap fit and its (lack of) predictive content

With the architecture fixed, the charged-lepton spectrum is reproduced by the outer mirror

$$H_{\text{out}} = \mu_d \, P_d + \mu_p \, P_\perp, \tag{2}$$

where P_d is the democratic singlet and $P_\perp$ the doublet. With the framework amplitude modes (the singular values a_e, a_μ, a_τ of the loaded circulant A) held fixed, (2) reproduces $[1 : 206.7683 : 3477.2283]$ = PDG to 3×10^{-16} — but via *two free gaps* $\mu_p = 1.5314$, $\mu_d = 0.9313$ fit to two ratios. [LIB2-133] An operator derivation plus a two-parameter fit to two numbers has *zero* predictive content for those numbers. [INPUT]. [LIB2-133]

Remark 5.1 (dual-route reconciliation with the compact Paper 2 ratio (owed, not closed))**.** Paper 2 carries a compact algebraic relation for the charged-lepton ratio $m_\mu/m_\tau = (\varphi/\sqrt{5})^{8/3}\sqrt{2}/10$ (*retiered Rev29*: STRUCTURAL / LOADED-CORRESPONDENCE / REPRODUCED, *not* zero-parameter — the +0.37% residual is the fit gap on C; Appendix X). That relation and the two-gap operator above are a *dual-route pair*, not a contradiction: the compact formula is an exact algebraic expression whose attachment is loaded, while the operator here is the attempted physical mechanism, and the two are reconciled only once one proves that the compact ratio in fact determines (μ_d, μ_p) (or exhibits why the two routes predict different objects). [LIB2-133] That proof is *owed* — routed to open items as a dispatch head — and until it lands neither route is a completed physical derivation of the individual charged-lepton masses. Neither route is demoted; the pairing is stated explicitly so the suite's two charged-lepton accounts are not read as a silent current-state contradiction.

The conditional 3|2 theorem (table above) gives the clean rational $\mu_d = 14/15$, $\mu_p = 23/15$ ($\mu_p - \mu_d = 3/5$, Tr $= 4$), but it is conditional on an *assumed* reciprocity and is inexact: the data require $\mu_p - \mu_d = 0.60012$ (not 0.6) and Tr $= 3.994$ (not 4), so the exact $(4, 3/5)$ law mispredicts at the 10^{-4} level (misses 5.9×10^{-5}, 3.3×10^{-4}). A fair census ranks $(4, 3/5)$ only 4th of 304 simple closures; 3/5 is merely the nearest simple rational to 0.60012. The residual 10^{-4} is localized ($\sim 97\%$) to a single $\sim 0.16\%$ renormalization of the mirror *scale*, with no parameter-free derived origin in hand. [OPEN].

Remark 5.2 (the exact additive-χ completion of the ratios is refuted; the order skeleton and shape survive (S245, `s894`/D1119))**.** A natural attempt to *replace* the two-parameter fit above with a single closed completion writes each mass ratio as $r = \chi^2/(\varphi^{2N} + \chi^2)$ on the occupancy depth support $\{0, 3, 11, 17\}$ (Appendix M) and asks whether one universal χ reproduces the charged-lepton and up-type ratios exactly. It does not: an exact single-χ closure *fails on every bijection* of the support (0/24 admissible; lepton pairs $r = -0.0795/ + 0.0518/ + 5217$), and no residual-derived dressing rescues it without an independent derivation (a residual-fitted χ is a fit, not a completion; the tautology guard holds). [LIB2-106] [CLOSED-NEGATIVE] at the declared ratio tier. What *survives* is exactly the structural content that was never in question: the ordered $\sim \varphi$-depth *skeleton* (the integer winding order of (1)) and the few-percent *shape* on it (central deviations u -0.21%, μ $+3.90\%$, τ -2.62%). The exact completion is dead at the ratio tier on every support; the order and the shape are unaffected. [LIB2-106] A named *watch item* is banked (Appendix X): a banked-period-70 two-parameter harmonic $a\cos(6\pi N/70) + b\sin(6\pi N/70)$ passes the 0.1% bar at $\max|\Delta d| = 3.04 \times 10^{-4}$ but only *fitted* (refused adoption); were its two coefficients ever *derived*, this row reopens. [LIB2-248] The blanket "$\leq$ 2-parameter completion is impossible" claim is therefore *withdrawn*; the honest statement is that surveyed one-parameter forms fail and no universal $\leq$ 2-parameter no-go is proved. [LIB2-106, LIB2-248]

6 The provenance wall: one valid S_{125} carrier

Every route to *deriving* the two gaps (rather than inserting them) reduces to one demand: a valid gauge-preserving S_{125} action that realizes the reciprocal central channel, the rank-32 carrier with its degree-3 $B-L$ cocycle, and the quantum $\mathbb{Z}_2$ gauging. The registered carrier is *not* such an action:

`FAIL_INVALID_MAJORANA_FOLD`

($\|M - M^\top\|/\max = 1.39 \times 10^{-5}$ vs 2×10^{-8}; electric Ward 1.107;
gauge-carrier residual 0.635).

A direct in-house test makes the wall concrete: feeding the registered physical arrays into the basis-invariant terminal/channel runner returns a *rank-defective positive operator* ($\sigma_{\min} = 8 \times 10^{-15}$) — the reciprocal channel cannot even form, so the loaded action never reaches the gap-extraction step. The corrected, gate-passing array does not exist; constructing it *is* the open problem, and it is the same object that gates the SM colour charges. [OPEN].

Remark 6.1 (what *is* closed on the wall: the minimal quantum $\mathbb{Z}_2$ contract (`s445`))**.** The diagonal $\mathbb{Z}_2$ generated by $G_{\text{diag}} = \tau_z \otimes C_{16}$ is gaugeable at the finite-representation level: a full-rank symmetry-preserving regulator exists (rank 192, $\sigma_{\min} = 1$, SM Ward 1.33×10^{-15}, $\det G_{\text{diag}} = +1$), the mixed-SM anomaly traces vanish ($O(10^{-15})$), and $\Omega_5^{\text{Spin}}(B\mathbb{Z}_2) = 0$ for an ordinary internal $\mathbb{Z}_2$, so the old absolute $\pm$ local-system choice is replaced by the minimal flat-bundle sum. [DERIVED]

(minimal closure; the propagating discrete gauge sector and any topological weight are left open). This is a genuine increment beyond the classical descent.

7 The in-house decider and the coalition verdict

Running the gauged $\mathbb{Z}_2$ all the way through on the loaded parent action (kernel `s446`, in-house): the gauging plus the polar–Möbius quotient plus the zero-momentum Schur complement *force* the gap block to the two-parameter form $\mu_d P_d + \mu_p P_\perp$ — the reality/CP-restricted commutant on the three outer modes is exactly two-dimensional, $\mathrm{span}\{P_d, P_\perp\}$, and the mode-to-gap assignment is fixed (the democratic mode carries a_τ, the doublet carries a_e, a_μ). That much is *derived*. But the as-built loaded action has gap block $= I$ exactly ($\mu_d = \mu_p = 1$), giving ratios $200.6 : 2201.6$ versus PDG $206.8 : 3477.2$ — the loaded action does *not* reproduce the leptons; the split must be inserted. Extracting the data-required gaps gives $\mu_d = 0.93128$, $\mu_p = 1.53139$, which satisfy neither $\mathrm{Tr} = 4$ nor split $= 3/5$ exactly. The two gap values are free. [INPUT].

Remark 7.1 (D930 closed 2/2; D931 dispatched)**.** A cold-context coalition review (D930) closed 2/2 (two independent reviewers), both converging on the conservative reading: the parameter-free Koide *shape* is the genuine result; the two gaps are irreducible Yukawa-like *inputs*; "extraction from S_{125}" is likely a category error (the appended minimal action *is* the physical carrier). The follow-on dispatch D931 asks for the one missing object — a valid corrected S_{125} carrier passing all six gates — *or* a proof that the obstruction is fundamental (i.e. identical to the colour-charge wall), which would settle the program by retiring the extraction demand.

The three live readings (pre-registered). The evidence currently favors (i). *(i) The values are input, the shape is the result* — the defensible achievement is the parameter-free Koide shape plus the operator architecture; the two gaps are irreducibly input, and chasing them is a category error. *(ii) The carrier is everything* — a valid gauge-preserving S_{125} and a forward functional, once found, fix the gaps and the colour charges in one stroke. *(iii) A different mediator* — WH/BH radial-ratio dynamics is the wrong engine for the residual. The single decisive test is the D931 carrier construction (or its impossibility proof).

8 Status ledger

Result	Status
Koide *shape* $Q = 2/3 \Leftrightarrow r = \sqrt{2}$ (circulant null)	[DERIVED] as shape; *value* 2/3 [INPUT] (*Rev29*)
WH/BH operator architecture (A1169–A1175)	[DERIVED] (complete, value-free; *models* the mediator — *Rev29*)
$\sqrt{m}$ amplitude-square mirror rule	[DERIVED]
Minimal quantum $\mathbb{Z}_2$ gauging contract (`s445`)	[DERIVED] (minimal)
Forced two-gap structure + mode assignment (`s446`)	[DERIVED]
The two gap *values* μ_d, μ_p	[INPUT]
Exact additive-χ ratio completion (§5.2)	[CLOSED-NEGATIVE] (ratio tier; order + shape survive)
Conditional 3|2 reciprocity $(14/15, 23/15)$	[CONDITIONAL], inexact
$\delta = 2/9$ "phase = weight" derivation	[CLOSED-NEGATIVE]
Static direct-mass contract; one-body $B-L$	[CLOSED-NEGATIVE]
Valid gauge-preserving S_{125} carrier	[OPEN] (= SM colour-charge wall)
Integer windings N; overall mass scale	[INPUT]

Bottom line (Rev23 state; superseded in part by §10). The rest-mass program is a built, self-consistent, minimal operator architecture that derives the Koide *shape* and the forced two-gap structure, but derives *no* mass or phase value: the two gaps are inputs, the conditional 3/5 is not exact, and everything else routes to one unbuilt object — a valid gauge-preserving S_{125} carrier, the same gate as the SM colour charges. An honest, fully-mapped plateau. Nothing is promoted; the engine is unchanged; α stays fenced; no public observable moves.

9 Three recorded negatives from the S197/S208 reconciliation (Rev30)

The S208 reconciliation ledger homes three of its Tier-C clean negatives in this appendix specifically. They were verified and grep-anchored at the time and then left unbanked for sixty-five sessions — not because they were doubted, but because none of them moves an observable. They are folded here.

C2 — the charged action-level Dirac mass is forbidden

A no-Q theorem forbids a charged Dirac mass *at action level*; the neutral 2×2 Majorana channel is not forbidden. The result is triple-confirmed — house kernels `s364` and `s391` plus an independent external derivation.

This is the reason behind a statement the ledger above already makes but does not explain: the $16 \oplus \overline{16}$ vectorlike block (A1172) records that *charged Dirac masses require the doubled representation.* The no-Q theorem is why they require it. Without the doubling there is no gauge-consistent charged

Dirac mass to write down, so the doubled rep is not a modelling convenience — it is the only surviving channel.

C8 — the selector Σ parity-split end-state, and what it honestly is

Frozen at A1208, the selector Σ splits by parity into

$$\text{EVEN} \;=\; U_{\text{rec}} + K_* + \text{gaps}, \qquad \text{ODD} \;=\; \kappa_\perp + \varepsilon.$$

The honest end-state. Rest-mass together with mixing is **a named section, not a derivation**. The parity split organizes the content and tells you which pieces are recovered (U_{rec}), which are structural constants (K_*, $\kappa_\perp$), and which are gaps or residues; it does not produce a mass. Reading the split as a derivation is the error this entry exists to prevent, and it is the same plateau the status ledger above describes in different words.

C9 — the winding-monotone driver is falsified by the leptons

The hypothesis that Σ_{wb} acts as a *winding-monotone* rest-mass driver is **falsified**, 3/3: across the charged leptons the residuals *alternate in sign* rather than decreasing monotonically with winding.

What is falsified, and what is not. Kernel `s365` establishes an exact closed form for the white/black winding sums, $\Sigma_w = \varphi^{-N}$ and $\Sigma_b = \varphi^{N}$, reproduced to a maximum absolute error of 1.42×10^{-14}. That closed form *stands*. What fails is the physical reading laid on top of it — that monotone winding drives the rest-mass ordering. An exact identity and a false interpretation of that identity can coexist, and here they do; the record separates them so a later reader does not retire the identity along with the reading. This negative is also what the reframe in §2 already assumes in substance (winding primary, mass a self-energy residual); the negative itself had simply never been printed.

10 The scale endgame (Rev25): ratios-complete, exactly one registered input

The S225–S226 arc (assessments A1351–A1355; kernels `s715`–`s748`) closed the program's scale question by theorem chain rather than by derivation of a value. The chain, in order:

1. **Rescaling no-go** (A1351, kernel-verified at deviation 0.0): the positive monoidal rescaling leaves every banked dimensionless output invariant — any claimed absolute-unit derivation from the banked structure alone has smuggled an input. Sharpened (A1352): monoidally natural boundary-trace functionals are homogeneous and also rescale freely.
2. **Two-parameter no-go** (A1353): two independent rescalings $(\lambda_D, \lambda_\sigma)$ leave every banked output invariant while $m_e/\sqrt{\sigma} = L_\sigma/L_{\text{depth}}$ moves. *Theorem: $m_e/\sqrt{\sigma}$ is underivable without a new non-homogeneous metric datum* — an input is *required*, not merely missing.
3. **Two doors** (A1354): the missing metric hub splits into (i) the scale datum — non-homogeneous, Bargmann-class — and (ii) the charge coherence kernel — *dimensionless*, Green-function-class. Two different missing objects; only door (i) carries the unit.

4. **The count** (A1355 + `s741`): $H^2(\mathrm{Sym};\mathbb{R}_+) = \mathbb{R}$, generated by the area cocycle; the census is unconditional (the declared-choice flag was removed in-house), and $\dim H^2 = \binom{n_{\mathrm{cyl}}}{2} = \binom{2}{2} = 1$ — *one H^2 class in the selected symmetry model* Sym(Wind), whose census has $n_{\mathrm{cyl}} = 2$ (`s697`). *Rev29 fence (F04)*: the earlier phrasing "one input *because* two cylinders" is withdrawn as an overclaim. The count is a theorem *about the selected model*; **exhaustiveness is open** — nothing here establishes that the two-cylinder census is the only admissible one, so the causal "because" is not carried. The input's identity is the Bargmann central charge of the winding category — the classical pedigree is exact: as in Bargmann's treatment of the Galilei group, the physical scale enters as a central extension class, not as a dynamical output.

Declaration (Rev25). The rest-mass program is **ratios-complete**: every dimensionless ratio in its scope is derived or typed-open, and the program requires *exactly one* dimensionful input — the depth-cylinder class $L_{\mathrm{depth}}/L_\sigma$, registered with value, ladder position, and sealed null-model guard in Appendix X (whose Rev32 input-count convention box reconciles this *one-class* count with the two-calibration and three-anchor usage inventories printed elsewhere). The registration is an administrative act sitting on mathematics: the theorem chain above says an input of exactly this class must exist and that no second one may.

Door 2, honestly. The charge coherence kernel (door ii) has a silhouette only: dimensionless, junction-resolved, configuration-dependent ("coherence multiplies by φ," A1354); the radion-first-harmonic candidate for its organization was tested and *killed* (`s743`). It remains the named owner of the charge-magnitude defects (Paper 9, §8) and is not the scale input.

Remark 10.1 (the one-input obligation — the hierarchy as a falsifiable prediction)**.** $\dim H^2 = 1$ *forbids* a second independent dimensionful input. Every other dimensionful ratio in the suite is therefore theorem-mandated *derivable* — in particular

$$\frac{v_{\mathrm{EW}}}{\sqrt{\sigma}} = 553.3026 \qquad (\log_\varphi = 13.1250;\ \text{not } \varphi\text{-menu-clean, consistent with a dynamical origin}),$$

which is now an *obligation* the framework must pay, not a hope: the electroweak hierarchy is converted into a falsifiable structural prediction (paid at conditional/loaded-depth tier in §11, $v_{\mathrm{EW}}/\sqrt{\sigma} = \varphi^{105/8}$). Exhibiting a *second* underivable dimensionful ratio would falsify the one-input theorem chain outright. The σ-bridge (v_{EW}-derivation) program is the designated post-freeze flagship, alongside the island-of-stability program (Paper 7).

Remark 10.2 (the Bergman kernel homes the φ^N ladder (`s594`/`s595`, S214))**.** A third object touches the rest-mass narrative and is recorded here at remark tier: the Bergman kernel of the golden domain evaluates exactly to $K(\text{golden}) = 1920\,\varphi^{10}/\pi^5$ — the φ^N ladder appearing in a canonical reproducing-kernel normalization. Suggestive of where the ladder *lives*; no number is scored on it.

Bottom line (Rev25). The scale question is finished mathematics: an input is required (two-parameter no-go), it is unique (cohomology count, unconditional census), its class is Bargmann, its value is registered with a sealed null guard (Appendix X) — and the same theorem that grants the one input demands that $v_{\mathrm{EW}}/\sqrt{\sigma}$ be derivable, making the hierarchy problem a prediction the framework must now pay or be falsified by.

11 The σ-bridge scale registration (Rev26): $v_{\rm EW}/\sqrt{\sigma} = \varphi^{105/8}$

Posture (binding, Rev26). This section *pays* the obligation of Remark 10.1 at **conditional / loaded-depth** tier. It is the suite's first foregrounded dimensionful result — but a *framed registration*, not a frame-free prediction: the value is fixed *relative to the marked compact-positive Higgs carrier* (the Marked Carrier Theorem, Paper 4), and frame-free selection of the electroweak line, level, and refinement is closed *negative* (the $E_{7(7)}$ no-go of Appendix F; the permanence obstruction). What is promoted is the *derivation of the registered value*, from bare obligation to a stated conditional registration on named loaded data; it is NOT a from-nothing prediction. No dimensionless observable moves, and α, colour, and $V_{\rm CKM} = I$ are untouched.

The framed-depth operator on the annular / beat-depth carrier evaluates to

$$D_{\rm depth} = N_{\rm ann} + \tfrac{\beta}{8} = 13 + \tfrac{1}{8} = \tfrac{105}{8}, \qquad N_{\rm ann} = 13 = F_7 = a_8, \quad \beta = 1, \tag{3}$$

with the metaplectic eighth-tick $1/8$ and $N_{\rm ann} = 13$ the eighth Fibonacci / annular index (homed at $\tau^{\otimes 8}$ level $n = 8$; competitors $10, 12, 14$ structurally excluded). This gives the electroweak registration

$$\boxed{\frac{v_{\rm EW}}{\sqrt{\sigma}} = \varphi^{105/8} = 553.30} \qquad (\log_\varphi = 13.1250), \tag{4}$$

paired with the companion *dimensionless* identity (exact, no scale content)

$$\frac{m_\tau}{\sqrt{\sigma}} = 4 = |J_{\rm vac}|^2 = \varphi^2 + 1 + \varphi^{-2}, \qquad J_{\rm vac} = \mathrm{diag}(\varphi, 1, \varphi^{-1}), \tag{5}$$

so that $v_{\rm EW}/m_\tau = \varphi^{105/8}/4$, which the registered inputs reproduce to 0.17%.

Remark 11.1 (what is derived, what is loaded)**.** The permanence theorem (Appendix F) closes the frame-free route: on the bare Freudenthal system $E_{7(7)}$ fixes no Higgs line (LOAD-EW-LINE) and no canonical quadratic-refinement basepoint (LOAD-EW-FRAME, $\beta = 1$), and the level $N_{\rm ann} = 13$ is homed but unattached (LOAD-EW-INDEX). Hence $\varphi^{105/8}$ is the *unique visible framed-depth registration*, relative to the marked compact-positive carrier of Paper 4 — a loaded-depth theorem *candidate*, not a frame-free scale theorem. Honest over-determination count (correcting an earlier house claim): the $L_{\rm depth}/L_\sigma$ registration and the $v_{\rm EW}$ route are mutually *circular*, so they do **not** constitute independent over-determinations of the scale — the framed registration carries *no* triangulation surplus from that pair. The one genuinely independent cross-check is the dimensionless $v_{\rm EW}/m_\tau = \varphi^{105/8}/4$ (using the exact identity $m_\tau/\sqrt{\sigma} = 4$), which the registered inputs meet at 0.17%; the dimensionful $v_{\rm EW}/\sqrt{\sigma} = \varphi^{105/8}$ itself is a loaded-depth registration, not a zero-input prediction.

Falsifier (obligation-as-threat). The one-input theorem chain (§10) forbids a second independent dimensionful input, so $v_{\rm EW}/\sqrt{\sigma}$ *must* be structurally fixed; $\varphi^{105/8}$ is the value it is fixed to. The prediction is falsified by either (i) a second genuinely underivable dimensionful ratio in the suite's scope, or (ii) a determination of $v_{\rm EW}/\sqrt{\sigma}$ inconsistent with $\varphi^{105/8}$ beyond the $\sqrt{\sigma}$ registration uncertainty. The framework pays this or is broken by it.

12 The authoritative charged-lepton ledger (entered Rev28; tiers as re-tiered Rev29)

This section is the *one* authoritative statement of charged-lepton provenance in the suite. Papers 0, 2, 4, 7 and Appendix A point here; where any of them carries a shorter form, this table governs.

Quantity	Provenance	Tier	What is *not* claimed
m_τ	external measured anchor (Λ_{G_2} dimensional-transmutation anchor at the heavy-lepton end)	INPUT	not derived; not a prediction
m_μ/m_τ	$(\varphi/\sqrt{5})^{8/3} \cdot \sqrt{2}/10$; provenance target-first [D616, A616, A618]	STRUCTURAL / LOADED-CORR. / REPRODUCED (*re-tiered Rev29*)	not proved and not zero-parameter: the +0.37% residual *is* the fit gap on C (required $C = 0.140894$ vs. $\sqrt{2}/10 = 0.141421$; Appendix X)
m_e	QED-corrected Koide chain under the *external* empirical rule $K = 2/3$ [A636, A637 — A637's internal "proved at every step" rating is not the tier carried here; see App. A]	KOIDE-CONSISTENT ⋆⋆⋆	conditional on Koide; not a $J_3(\mathbb{O}_s)$-internal derivation
Joint ledger	anchor + structural ratio (re-tiered Rev29; see the tier rows above) + external-rule closure	algebraic mass *correspondence*	the three masses are **not jointly derived** from $J_3(\mathbb{O}_s)$ alone
Dual route	the compact ratio vs. the two-gap operator of §4/§5	RECONCILIATION OWED	the two routes are not yet shown to be the same object

One-line posture. The charged-lepton sector is an algebraic mass correspondence, structurally pinned with mixed provenance, with a dual-route reconciliation owed. *Rev29*: the phrases "closed", "proved zero-parameter" and "zero-free-parameter" are **withdrawn for the m_μ/m_τ ratio as well**, which was the last place they applied; the ratio is STRUCTURAL / LOADED-CORRESPONDENCE / REPRODUCED, and no quantity in this appendix may now be cited as zero-parameter.

Leibniz Quantum Beats Newton

Appendix P — The Fifth Direction: Two-Sided Necessity, the Tick's Exact $\mathbb{Z}_4$ Momentum-Like Grading, and the NS/R Parity Pattern of the Flicker (Rev33.2; entered Rev28, claim wording corrected *Rev29*)

Five kernel-verified theorem-tier statements (S226: `s734`, `s740`, `s741`; A1356), with the Kaluza–Klein/Einstein–Bergmann identification confined to a labeled interpretive subsection. No observable moves; nothing dynamical is claimed.

Tom O'Sieg

August 2026

Posture (binding). This appendix promotes exactly five statements, each kernel-verified or hostile-lane-proved, and none of them an identification of the framework with a Kaluza–Klein spacetime. The identification *narrative* is quarantined in §3 (labeled interpretive) and earns no theorem status. The colour *carrier* question stays open (the loaded norm is missing); colour promotion remains fenced; structural statements only.

Contents

1 What is being claimed **1**

2 Theorem tier **2**

3 Interpretive subsection (labeled; discussion tier — no theorem status) **3**

4 Status ledger **4**

1 What is being claimed

The banked winding category (A1347's object, not $\mathrm{Wind}_{\mathrm{ev}}$; `s1341`) carries an internal direction that behaves, in five independently verified respects, like a compact fifth coordinate: its clock weight carries an exact $\mathbb{Z}_4$ momentum-like grading, its parity flicker reproduces the NS/R parity pattern under the signed trace, its scale slot is a one-dimensional cohomology class in the selected symmetry model, a flat (cylinder-condition) reduction is dimensionally impossible, and no purely four-dimensional model reproduces its three signatures. Each statement is a theorem about *banked objects*; none asserts that spacetime has five dimensions. [LIB2-055]

Conflation fences (*Rev29*, families F02–F04). Three "is"-statements carried by earlier revisions are corrected here and are not restored anywhere below. (F02) The tick is *not* asserted to *be* a fifth momentum: what is proved is an exact $\mathbb{Z}_4$ momentum-*like* grading, and **no compact-coordinate map exists** — no internal coordinate x^5, no periodic identification, and no Fourier dual is constructed anywhere in the banked material. (F03) The flicker is *not* asserted to *be* the NS spin structure: the signed trace *reproduces the NS/R parity pattern*, and **no Spin-bundle map exists** — no bundle, no manifold, and no spin structure on one is exhibited. (F04) The H^2 count is a theorem about the *selected* symmetry model, not a causal derivation of the input count; **exhaustiveness is open**.

2 Theorem tier

Novelty label: new specialization proved here.

Theorem 2.1 (the tick carries an exact $\mathbb{Z}_4$ momentum-like grading; `s740`)**.** *The $\mathbb{Z}_4$ clock weight of the banked grading behaves as a momentum-like quantum number modulo 4: the grading law, additivity under composition, and reversal under the mirror are all* exact *on the full generator set. [LIB2-054] (Kernel-verified; deterministic; zero deviations.)* Fence (F02, *Rev29*)*: this is a statement about a grading on banked objects, not an identification with a momentum conjugate to a compact coordinate.* ***No compact-coordinate map is constructed*** *— there is no internal x^5, no periodic identification, and no Fourier duality between the grading and any coordinate anywhere in the banked material. [LIB2-054] The word "momentum" is used for the formal properties (additivity, mirror reversal, $\mathbb{Z}_4$ valuedness) and for nothing else.*

Novelty label: new specialization proved here.

Theorem 2.2 (the cylinder condition fails by dimension count; `s740`)**.** *The classical Kaluza–Klein* ***cylinder condition*** *(no dependence on the internal coordinate) is inconsistent with the banked content: the required invariant decomposition has dimension signature $(1, 27, 27, 1)$, and the count does not close on the cylinder-invariant subspace. [LIB2-055] Any five-dimensional reading of the framework is therefore forced into the* Einstein–Bergmann *class (periodic internal dependence), not the Kaluza–Klein cylinder class. Moreover the loaded fiber admits no dynamical stabilization from the banked structure alone — the radion problem is met honestly, not hidden (`s740`; cf. the registered-input theorem chain, Appendices O/X).*

Novelty label: new specialization proved here.

Theorem 2.3 (the signed trace reproduces the NS/R parity pattern; `s734`)**.** *The signed trace over the banked mirror (Lucas $\pm$) ladder reproduces the antiperiodic (NS) / periodic (R) parity pattern of the three-torus sector T^3_S, with the Freudenthal bit playing the role of the holonomy sign. [LIB2-084] Exact at both levels on the banked data ($N = 24$ check exact).* Fence (F03, *Rev29*)*: what is exhibited is the* parity pattern *a spin structure would produce, not a spin structure. [P-016]* ***No Spin-bundle map is constructed*** *— no manifold, no principal bundle, and no lift of a frame bundle appears anywhere in the banked material, so "the flicker* is *the NS spin structure" is withdrawn. [LIB2-084, P-016]*

Novelty label: new specialization proved here.

Theorem 2.4 (one scale slot, counted; A1355 + `s741`)**.** $\dim H^2(\mathrm{Sym};\mathbb{R}_+) = \binom{n_{\mathrm{cyl}}}{2} = 1$ *for the banked symmetry census* ($n_{\mathrm{cyl}} = 2$)*: the generator is the area cocycle; the physical (both-flip) mirror preserves the class; single-cylinder flips* ($\det = -1$) *are forbidden by consistency, and the census is* unconditional *— the earlier declared-choice flag on the* Sym(Wind) *identification was removed by in-house re-derivation with the actual banked generator set (`s741`). [LIB2-056] "Exactly one dimensionful input" is then a counting statement* about this model*: one H^2 class in the selected symmetry model, whose census has $n_{\mathrm{cyl}} = 2$. [LIB2-056]* Fence (F04, *Rev29*)*: the earlier phrasing "one input* because *two cylinders" is withdrawn as an* ***overclaim****. The theorem computes* $\dim H^2$ given *the* Sym(Wind) *census; it does not establish that two cylinders are the only possibility, and* ***exhaustiveness is open****. [LIB2-056] The census being unconditional* within *the identification is not the same as the identification being forced, and the causal "because" is not carried. [LIB2-056]*

Novelty label: new specialization proved here.

Theorem 2.5 (no 4D counter-model *without equivalent internal structure*; A1356, hostile lane)**.** *A four-dimensional model that lacks the equivalent internal structure — a noncompact internal modulus, an internal $H^1(-;\mathbb{Z}_2)$ / $w_1 \neq 0$ twisted bundle, and an internal grading circle — cannot reproduce the three banked signatures, for three named reasons: (i) the parity ladder requires an independent internal $H^1(-;\mathbb{Z}_2)$ class — universal and particle-blind — which pure 4D supplies only as a spacetime loop (path dependence) or a hidden circle; (ii) compact 4D data yields* elliptic *holonomy (phases), never a positive hyperbolic φ^N dilation, absent a noncompact scale direction; (iii) the half-twist colour structure needs a $w_1 \neq 0$ transverse cycle that pure 4D does not possess. [LIB2-085] (Scored at the frozen binary; full credit.)*

Remark 2.6 (scope — the fifth direction can be traded, not eliminated)**.** The theorem is a *necessity of the structure*, not a spacetime-dimension count: a four-dimensional model *can* reproduce the signatures if it imports a noncompact internal target modulus, a discrete/twisted internal bundle ($w_1 \neq 0$), or an internal grading variable — but doing so trades the fifth direction for the equivalent internal data rather than evading it. The claim is that the three signatures cannot arise from *bare* four-dimensional spacetime data with no such internal structure; they demand either a fifth direction or its internal proxy. [LIB2-085]

Remark 2.7 (two-sided necessity)**.** Theorems 2.1–2.2 argue from *inside* (the internal direction is really there, and it cannot be flattened away); Theorem 2.5 argues from *outside* (its signatures cannot be faked in 4D). The necessity argument is therefore two-sided — the strongest epistemic form available short of a scored observable, which the fifth direction does not yet have. That lack is exactly why §3 is interpretive.

3 Interpretive subsection (labeled; discussion tier — no theorem status)

> **Label.** Everything in this section is *interpretive* narrative: a reading of the theorems above, adopted for orientation and program design, not banked as a result. This is the discussion layer, and it is fenced.

Read together, the five theorems say the framework has arrived at the classical Kaluza–Klein frontier in its Einstein–Bergmann form: holonomy (circulation) data is derived — charge signs,

parities, the nuclear circulation ladder of Paper 9 — while the modulus is not derived but *counted*: exactly one Bargmann/radion class, its value registered as the framework's single dimensionful input (Appendices O/X). "Gauge data derived, modulus registered" has been the generic state of every KK theory since 1921; what is new here is that the radius problem is met by *theorem* (the count of Theorem 2.4, the honest no-stabilization statement of Theorem 2.2) instead of by hope. In this reading the moment-thickness of the time layer (Appendix H) is the depth length L_{depth} itself; the universe is one *section* of the loaded bundle; and the tick-grading theorem *echoes* the 1926 Klein quantization argument in golden form (an echo, not an identification — see the F02 fence). If the identification ever earns more than narrative status, it will be because the two-circulation sector or the carrier norm produces a scored number of its own — until then, this section carries no weight in the ledger.

Remark 3.1 (pencil paragraph (in-house, unreplicated — recorded, not promoted))**.** Two fresh in-house diagnostics are noted for completeness, both GRAY: `s744` (signature appears to track stiffness: soft tree-Hessian modes, including the negative one-loop mode, load the $(2,2)$ sheet; translators and generic modes load $(1,3)$ — one session old, unreplicated) and `s736` (n-face defects proportionally φ-quantized at $\sim$8% significance after trials — suggestive only). One clean negative is quotable: `s743` — charge-face defects are *not* tick-organized; the radion-first-harmonic hypothesis is killed. None of the three enters any ledger.

4 Status ledger

Statement	**Status**	**Source**
Tick: exact $\mathbb{Z}_4$ momentum-like grading (*no* coord. map)	DERIVED (exact)	`s740`
Cylinder condition fails; $(1, 27, 27, 1)$; EB forced	DERIVED	`s740`
Signed trace reproduces NS/R parity pattern (*no* Spin-bundle map)	DERIVED (exact)	`s734`
$\dim H^2 = \binom{n_{\text{cyl}}}{2} = 1$ in the selected model	DERIVED; exhaustiveness OPEN	A1355 + `s741`
4D-only counter-model refused (three walls)	DERIVED (hostile lane)	A1356
Golden-KK/EB identification	INTERPRETIVE	§3
`s744`/`s736` diagnostics	GRAY (unreplicated)	tracker tier
Radion first harmonic	CLOSED-NEGATIVE	`s743`

Bottom line (*Rev29* wording). The fifth direction is necessary from both sides, its tick carries an exact $\mathbb{Z}_4$ momentum-like grading (with no compact-coordinate map), the signed trace of its flicker reproduces the NS/R parity pattern (with no Spin-bundle map), and its one missing number is counted — in the selected symmetry model, exhaustiveness open — and registered rather than fitted. The identification with an Einstein–Bergmann spacetime remains a labeled narrative — and will stay that way until it pays in a scored observable.

Leibniz Quantum Beats Newton

Appendix P Sidebar — The Einstein–Bergmann–Bargmann–Pauli Five-Dimensional Program and Its Ledger Descendants (Rev33.2; drafted Rev29-WIP)

A historical companion to Appendix P. Tier: HISTORICAL throughout — no theorem is proved here, no observable moves, and no physical five-dimensional spacetime is claimed. Every framework statement cited below carries its own tier in its own assessment; this sidebar adds commentary, not status. Sources: the 1938/1941/1943 papers; assessments A1452, A1462–A1466; kernels `s942`–`s949`.

Tom O'Sieg

August 2026 (sidebar drafted S256, Rev29-WIP)

Posture (binding). This sidebar is HISTORICAL commentary. It promotes nothing. The correspondences drawn between Einstein's five-dimensional program and the suite's banked results are *structural analogies with exact citations*, not identifications: the suite's fifth direction remains the internal tick/flicker carrier of Appendix P, and the identification with a physical Kaluza–Klein/Einstein–Bergmann spacetime remains interpretive and quarantined there. The S255–S256 results cited by assessment number (A1462–A1466) are fold-pending: their full appendix treatments do not yet exist in this revision, and the assessment files are the citable artifacts of record (Appendix Y discipline).

Contents

1 Why this sidebar exists **1**

2 The three acts, 1938–1943 **2**

3 The failure, retold as banked theorems **2**

4 On the suite's "reluctance" toward 5D **3**

5 The Bargmann class: dimensionality from two structures **4**

1 Why this sidebar exists

The suite's one registered dimensionful input is called the *depth-cylinder Bargmann class* (Appendices O/X). Appendix P confines the Kaluza–Klein reading of the fifth direction to a labeled interpretive

subsection and names Einstein–Bergmann in doing so. Those two names are not decoration. Between 1938 and 1943 Einstein, with Peter Bergmann, Valentine Bargmann, and finally Wolfgang Pauli, ran the closest historical ancestor of the program this suite is now executing: a finite-width fifth dimension, particles as regular field concentrations, and a scalar width modulus nobody could fix. The program failed, and Einstein said so plainly. The purpose of this sidebar is to record *why* it failed, in his own terms, and then to show that the S255–S256 mass-frontier arc has re-derived each of his failure modes as an *exact banked theorem* — while adding, with exact constructions, the two ingredients his setup could not contain. The wall he died on is real; the ledger has now proved it is real; and proving a wall real is different from, and better than, running into it.

2 The three acts, 1938–1943

Act I — the thick cylinder (1938). Einstein and Bergmann, "On a Generalization of Kaluza's Theory of Electricity" (Ann. Math. **39**, 683). Kaluza's original theory imposed the *cylinder condition* — no field depends on x^5 — by fiat, which reduced the fifth dimension to bookkeeping. Einstein and Bergmann relaxed fiat to *periodicity*: a fifth direction of genuine finite width, fields expandable in x^5-harmonics, spacetime as a sheet bounded by two hypersurfaces. In modern language they introduced the Kaluza–Klein tower; in this suite's language they were the first to treat the *thickness* of the extra direction as physical data. Their stated hope was that particles would arise as stable, everywhere-regular concentrations of the five-dimensional field.

Act II — the particle hunt (1941). Einstein, Bargmann, and Bergmann, "On the Five-Dimensional Representation of Gravitation and Electricity" (Theodor von Kármán Anniversary Volume). The search for non-singular particle-like solutions of the periodic theory. The result was deflationary: the construction kept collapsing back to general relativity plus Maxwell, with nothing left over to *be* the electron.

Act III — the no-go (1943). Einstein and Pauli, "On the Non-Existence of Regular Stationary Solutions of Relativistic Field Equations" (Ann. Math. **44**, 131): the five-dimensional vacuum theory admits *no* regular stationary particle-like solutions with mass. This theorem is the program's tombstone. Einstein's own post-mortem, stated in correspondence and in his later survey remarks, had four counts: the theory produced nothing beyond GR + Maxwell; it could not produce a particle; it said nothing about the quantum; and the g_{55} component — the width of the fifth dimension — was either frozen arbitrarily or ran loose as an unwanted scalar with no principle to fix it. A fifth count sat unspoken until decades later: a smooth five-dimensional circle cannot produce chiral fermions at all.

3 The failure, retold as banked theorems

The correspondence below is the sidebar's content. Each row pairs one of Einstein's failure modes with the exact result that is its modern descendant in this ledger. The direction of the analogy matters: the ledger results are not *about* Einstein's theory; they are theorems of this framework's conditional 5D attachment which happen to close, with proofs, the questions his program left as frustrations.

Einstein's failure mode	**Ledger descendant (exact, banked)**
Pure geometry yields no particle structure; the periodic theory collapses to GR + Maxwell (1941).	Free compactification is HIERARCHY-BLIND (A1462, `s942`); the free mouth-Real determinant is *globally* hierarchy-dead by strict concavity and weighted Jensen (A1465, `s947`); every universal spectral action selects only hierarchy-free rays (A1464/A1465). "Cheap geometry gives nothing" is now a theorem family, not a disappointment.
No regular stationary particle solutions exist (Einstein–Pauli 1943).	The evasion is exactly what the 1943 hypotheses exclude: a scalar sector with a potential and a topological charge. The BPS kink wall $\sigma = v\,n\tanh(v\sqrt{\lambda/2}\,y)$, tension $\frac{2}{3}\sqrt{2\lambda}v^3$, is derived, regular, and stable (A1464, `s945`) — a soliton living precisely outside the Einstein–Pauli no-go's scope.
A smooth S^1 gives vectorlike matter — no chirality (the unspoken count).	The scalar-twist censuses are zero-mode-dead, and the mouth-Real matrix twist $T_{\mathrm{m}} = -\tau_1$ produces exactly one branch-Real chiral zero channel with NS partner gap $\lvert p_5\rvert R = \frac{1}{2}$ (A1463, `s944`). Chirality enters through the mouth exchange — an ingredient with no 1938 analogue.
g_{55}: the width modulus can be neither fixed nor banished.	The radial-shadow theorem (A1466, `s948`): the compact phase of the complexified X line is fixed by the center/semion structure, while the radial magnitude s is provably blind to center, semion, anomaly, Yukawa invariance, and wall topology. Einstein's frustration is now a proved independence, with a countermodel pair. The wall is real.
Particles as "bridges" (Einstein–Rosen 1935), killed by singularities and instability.	The WH/BH mouth pair, algebraized: the branch-Real Majorana clause (A1452, one zero-dimensional clause, priced not derived). The bridge survives by not being a geometric bridge — conditional per Appendix P.

Two ingredients Einstein lacked, one wall he could not name. The ingredients — chiral matter via the mouth twist, and a scalar/topological sector for solitons — are now exact constructions. The wall — the width modulus — is now an exact independence theorem, and it is the live head object (P0-24, Q2: does the parent action fix s?).

4 On the suite's "reluctance" toward 5D

It is fair to ask why a framework with a fifth-direction theorem, a KK-tower annulus appendix, and a depth-cylinder input has been slow to simply *be* a Kaluza–Klein theory. The answer is that the reluctance is Einstein's failure, encoded as discipline. Einstein committed the interpretation first —

a real physical fifth dimension — and spent fifteen years hunting mathematics to justify it. The suite runs the arrow the other way: the internal statements (the tick's exact $\mathbb{Z}_4$ momentum-like grading; the signed trace reproducing the NS/R parity pattern) are theorem-tier and closed *as stated* — and, per the *Rev29* F02/F03 fences in Appendix P, they are **not** identifications: there is no compact-coordinate map and no Spin-bundle map. The spacetime identification is held interpretive in Appendix P, where it can wait indefinitely without collapsing. The step has, in fact, now been taken — in the only form the two-axis provenance rule permits: $\mathfrak{L}_{\mathrm{MRSS}}$ is a *named candidate 5D attachment*, constructed-conditional, its caveat printed in every round that uses it (A1463 onward). This is the Einstein–Bergmann program resumed, with a ledger where he had only conviction.

5 The Bargmann class: dimensionality from two structures

The deepest correspondence is the one the suite's own nomenclature made before this sidebar was written. After leaving Einstein's program, Bargmann proved (1954) that quantum mechanics represents the Galilei group only through a *central extension* — and the central charge is *mass*. A central extension is precisely the mechanism of the "T-bridge" intuition: a flat structure acquires one new dimension not by adding geometry but by a cocycle measuring the failure of two structures to compose flatly. The suite's registered dimensionful input is the *depth-cylinder Bargmann class*: the extension class that welds the internal clock (flicker/NS) to internal translation (tick/p_5) and turns a flat boundary into a cylinder. Einstein and Bergmann carried the width as a free parameter with no principle; Bargmann's own later mathematics explains why kinematics alone could never have fixed it — central charges are inputs *to* a symmetry, not outputs *of* it. Whether the class is derivable from the parent action or remains the honest registered input is exactly the open radius-weld/X-magnitude question (P0-24, Q2/Q3). Einstein discovered that question the hard way between 1938 and 1943. This ledger has it stated, fenced, and gated.

Fences (carried). No physical five-dimensional spacetime is claimed anywhere in this sidebar. The Einstein–Pauli evasion row does not assert that the BPS kink is a particle. The Majorana-clause row remains one priced zero-dimensional clause. The radial modulus s and the finite thresholds K_3, K_1 remain underived; the mechanism of A1466 *can span* the hierarchy and has not been claimed to explain it. Historical attributions follow the primary papers cited in §2; secondary paraphrases of Einstein's post-mortem are marked as such.

Leibniz Quantum Beats Newton

Appendix Q — The Golden Vacuum's Classical Spectrum: Machine-Zero Closures from the Dynamics Lane (Rev33.2; entered Rev25; bounded, conditional tier)

A bounded window onto the S209–S223 dynamics arc: only the finished, machine-precision items (the `s652c` spectrum closure and the Gate-arc receipts, including one recorded blind-prediction loss). Everything here is conditional-tier; the live lane ($\kappa(w)$ anatomy, B-dressing, sealed windows F/W) is explicitly out of scope. The dynamics revision proper is Rev26.

Tom O'Sieg

August 2026

Posture (binding). This appendix exists for cross-pollination: the lab notebook informs the coalition lanes, and these closures were invisible behind the dynamics fence. Scope is deliberately *bounded*: finished, deterministic, machine-zero items only. Every statement is **conditional**-tier — conditional on the κ_{phys} mechanism, whose status is STRUCTURAL CANDIDATE (the S220 wording), *not* derived. [LIB2-196] Nothing here is a scored world-facing observable; nothing moves the engine; the epistemic class is internal machine-zero closure, which is deliberately kept separate from the frozen-bar world-scored ledger of Paper 9.

[LIB2-196]

1 The spectrum closure (`s652c`, S220)

On the CDIZ locus $x_i = 1$ with STU eigenvalue data $e = (0.99471, 0.96943, 0.94410)$ (and $s^4 = 1$ to 2×10^{-15}), the golden vacuum's *entire* classical spectrum closes as CSS charge sums of the four gravitino masses:

Novelty label: new specialization proved here.

Proposition 1.1 (conditional tier)**.** *All* 8 *gravitini,* 28 *vectors,* 56 *fermions (including the goldstini), and* 70 *scalars of the $N = 8$ classical spectrum at the golden point are reproduced at machine precision as CSS charge sums of the four gravitino masses on the CDIZ $x_i = 1$ locus. [LIB2-196] Derived in-house* before *the lane's matching return arrived — an independent-convergence receipt, not a fit.*

This is the arc's crown jewel and the reason this appendix exists: a complete classical spectrum organized by one four-number seed, at machine zero, twice, independently.

2 The Gate-arc receipts

The supporting closure chain, all deterministic and manifested:

- **A-tensor exact rebuild** (`s638`): GAP $= 0.0$; Gate C at 1.1×10^{-15}.
- **Gate D machine closure**: R6 residual 1.05×10^{-15}.
- **Vector Ward identities**: trace 1.4×10^{-14}; derivative form 5.9×10^{-11}.
- **Gate H — Coleman–Weinberg inertia, with the loss on the books**: the computed CW inertia signature is $(9, 0, 1)$. The pre-registered blind prediction (A1329) was $(10, 0, 0)$ and was scored per protocol as a **recorded loss**. It is kept here deliberately: the blind ledger's credibility rests on its scored misses as much as its hits.

3 Two negatives at the golden boundary point (Rev30, A1481)

Two independent results constrain what can be built at the golden boundary point z_0. Both are house-verified; both are negatives.

(i) z_0 is not a critical point of the bare quartic. Direct computation gives $\mathrm{d}I_4(z_0) \neq 0$, with $\|\mathrm{d}I_4\| = 8.944272$ and the α-component exactly -4. [LIB2-104] The consequence is immediate and fatal to a class of proposals: **a parent action whose candidate vacuum is not a critical point has no vacuum there.** This does not touch the stationarity results proved elsewhere for the constrained objects of this appendix — it says that the *bare* quartic does not have a stationary point at z_0, and therefore cannot by itself be the action that selects it. [LIB2-104]

(ii) No positive equivariant polarization exists in the split-G_2 family. [LIB2-105] With $\dim \mathrm{Comm}(\mathrm{Der}(\mathbb{O}_s))$ in $\mathrm{End}(\mathbb{O}_s)$ equal to 2, and the split norm on $\mathrm{Im}\,\mathbb{O}_s$ of signature $(3, 4)$, all four equivariant involutions of the form $J = s\,P_1 + t\,P_{\mathrm{Im}}$ give $\mathrm{sig}(G \cdot J) \in \{(4, 4), (5, 3), (3, 5), (4, 4)\}$ — **none positive**. [LIB2-105] The no-go stands as stated.

Scope, and one arithmetic caution. Both results are computed in the *split* real form and inherit the real-form declaration of Appendix X: signatures in this family are a consequence of a declared input, not an intrinsic feature, and must be quoted with that premise named. [LIB2-105] Relatedly, a parameter chain that has been reported as a narrowing — $196 \to 52 \to 20$ — is **arithmetic, not physics**: neither endpoint is derived, and 20 is still a twenty-parameter family. Nothing in this section promotes an observable, moves a tier, or touches the engine.

4 Status language (strict)

- Everything in this appendix: **conditional tier**, on the κ_{phys} candidate mechanism. [LIB2-104]
- $\mathrm{sign}(c)$: STRUCTURAL CANDIDATE (S220 PI wording) — not settled, not promoted.
- The $\kappa(w)$ anatomy, the B-dressing program, and the sealed blind windows F/W: **live lane, out of scope here** — deliberately not papered while mid-stream.

- The dynamics revision proper — where this tier either promotes or dies — is **Rev26**, gated on $\mathrm{sign}(c)/\kappa(w)$ settling.

Bottom line. One bounded window: the golden vacuum's classical spectrum closes at machine zero from a four-number seed, the gate chain behind it is receipt-complete, and the one blind prediction it ventured lost and is recorded as lost. Conditional tier throughout; the live dynamics lane stays fenced until Rev26.

Leibniz Quantum Beats Newton

Appendix R — The Interacting Vertex: from the Threshold Measure to the First Derived Relative Vertex Sign (Rev33.2)

Rev26 dynamics frontier (S238–S240). Conditional/candidate tier throughout. The head wall "no parent action / physical projection map" attacked directly for the first time: the transfer question is now a package with an exact parts count — one merged loaded import, three continuous data, and one named non-equivariant placement kernel — and it has produced its first derived, zero-parameter vertex sign. *Nothing here is a world-scored observable; the vertex sign is* ***relative and amputated*** *(a $g \to 0^+$ doorway ratio, structurally a trilinear antisymmetry), not an interaction strength or S-matrix prediction; the F4 wording fence ("mass shell"/"amplitude" stay out of public claims) applies to every statement below.*

Tom O'Sieg

August 2026

Posture (binding). This appendix papers the S238–S240 arc at the *frontier*, not the observable tier. Its epistemic class is **conditional** (on the loaded boundary $\mathfrak{B}_{\text{phys}}$ and its holonomy class $[\sigma_{\text{CD}}]$) and **candidate** (the interpretive readings — dominoes $\leftrightarrow$ generations $\leftrightarrow$ arms $\leftrightarrow$ placements — stay explicitly distinct and unmerged). Every quantitative closure below was independently reproduced in-house (house double Cayley–Dickson octonions, house group theory, independent eigensolvers), byte-identical to the executed external kernel, before being banked. **No public observable moves** in this appendix; α and colour stay fenced; $V_{\text{CKM}} = I$; the engine is unchanged. The single derived number, the relative sign -1, is a *sign of a bare/amputated doorway ratio*, tiered below any width or scale claim.

1 The interacting contact measure

Provenance. This section carries the S237–S238 measure rounds: the measure theorem under attack (A1386), the Horn D discriminator (A1387), the order-selection vertex (A1388), Σ analytic then executed (A1389, A1390), $\mathcal{P}_{\text{th}}$ (A1391) and the interacting measure (A1392). Round-by-round index: Appendix Y.

The wall left standing after the geometry arc was the physical readout of the threshold-residue projection $\mathcal{P}_{\text{th}}$. The finite-circumference spectral family is

$$H_{c,R} \;=\; E_{\text{th}} \;+\; c\,(-i\partial_\theta)^2 \quad \text{on} \quad L^2(S^1_R) \otimes P_z\mathbb{R}^8, \qquad X(\theta + 2\pi R) = \sigma_{\text{CD}}\, X(\theta), \tag{1}$$

with σ_{CD} the deck holonomy. Two exact results bracket its readout.

Novelty label: new specialization proved here.

Theorem 1.1 (Finite-R Riesz edge projector; conditional on the S^1 family)**.** *At finite circumference the periodic $E_{\rm th}$ eigenvalue is isolated and* $\mathrm{Res}_{z=E_{\rm th}}(z-H_{c,R})^{-1} = P_{E_{\rm th}}$ *exactly; the odd sector carries no residue there. Thus $\mathcal{P}_{\rm th}$ is exact mathematics* if *the physical readout is "take the isolated pole residue."*

Sketch. On S^1_R the Laplacian $(-i\partial_\theta)^2$ has discrete spectrum k_n^2 with k_n set by the $\sigma_{\rm CD}$ deck twist; the even sector contains the constant mode $k=0$, giving the isolated eigenvalue $E_{\rm th}$ with a rank-one spectral projector, while the twisted (odd) sector has $k_n \neq 0$ for all n, so no mode sits at $E_{\rm th}$. The resolvent $(z-H)^{-1} = \sum_n |n\rangle\langle n|/(z-E_{\rm th}-ck_n^2)$ has a simple pole at $z=E_{\rm th}$ only from $k=0$; its residue is $P_{E_{\rm th}}$, and the contour integral $\oint_{E_{\rm th}}(z-H)^{-1}dz/2\pi i$ reproduces P_0 to $< 10^{-10}$ (house *s*842-V8). □

Novelty label: new specialization proved here.

Theorem 1.2 (Free-measure no-go)**.** *On the free contact problem the readout does* not *follow from the spectral measure. As $R\to\infty$ the edge-atom mass $1/(2\pi R)\to 0$ while the off-edge tower converges to the threshold continuum $\rho(E) = \left(2\pi\sqrt{c}\,\sqrt{E-E_{\rm th}}\right)^{-1}$, and*

$$\int \frac{\rho(E)}{E}\,dE \;=\; \frac{1}{2\sqrt{c\,E_{\rm th}}} \tag{2}$$

exactly — *the continuum carries the full $R\to\infty$ point resolvent; $E_{\rm th}$ is a branch point, not a pole. Atomization (route (a)) and the KV-flow route (c) are dead at shipped tier; the wall relocates onto the* interacting *width-dressed measure rather than softening.*

Sketch. The finite-R edge atom has weight equal to the constant-mode normalization $1/(2\pi R)$, which vanishes as $R\to\infty$. The off-edge modes condense into the continuum $\rho(E)\,dE$ with $\rho(E) = (2\pi\sqrt{c})^{-1}(E-E_{\rm th})^{-1/2}$ (density of states of a 1D quadratic band). The point resolvent at the origin is $\langle 0|H^{-1}|0\rangle = \int \rho(E)E^{-1}dE$; substituting and integrating $\int_{E_{\rm th}}^{\infty}\left(E\sqrt{E-E_{\rm th}}\right)^{-1}dE = \pi/\sqrt{E_{\rm th}}$ gives exactly $1/(2\sqrt{c\,E_{\rm th}})$, i.e. the continuum alone saturates the full resolvent weight and the edge atom contributes nothing in the uncompressed limit — so no measure-atomization readout survives freely (house *s*842-V3, analytic + quadrature at three c-values). □

The dressing is furnished by the OOO source dictionary, now derived at the algebra level: $B: z\mapsto \mathrm{vec}(M_z)$ with

$$B^{\mathsf T}B = 32\,I_8, \quad \mathrm{rank}\,B = 8, \quad M_{e_k}^{\mathsf T}M_{e_k} = 4\,I_8 \text{ (all singular values 2)}, \quad (\sigma_{\rm CD}\otimes\sigma_{\rm CD})\,B = B\,\sigma_{\rm CD}, \tag{3}$$

four even source channels (e_0–e_3) and four odd (e_4–e_7). The structural hope that the odd channel is protected ($\chi_{\rm odd}\equiv 0$) is *refuted*: $|\chi_-| = |\chi_+|$ exactly (democratic split 16/16). The exact branch map $\{2\varphi^{-2}, 0\}$ is therefore typed as a $g\to 0^+$ pole-residue *limit*, with $g_{\rm crit} = 2c/(\pi R)$ a genuine predicted failure regime, not an all-coupling theorem. Attraction derives conditionally on a positive exit gap (Schur complement $-B^\dagger(H_{\rm exit}-E)^{-1}B \preceq 0$, isotropic $-(32/\mathrm{gap})I_8$), and the radial null closes for this dressing: $B\cdot r_{\rm radial} = 0$ identically, so $P_{\rm contact} = P_z$ (dressing-relative).

2 The chamber atlas and the decomposed transfer functor

Provenance. The chamber functor round (A1394, D1096): the L3→L4 shortcut was KILLED there and the functor decomposed into typed data; the linear zero and exit gap come from A1393 (D1095). Appendix Y.

The boundary side organizes into a computed object: the quaternionic chamber atlas — seven chambers (one division-$\mathbb{H}$ marked, six split $M_2(\mathbb{R})$), 21 typed lines, edge automorphism orbits $\{6, 3, 12\}$, a domino decomposition $\{1\} + \{3 \times 2\}$ with a genuine C_3 action, and σ_{CD} fixing every chamber setwise. Its symmetry group is identified exactly:

$$G_{\mathrm{ch}} \;\cong\; S_4 \qquad (\text{chambers} = \text{tetrahedron edges};\;\; \mathbb{R}^6 = \mathbf{1} \oplus \mathbf{2} \oplus \mathbf{3}), \tag{4}$$

with order profile $\{1{:}1, 2{:}9, 3{:}8, 4{:}6\}$, trivial centre, A_4 witness of order 12. The exit continuum aligns with the threshold on the nose:

Novelty label: new specialization proved here.

Theorem 2.1 (Exit-edge alignment; conditional on additive two-tube locality)**.** *The stable positive branches of the shipped wall Hessian are $E_x^+ = 2\varphi\, I_4$ and $E_y^+ = 2\, I_4$; the additive two-tube pair edge is*

$$E_{\mathrm{edge}} \;=\; 2\varphi + 2 \;=\; 2(\varphi + 1) \;=\; 2\varphi^2 \;=\; E_{\mathrm{th}}, \tag{5}$$

using $\varphi + 1 = \varphi^2$ — the threshold is *the sum of the two occupied golden tubes. Projecting the dictionary onto the stable sector, $B_+ = P_{ss} B$ obeys $B_+^{\mathsf{T}} B_+ = 16\, P_{\mathrm{even}}$, rank 4, $\chi_{\mathrm{odd}}^{\mathrm{stable}} = 0$ (odd protection restored, conditional on the positive-exit projection).*

Sketch. Diagonalizing the shipped wall Hessian on the split-x and split-y tubes gives stable positive branches 2φ and 2 (fourfold each). Their additive combination on the two occupied tubes is $2\varphi + 2$; the golden identity $\varphi^2 = \varphi + 1$ collapses this to $2\varphi^2$, which is precisely the banked threshold E_{th}. For the doorway, P_{ss} projects B onto the four stable even directions; $B_+^{\mathsf{T}} B_+$ inherits the $32\, I_8$ normalization halved on the retained even block ($16\, P_{\mathrm{even}}$, rank 4) and annihilated on the odd block, because the odd source channels live in the compact–split pairs carrying the negative transverse branch (house *s*849-V1/V2, exact to $< 10^{-14}$). □

Two negatives sharpen the object. The raw transverse Hessian has inertia $(8, 8, 0)$, so global positivity is *false*; below the edge the projected gap equals δ (energy-dependent, closing at threshold), with exact failure locus $t^\star = \sqrt{\varphi}$ — the banked $\sigma_2 = 0$ escape edge. And the induced magnitude cannot be discrete: $g_{\mathrm{ind}}^+ / g_{\mathrm{crit}} = 8\pi R/(c\Delta)$ grows with R, so no single Δ fixes the coupling; a renormalized subtraction is required and the $g \to 0^+$ reading stays a typed limit. The attempted chamber functor therefore resolves, with incidence forcing none of it, into four typed action-side data:

$$\mathfrak{F}_{\mathrm{ch}}^{\mathrm{phys}} \;=\; (\,\eta_+,\; \mathcal{T}_{\mathrm{ch}\to\partial},\; C_{\mathrm{sub}},\; r_C\,), \tag{6}$$

under the standing rule (adopted verbatim as house law) that *incidence multiplicities are not normalized transition amplitudes*: incidence $\not\Rightarrow$ positive metric; line counts $\not\Rightarrow$ subtraction; shared line $\not\Rightarrow$ locking amplitude.

3 The η_+ weld

Provenance. A1395 (D1097), where the weld landed *conditional* and two house overclaims were refuted in the same round (casualty register, Appendix Y §7).

The first of the four data collapses onto the oldest standing import. Among all diagonal sign maps on $\mathbb{O}_s$ exactly eight are automorphisms, and exactly one yields a positive twisted form:

Novelty label: new specialization proved here.

Theorem 3.1 (The η_+ weld; derived-conditional)**.** *The σ_{CD}-twisted conjugation form $\langle a, b\rangle_\eta = \mathrm{Re}(\bar{a}\,\sigma_{\mathrm{CD}} b)$ has Gram $= I_8$ exactly on all of $\mathbb{O}_s$ (both chamber types); the positive twisted metrics form one* Aut*-conjugacy class. Adopting it as the physical kinetic metric* merges *the newest import with the oldest: the boundary loads the holonomy class $[\sigma_{\mathrm{CD}}]$, and η_+ is the Cartan metric induced by that class — one conditional import in place of two. In the welded frame the even-only stable doorway $B_+^{\mathsf{T}} B_+ = 16\, P_{\mathrm{even}}$ is a* theorem *of the welded metric (given the positive-energy transfer rule), and the eight diagonal automorphisms coincide with the chamber-setwise-fixing kernel — the weld, the atlas, and the Y-spine probe meet on σ_{CD}.*

Sketch. A diagonal sign map $\mathrm{diag}(\epsilon_0, \ldots, \epsilon_7)$, $\epsilon_i = \pm 1$, is an automorphism of $\mathbb{O}_s$ iff it respects the split-octonion multiplication table; an exhaustive scan of the 256 sign patterns leaves exactly 8 automorphisms (house s855-V1). For each, form the twisted bilinear $\langle a, b\rangle = \mathrm{Re}(\bar{a}\,\sigma\, b)$ and Gram it against the standard basis; exactly one — $\sigma = \sigma_{\mathrm{CD}}$ — is positive definite, with Gram $= I_8$ identically. These eight automorphisms are exactly the kernel of the S_4 chamber action (order 8, the setwise-fixing subgroup of s856), which is why the weld object and the atlas object coincide.□

4 The first derived relative vertex sign

Provenance. A1395 (D1097). The sign is amputated and F_4-fenced; it is not an amplitude. Appendix Y.

What this is, structurally. The result below is the antisymmetry of a forced alternating trilinear under the coordinate swap $x \leftrightarrow y$: a fixed component and its argument-swapped partner differ by a sign, so their ratio is -1 and the normalization g_3 and subtraction C_{sub} divide out. It is a derived *relative sign* of the amputated doorway vertex, not an interaction strength, width, or amplitude — the value of naming it is that the sign is *forced* (not conventional) by the welded metric and the census, and it is the first vertex quantity the program derives at zero parameters.

On the trace-free (off-diagonal) doorway the two lower invariants $I_1 = I_2$ vanish identically and the Albert cubic survives alone, forcing the cubic vertex $V_{g_3}(x, y, z) = g_3\, T_{\det}(x, y, z)$ there. (The stronger full-**27** uniqueness claim is *refuted*: $\mathbf{27} = \mathbf{1} \oplus \mathbf{26}$ is reducible, with at least three independent invariant trilinears; the repair holds on the carrier that matters.) From this forced vertex the program banks its first derived interaction quantity:

Novelty label: new specialization proved here.

Theorem 4.1 (First derived relative vertex sign)**.** *In the house Fano basis, with the welded metric and the amputated $g \to 0^+$ doorway,*

$$\boxed{\frac{\mathcal{A}(4_x, 5_y; 1_z)}{\mathcal{A}(5_x, 4_y; 1_z)} \;=\; -1} \tag{7}$$

zero-parameter*: the cubic normalization g_3 cancels and the common subtraction C_{sub} cancels (checked at $\delta \in \{0.01, 0.1, 1\}$). Supporting census: the* 64*-channel census $\{1, 21, 42\}$ is entirely σ_{CD}-even; the* 16*-channel stable selection and the* 12*-row Fano sign table are entry-exact and antisymmetric in the house basis.*

Census and antisymmetry (house s855-V3/V4). The 64 nonzero trilinear channels partition as

$$64 \;=\; \underbrace{1}_{\text{scalar}} \;+\; \underbrace{21}_{\text{diagonal (0pp)}} \;+\; \underbrace{42}_{\text{Fano}}, \tag{8}$$

all σ_{CD}-even (the scalar rows carry $+2$). Restricting to the stable split⊗split sector leaves 16 channels; of these the 12 Fano-line rows form the sign table, each row exactly antisymmetric under the coordinate swap $x \leftrightarrow y$ ($\mathcal{A}(a_x, b_y; c_z) = -\mathcal{A}(b_x, a_y; c_z)$ entrywise, verified on all 12). The quoted ratio is the $(a, b) = (4, 5)$ instance of that antisymmetry; because it is a pure sign flip, the cubic normalization g_3 and the common subtraction C_{sub} divide out identically, which is what makes the number zero-parameter. The external kernel then checks the same -1 survives the amputated $g \to 0^+$ propagator dressing at $\delta \in \{0.01, 0.1, 1\}$; the house reproduces the algebraic sign directly and byte-identically (result hash `ddbede9d`, bit-portable).

*The full-**27** refutation (house casualty, s855-V5).* The dispatch's premise that the Albert cubic is the *unique* invariant trilinear on the full **27** is false: $\mathbf{27} = \mathbf{1} \oplus \mathbf{26}$ is reducible, and the invariant-value matrix of $\{\mathrm{tr}^3,\ \mathrm{tr}\cdot q_2,\ \det\text{-polarization}\}$ has rank 3 — at least three independent invariant trilinears (q_2 here is the quadratic trace invariant $\mathrm{tr}(x^2)$, a form, not a constant; it is unrelated to the constants q_0^{KK}, q_0^{clk} of Appendices G/H). The repair: on the trace-free / off-diagonal doorway the two lower invariants $I_1 = I_2$ vanish identically (self-test $T_{xxx} = 6N$), so $V_{g_3} = g_3\, T_{\det}$ is forced *there* and the -1 is unaffected.

Fence (binding, F4). The ratio is a bare/amputated doorway sign, not a finite-width S-matrix prediction. It is *relative*, not absolute: it is a **sign**, and every magnitude stays behind C_{sub}. It becomes a candidate public falsifier only once an amplitude theorem lifts the F4 wording fence; until then "mass shell" and "amplitude" stay out of world-facing prose. This is nonetheless the deliverable both external reviewers demanded — "derive one vertex" — at its first honest rung.

5 The equivariant transfer, the invisible mode, and $\Pi_{5\leftarrow 6}$

Provenance. A1396 (D1098, the ten-dimensional deficit: 10=10 dead beyond the orientation fibre, the readout built at rank 7, g_3 structurally free) and A1400 (D1100, the five-state closure REFUTED: the unique fully equivariant unital kernel is the uniform rank-1 U). Both are *respected, not overturned*, by the Rev28 observation quotient $\Pi^{\mathrm{MOS}}_{5\leftarrow 6}$ (Appendix T). Appendix Y.

The smallest missing object — the chamber-to-boundary spectral transfer $\mathcal{T}_{\mathrm{ch}\to\partial}$ — splits into a built half and a loaded remainder. First, the graded coincidence dies the right death:

Novelty label: new specialization proved here.

Theorem 5.1 (The 10=10 coincidence, adjudicated dead beyond the fibre)**.** *The graded dimensions match ($5 + 5$ both sides), but the representations do not:* $(\mathbf{2} \oplus \mathbf{3})_{S_4} \ncong (\mathbf{1} \oplus \mathbf{2} \oplus \mathbf{2})_{D_5}$. *Since* $W_0^{A_4} = 0$ *and every homomorphism* $S_4 \to D_5$ *factors through* $\mathbb{Z}_2$*, the equivariant intertwiner space* $\mathrm{Hom}_{S_4}(K_{\mathrm{ch}}, \mathbb{R}[D_5]_\varphi) = 0$*: there is* no nonzero equivariant map*, not merely no isomorphism. Only the* $\sigma_{\mathrm{CD}} \leftrightarrow$ *Möbius two-state orientation fibre survives as genuine common structure.*

Sketch (house s858-U1–U3). $G_{\mathrm{ch}} \cong S_4$ (order profile $\{1{:}1, 2{:}9, 3{:}8, 4{:}6\}$, trivial centre, A_4 witness of order 12). The chamber deficit restricted to A_4 has no invariants: $W_0^{A_4} = 0$. Any equivariant $K_{\mathrm{ch}} \to \mathbb{R}[D_5]_\varphi$ composed with an $S_4 \to D_5$ map would carry an A_4-invariant into the image; but the brute-force enumeration gives exactly 6 maps $S_4 \to D_5$ (and 10 reverse), *all* of image order ≤ 2, so each factors through $\mathbb{Z}_2$ and annihilates A_4. Hence Hom $= \{0\}$. The 5+5 graded coincidence is real but carries only the $\mathbb{Z}_2$ orientation bit. □

Constructively, the equivariant half is done: with column order (six chambers, then trace) the 16-channel readout has Fano block Gram $M_F^{\mathsf{T}} M_F = 8\, I_6$ (rank 6; every chamber label separately visible, all five relative weights recoverable) and scalar entry 16, i.e. spectrum diag(8×6, 16), so the full stable map has rank 7. (The house prediction "rank ≥ 8" was wrong — a casualty — the labels *are* the six columns, not a separate payload.) What provably cannot pass is the five-placement AnnTL coordinate and the absolute mode. After one source normalization exactly one kernel direction remains,

$$\mathcal{N}_{\mathrm{abs}} = g_3 \cdot Z_{\mathrm{sub}}(C_{\mathrm{sub}}), \qquad \delta \log |g_3| = -\,\delta C_{\mathrm{sub}} \tag{9}$$

(rank 7 of the 9-parameter Jacobian) — "the one number the boundary cannot see." And g_3 is proven structurally free: the kinetic UNINORM $c_{\mathrm{vec}} = 24$ fixes the tensor normalization ($2/24^{3/2} = 0.01701035\ldots$), *not* the action coefficient; a unit-channel convention $|g_3| = 24^{3/2}/2 = 58.78775383\ldots$ would be a new parent-action axiom, not a derivation. The remainder is named and typed:

Named datum 5.2 ($\Pi_{5\leftarrow 6}$, loaded)**.** The chamber-to-AnnTL placement/spectral kernel is *necessarily non-equivariant*: any construction must break S_4, so it is loaded data, not derivable atlas structure. It welds the transfer question to the S232 loaded-boundary frontier — the same loading that carries $[\sigma_{\mathrm{CD}}]/\eta_+$ must carry $\Pi_{5\leftarrow 6}$, or nothing does.

6 Status language (strict) and the closing ledger

- **Theorems (in-house, executed, bit-portable):** finite-R Riesz edge projector; free-measure no-go; exit-edge alignment $E_{\mathrm{edge}} = 2\varphi^2 = E_{\mathrm{th}}$; the even-only welded-metric doorway; the 10=10 equivariant no-go; g_3 structurally free; and the relative sign -1 (amputated, F4-fenced).
- **Derived-conditional:** the η_+ weld (on the loaded holonomy class); attraction (on a positive exit gap); the odd-protection projection (on the positive-exit metric).
- **Named-open data (the wall, final form this arc):** one rank-7 channel readout (built) + three continuous data $\{g_3, C_{\mathrm{sub}}, r_C\}$ (equivalently $\mathcal{N}_{\mathrm{abs}} + r_C$+ one convention) + the loaded kernel $\Pi_{5\leftarrow 6}$.

Object	Tier	Note
$[\sigma_{\mathrm{CD}}]$ holonomy + η_+ Cartan metric	one merged conditional import	weld, §3
additive locality; positive-energy transfer	standing structural imports	§1–2
g_3 (cubic normalization)	continuous, structurally free	§5
C_{sub} (additive subtraction)	continuous; $\mathcal{N}_{\mathrm{abs}}$ invisible	§5
r_C (locking form factor)	continuous, open	$\kappa = 1 + \lvert r\rvert$, $t_C(\delta) = r_C\delta$
$\Pi_{5\leftarrow 6}$ (placement kernel)	loaded, non-equivariant	§5

Bottom line. The head wall ("no parent action") was attacked head-on and cracked along its typed seams. The transfer question is no longer one fuzzy object: it is a merged loaded import ($[\sigma_{\mathrm{CD}}]$ holonomy welded to its induced metric η_+), three continuous constants, one of which ($\mathcal{N}_{\mathrm{abs}}$) is provably invisible from the boundary, a built rank-7 equivariant readout, and one loaded non-equivariant placement kernel $\Pi_{5\leftarrow 6}$. On the way it produced the program's first derived, parameter-free (discrete, derived) relative vertex sign — the -1 — kept strictly amputated and F4-fenced. The "no dynamics" category is no longer a category; what remains is one loaded kernel and three constants, each with a name, a type, and a test. Conditional/candidate tier throughout; nothing here is a world-scored observable.

Leibniz Quantum Beats Newton

Appendix S — The Priced Selector: the Mixed Source, the Independence Ladder, and the Closed Arrow-Price Ledger (Rev33.2)

Rev27 selector frontier (S244–S245, the D1116–D1121 hexad). Conditional/candidate tier throughout. This appendix does not add an observable; it characterizes the wall. *Where Appendix R left the parent-action question as "one loaded kernel and three constants," this arc prices the discrete half of the same wall exactly: the missing selector is now a* ***closed ledger*** *of named, tiered objects, none derived beyond a single conditional quarter-turn that the candidate action itself pays for. Nothing here is world-scored; no public observable moves; α and colour stay fenced; $V_{\mathrm{CKM}} = I$; the engine is unchanged; KILL-NODE 2 remains fired.*

Tom O'Sieg

August 2026

Posture (binding). Every equation below is *conditional* on the candidate shape action S_{shape} of §1, which is itself banked as a zero-parameter *candidate* and is **not** promoted (no microscopic theorem forces it; KILL-NODE 2 remains fired). The value of the arc is not a new number but a change of epistemic status for the wall: the discrete selection datum, previously "some mixed tensor," is resolved into an exactly typed object $[j_A] \in \mathbb{R}\mathrm{P}^2$ and a short shelf of *priced* candidate axioms. Every quantitative closure was independently reproduced in-house (house double Cayley–Dickson octonions, house S_3/C_5 character arithmetic, independent eigensolvers), byte-identical to the executed external kernels, before being banked; the six return round-trips (D1116–D1121) are receipt-verified and bit-portable. Four house over-claims were caught and repaired inside the arc (casualties #4–#7, §7); they are logged, not hidden — the adversarial design is the point. **No public observable moves.**

1 The mixed source and its exact typing

Provenance. The parent-action round **A1416** (D1116) supplied the first two exact equations on C with S_{shape} typed as a conditional candidate; the mixed-tensor typing is **A1417** (D1117). Both rounds had no ledger row before S254 and are recorded in Appendix Y §4.

The dictionary datum that the interacting vertex (Appendix R) left continuous is the placement source C — ten real amplitudes on which no banked principle had yet written an equation. The candidate shape action closes the first two, conditionally.

Novelty label: new specialization proved here.

Theorem 1.1 (First two exact equations on C; conditional on the candidate)**.** *The zero-parameter Gram-shape action built from the banked tensors,*

$$S_{\text{shape}}[C] \;=\; \tfrac{1}{4}\|CC^{\mathsf{T}} - \tfrac{\operatorname{tr} G}{2}\, I_2\|_F^2, \qquad \textit{Euler equation } (CC^{\mathsf{T}} - \tfrac{\operatorname{tr} G}{2} I)C = 0, \tag{1}$$

has, on its nonzero critical stratum, exactly the two nonlinear conditions

$$\|C_1\|^2 = \|C_2\|^2, \qquad C_1 \cdot C_2 = 0 \tag{2}$$

(the two rows equinormal and orthogonal). The nonzero critical set is $\mathrm{Crit}_{\neq 0} = \mathbb{R}_+ \times V_2(\mathbb{R}^5)$ *(dimension* 8*, Hessian rank* 2*); the* κ*-completed action reaches dimension* 7 *(rank* 3*). The selection is thus* $10 \to 8 \to 7$*, not a point.*

Status (strict). The equations are *derived from the candidate*; no microscopic theorem forces S_{shape}. The linear-tier rank-0 theorem and this quadratic-tier Gram probe are compatible: the rank-0 result lives on the linear invariants, S_{shape} on the quadratic Gram (house *s*890-J1, finite-difference Euler check; five-fold reproduced). The next theorem says why no *quadratic* action can finish the job.

Novelty label: new specialization proved here.

Theorem 1.2 (Quadratic-action no-go; the invariant-action census)**.** *For the placement representation the invariant dimensions are*

$$\dim(\textit{linear invariants}) = 0, \quad \dim(\mathrm{Sym}^2\ \textit{invariants}) = 5, \quad \dim(\mathrm{Sym}^2\ \textit{invariants})\big|_{\textit{domino parity}} = 3, \tag{3}$$

(direct 30/60*-element group averaging, house s*890*-J2). Consequently every positive invariant quadratic action selects* $C = 0$ *(the dead uniform dictionary* $\Phi = U$*); every semidefinite one leaves whole blocks flat.* A selected nonzero dictionary requires quartic-or-higher dynamics or a mixed source.

This is the exact reason the wall is hard, stated as character arithmetic. It forces the attention onto the mixed source $J(A)$, which the next round types to the coefficient.

Novelty label: new specialization proved here.

Theorem 1.3 ($J(A)$ typed to $\mathbb{C}^2$ with an exact rank criterion)**.** *The minimal rescue* $S_{\text{int}} = \frac{1}{2}\|C - q(A)a(A)^{\mathsf{T}}\|^2$ *has full Jacobian rank* 10 *and unique selection* $C = q(A)a(A)^{\mathsf{T}}$ iff *the microscopic datum* $J(A) \in \mathrm{Hom}(W_0, V_{\pm 2})$ *exists. Under the placement/charge structure of §2 this datum is exactly a pair*

$$(z_+, z_-) \in \mathbb{C}^2\ (\dim_{\mathbb{R}} 4), \qquad \textit{basis } \{P_2 \otimes I_\pm,\ JP_2 \otimes I_\pm\}, \qquad \boxed{\operatorname{rank} C_z = 2 \iff \operatorname{Im}(\bar{z}_+ z_-) \neq 0}. \tag{4}$$

The factorized $q(A)a(A)^{\mathsf{T}}$ *is the rank-one locus (dimension* 3*); a full-row-rank dictionary requires a* nonzero relative phase *between the two* S_3 *channels. The* 4*-dimensional price is symmetry-protected, not accidental:* $\mathrm{End}_{C_5}(V_{\pm 2}) \cong \mathbb{C}$ *and the two recipient channels are isomorphic, so nothing banked selects the coefficients (house s*891*-M2/M3/M5).*

2 The placement–charge table and the free attachment

Provenance. A1417 (D1117): the placement-charge table $\iota = 16p$ and the $V_{\pm 1}$ exclusion. The odd-sector quantization it rests on is A1407 (D1107). Appendix Y.

Underneath $J(A)$ sits a complete, exact placement bookkeeping.

Novelty label: new specialization proved here.

Theorem 2.1 (Canonical section and charge content)**.** *The canonical section $\iota(p) = 16p$ is the unique additive order-five right-inverse of $\mathbb{Z}_{20} \to \mathbb{Z}_5$; the charge map is $r(q) = 4q \equiv -q \pmod 5$; the even sectors carry $(0, +2, -2, 0)$ and the odd octet carries* only $0, \pm 2$*,*

$$\mathcal{O}_{\mathrm{odd}}\big|_{C_5} \;\cong\; 4 \cdot \mathbf{1} \oplus 2 \cdot V_{\pm 2}, \tag{5}$$

with no object descending onto $V_{\pm 1}$ ($4 \nmid q$) — the fifth exclusion *of $V_{\pm 1}$ (house s*891*-M1).*

Novelty label: new specialization proved here.

Theorem 2.2 (Attachment forced-free)**.** *The cover-to-placement attachment is forced at the already-loaded basepoint tier: it costs* 0 *new bits and* 0 *new dimensions. The census is exact — additive $4 + 8 = 12$, multiplicative $4 \times 8 = 32$, octet split $4 + 4 + 0$ (house s*892*-N6) — the odd carrier's* 8 *real dimensions become* available *multiplicities (unselected), $V_{\pm 1}$ stays absent (sixth exclusion, character-chain tier), and the central cover -1 is placement-invisible ($\chi_{10} \circ \iota = 1$), so the round-trip sign remains cleanly at the orientation/cover tier.*

3 The independence ladder

> **Provenance.** The ladder assembles A1419 (D1119, anti-alignment), A1420 (D1120, the j_A round: $[j_A] \in \mathbb{RP}^2$ and the null axis $(5, -18, 1)$) and A1428 (D1124, the kernel axis dying a second time). The V-side quadrature is **A1418** (D1118), the hexad's only blind-adjudicated round. Appendix Y.

The arc's structural spine is a ladder of *independence* statements: at each new tier one asks whether the banked structure at that tier writes any equation on the mixed coefficients, and the answer is repeatedly *no*. Each rung is a theorem about the axiom stack, not a conditional on the candidate.

Novelty label: new specialization proved here.

Theorem 3.1 (Third tier — coherence and weld are silent)**.** *The half-ribbon coherence equations and the σ_{CD} weld contain no occurrence of* $\mathrm{Re}/\mathrm{Im}\, z_\pm$*: the constraint Jacobian has rank* 0 *exactly (discriminant examples $2, \sqrt{3}, 2\sin(\pi/7)$ verified). Neither the $\pm\pi/2$ quantization nor $z_- = \pm\bar{z}_+$ is derivable from them.* The axiom stack is certified minimal at three tiers *(tube · action · weld). The named conditional that* would *buy the quarter-turn is the common-lift factorization axiom*

$$\omega_{\mathrm{mix}} = \omega_V \otimes (1_{E_+} \oplus -1_{E_-}) \qquad (= \textit{CASE D}), \tag{6}$$

whose adoption prices the mixed tensor at 2 *amplitudes + the standing orientation $\mathbb{Z}_2$ (house s*892*-N5; PI decision, not house).*

Novelty label: new specialization proved here.

Theorem 3.2 (Fifth tier — the pulled-back shape and the flat source axis)**.** *Through the actual mixed map $C_z(x) = u_+ w_+(x)^{\mathsf{T}} + u_- w_-(x)^{\mathsf{T}}$, the S_{shape} action of Theorem 1.1 pulls back to the exact symbolic identity*

$$S \;=\; \frac{|x|^4}{8}\Big[(|z_+|^2 - |z_-|^2)^2 + 4\big(\mathrm{Re}\,\bar{z}_+ z_-\big)^2\Big]. \tag{7}$$

Since S depends on x only through $|x|^2$, the source axis $[x] \in \mathbb{R}\mathrm{P}^1$ is exactly flat *(the κ-completion is equally blind).* The axiom stack is minimal at five tiers *(tube · action · weld · geometry · parent-action axis) (house s*896*-V2).*

> **Source side vs target side (*Rev29*, confirmation and fence).** The source-axis independence just stated **stands**: an S260 report that it had been broken was itself in error, and nothing in the Rev28–Rev29 boundary arc disturbs the flatness of $[x] \in \mathbb{R}\mathrm{P}^1$ under S_{shape}. Separately, and to fence a distinct error: any canonical ray exhibited on the *target* side is a **target-side object only**. No projective identification of the source and target rays is claimed anywhere in this appendix, and the S260 line asserting that source and target were "projectively identical" was FALSE and is not to survive in any form. This appendix carries no canonical target ray; the reader is warned only so that the two sides are not silently welded.

Novelty label: new specialization proved here.

Proposition 3.3 (The V-side quadrature is derived-conditional)**.** *Every nonzero critical point of S obeys $|z_+| = |z_-|$ and $z_- = \pm i\, z_+$: the quarter-turn (the semion phase value) is* forced by the candidate action itself*, conditional on S_{shape}. This is the arc's one derived surprise — the action pays for a phase that was previously carried as an axiom — and it is the* only *datum in the ledger derived beyond counting.*

4 The paired-source theorem

The correct source model (correcting the retired house doorway, casualty #4) is a *single* common selector contrast, not independent channel patterns. [LIB2-261]

Novelty label: new specialization proved here.

Theorem 4.1 (Paired source; the CASE-D locus)**.** *With one selector contrast x ($\sum x = 0$) and $(w_+, w_-) = (I_+x, I_-x)$, CASE D is realized exactly by the* even-parity double-flips on the three A_2 root lines $[e_i - e_j]$ *— six actions, two per line, all substrate-induced; the plus-axes $[e_i + e_j - 2e_k]$ force the aligned CASE T; a generic $[x]$ admits the identity lift only. The CASE-D locus is codimension one (three $\mathbb{R}\mathrm{P}^1$ points), and the even-half / parity theorems of the geometry tier stand unchanged (house s*896*-V1).*

The trichotomy is now exact: $[x]$ on a root line ⇒ CASE D earned (the axiom retires); $[x]$ on a plus-axis ⇒ CASE T forced (the full-rank branch dies); generic $[x]$ ⇒ no substrate lift. Nothing shipped selects among the three natural encodings (one-hot → plus-axis; generation order $(-1, 0, 1)$ → root line; depth contrast → generic); the coincidence that the generation order lands *on* a root line is recorded as suggestive, *not* selected. [LIB2-261]

5 The evaluation relation and the sixth independence

The occupancy-to-chamber arrow j_A is counted exactly.

Novelty label: new specialization proved here.

Theorem 5.1 (Arrow space and the evaluation relation)**.** $H^{\mathrm{src}}_{\mathrm{occ}} \cong 4 \cdot \mathbf{1} \oplus 3E$ *($\chi = (10, 4, 1)$), and* $\mathrm{Hom}_{S_3}(H^{\mathrm{src}}_{\mathrm{occ}}, E) \cong \mathbb{R}^3$ *(character count 3 = nullspace dimension of the explicit 40×20 intertwining system). Schur gives one ray per copy but kills no mixture, so*

$$[j_A] \in \mathbb{R}\mathrm{P}^2. \tag{8}$$

The evaluation $\mathbb{R}^3 \to E$ has rank 2 with kernel $(5, -18, 1)$; at the banked contrasts $s = (2, -1, -1)$ (plus-axis), $o = (-1, 0, 1)$ (root line $e_1 - e_3$), $d = (-28, 5, 23)$ (generic),

$$\boxed{d = -5s + 18o} \qquad (\textit{exact}), \tag{9}$$

and every *source axis $[x] \in \mathbb{R}\mathrm{P}^1$ is reachable by an equivariant arrow (house s897-W2).*

Naming discipline: $H^{\mathrm{src}}_{\mathrm{occ}}$ is not H_{Cartan} (*Rev29*). Two distinct 10-dimensional S_3-modules have been carried in this suite under the single name "H_{occ}." They are now split by name and must never again be conflated:

- $H^{\mathrm{src}}_{\mathrm{occ}} \cong 4{\cdot}\mathbf{1}{\oplus}3E$, the **source-side occupancy carrier** of Theorem 5.1, with S_3-character $\chi(H^{\mathrm{src}}_{\mathrm{occ}}) = (10, 4, 1)$;
- $H_{\mathrm{Cartan}} = \mathfrak{so}(5) \cong 3 \cdot \mathbf{1} \oplus \mathrm{sgn} \oplus 3E$, the **Cartan carrier** of Appendix T (there $\mathfrak{so}(5) = \mathfrak{p}_4 \oplus A_3 \oplus B_3$ with $\mathfrak{p}_4 = 2 \cdot \mathbf{1} \oplus E$, $A_3 = \mathbf{1} \oplus E$, $B_3 = \mathrm{sgn} \oplus E$), with S_3-character $\chi(H_{\mathrm{Cartan}}) = (10, 2, 1)$.

Both have dimension 10 and both split as $4 + 3 + 3$, which is why the conflation happened; they are nevertheless *not isomorphic* as S_3-modules. The virtual difference is

$$H^{\mathrm{src}}_{\mathrm{occ}} \ominus H_{\mathrm{Cartan}} \;=\; \mathbf{1} \ominus \mathrm{sgn}, \qquad \chi(H^{\mathrm{src}}_{\mathrm{occ}}) - \chi(H_{\mathrm{Cartan}}) = (10, 4, 1) - (10, 2, 1) = (0, 2, 0), \tag{10}$$

and $(0, 2, 0)$ is exactly the character of $\mathrm{triv} - \mathrm{sgn}$. **That difference is exactly AX-COT**: the Appendix T axiom buys the (4+3+3) match only after the sign twist $B_3 \otimes \mathrm{sgn} = \mathbf{1} \oplus E$, i.e. AX-COT is precisely the undischarged cross-layer clause that absorbs $\mathbf{1} \ominus \mathrm{sgn}$. It is therefore a typing axiom with content, not bookkeeping, and no argument may pass between the two modules by calling them the same object.

Novelty label: new specialization proved here.

Theorem 5.2 (Sixth independence)**.** *No banked principle — equivariance, Kan extension, the paired-source theorem, the CRT tick classes, the weld, or the χ-verdict of Appendix O — selects j_A. Killing the depth ray does not select order (s and o alone span E); the root-line conditions are full coefficient hyperplanes, not coordinate rays. [LIB2-263]* The axiom stack is minimal at six tiers *(the five of Theorem 3.2 + the arrow). The order arrow stays suggestive, not selected. [LIB2-262]*

What the ARROW tier is (*Rev29*, tier correction). The arrow tier is to be read exactly as follows and in no other way: it is *an unattached pre-observation loading/faithfulness datum whose present export is blind.* It is **not** a gauge statement. Nothing here quotients by a redundancy, fixes a frame, or asserts that arrows related by anything are physically equivalent; $[j_A]$ is a loading datum standing *before* any observation functor is applied, and its export at the present tier is blind, i.e. it distinguishes nothing downstream. Any reading of the arrow row as gauge freedom — and hence as costless — is withdrawn.

The convention-independent phase law is $\delta_z = \Delta_\theta + \frac{\pi}{4}(\epsilon_+ - \epsilon_-) \pmod{\pi}$; with the relative lift axis Δ_θ unpinned, both character types admit nonzero action-critical equivariant points, so the action's "vote" for the root line is *common-lift-conditional only* (casualty #5, corrected in place). [LIB2-262]

6 The copy-blindness split

The final round splits the obstruction into a derived half and a named-open half.

Novelty label: new specialization proved here.

Theorem 6.1 (Copy-blindness; the multiplicity tensor)**.** $\mathrm{End}_{S_3}(E \otimes \mathbb{R}^3) = I \otimes M_3(\mathbb{R})$ *(dimension* 9*). Then:*

- ***Derived:*** *every target-Gram-only action $S(X) = F(XX^\mathsf{T})$ is exactly copy-blind (right-$O(3)$ invariance) — the $D1116$-class obstruction is now a theorem.*
- ***Refuted:*** *"every invariant Gram action is copy-blind" is false — $q_A(X) = \mathrm{tr}(AX^\mathsf{T}X)$ is S_3-invariant and selects a copy.*

The exact closure: the missing selector is a non-scalar multiplicity tensor $A \in \mathrm{Sym}_3$ *(*6 *coefficients;* 5 *after scale;* 2 *if type-diagonal). Symmetry permits it; nothing banked derives it (house s*901*-Z2).*

7 The closed arrow-price ledger

> **Provenance.** The price rows collate A1416–A1421 (the D1116–D1121 hexad) with A1426–A1429 (D1122–D1125). Minimal height was REFUTED-AS-CANONICAL in A1421; the one-row axiom compression is A1428; the D1125 candidate is A1429, KILLED as selector-deriving one round later by A1430 (§8). Round-by-round index: Appendix Y.

The arc's headline is not a value but a *closure*: the discrete half of the wall is no longer a growing list of hopes, it is a finite ledger of priced objects.

Named datum 7.1 (The full-rank rescue price, final at this tier)**.** A full-row-rank selected dictionary requires exactly

$$\underbrace{[j_A] \in \mathbb{R}\mathrm{P}^2}_{\text{or: sparsity axiom+branch bit}} \times \underbrace{(\text{common-lift axiom } \mathbf{OR} \text{ the phase } \Delta_\theta)}_{\text{one relative datum}} \times \underbrace{2 \text{ magnitudes}}_{|z_+|,|z_-|} \times \underbrace{\text{the standing } \mathbb{Z}_2}_{\text{orientation bit}}. \tag{11}$$

Everything is *counted*; nothing is derived beyond the V-side quadrature (Proposition 3.3, conditional).

The candidate-selector shelf that would buy the order arrow is likewise priced, and its cheapest-looking member is refuted as canonical:

Novelty label: new specialization proved here.

Theorem 7.2 (Minimal height refuted-as-canonical)**.** *The house ordering $1 < 2 < 28$ exists only in the ray-wise-primitive convention. In the common A_2 weight lattice the selector is shorter ($\|\mathrm{Pr}\|^2 = 2/3$ vs 2); common-numerator heights $(2, 3, 28)$ and spread/quotient heights $(1, 2, 17)$ also*

pick the selector; a fifth convention (min-representative ℓ^∞) ties at $(1, 1, 9)$. *The exact cause is that per-ray* gcd *division is nonlinear.* Lattice obstruction: *over the full arrow lattice, minimal output height* 1 *is attained on all three root rays by infinitely many mixed arrows (*13 *projective coefficient rays tie at height* 1*), so minimal height cannot even replace the sparsity axiom. The surviving form is a* new three-clause axiom *(single-summand + ray-wise normalization + minimal root height) buying the order arrow at* 2 *moduli* $+\log_2 3$ *bits* by assumption *(house s*901*-Z1).*

Object	Tier	Note
S_{shape} (candidate action)	candidate, unpromoted	§1; KILL-NODE 2 fired
first two equations $\|C_1\|=\|C_2\|$, $C_1 \cdot C_2=0$	derived-conditional	$10 \to 8 \to 7$, Thm 1.1
quadratic-action no-go (0/5/3)	theorem	Thm 1.2
$J(A) = (z_+, z_-) \in \mathbb{C}^2$, rank=2 $\iff$ $\operatorname{Im} \bar{z}_+ z_- \neq 0$	theorem (typing)	Thm 1.3
charge table ι=16p; $V_{\pm 1}$ excluded; attachment free	theorem	§2
independence at three / five / six tiers	theorems	§3,5
V-side quadrature $z_- = \pm i z_+$	derived-conditional	Prop 3.3 (the one surprise)
paired-source / CASE-D root-line locus	theorem	Thm 4.1
evaluation relation $d = -5s + 18o$, $[j_A] \in \mathbb{RP}^2$	theorem	Thm 5.1,5.2
ARROW tier ($[j_A]$ as a tier, *Rev29*)	an unattached pre-observation loading/faithfulness datum whose present export is blind	not gauge; §5
copy-blindness split; tensor $A \in \mathrm{Sym}_3$	theorem + named-open	Thm 6.1
minimal height refuted-as-canonical	theorem	Thm 7.2
{3-clause height · sparsity · tensor A}	candidate axioms, priced	the selector shelf

House casualties this arc (all lane-caught, all pre-flagged tiers). #4 the independent-channel doorway retired for the paired-source theorem (§4); #5 "the action votes for the root line" corrected to common-lift-conditional (§5); #6 the blanket "$\leq$ 2-parameter completion no-go" withdrawn — a banked-period-70 two-parameter harmonic passes the 0.1% bar at $\max|\Delta d| = 3.04 \times 10^{-4}$ *fitted* (refused adoption; watch item, Appendix O/X); #7 the depth-exclusion cut retired ($[M, Q_{\text{em}}] = 0$ forbids identifying the two $\mathbb{Z}_4$ labels but not depth sourcing). [LIB2-263]

Bottom line. The discrete half of the parent-action wall is now a *closed ledger*. What was "some mixed tensor" at the start of the arc is, at its end, a fully typed and priced object: a projective selection datum $[j_A] \in \mathbb{RP}^2$, a two-magnitude pair, a relative phase available either as a named common-lift axiom or as the unpinned lift Δ_θ, and one standing orientation bit — with a short shelf of candidate axioms, each named and priced, and its cheapest member (minimal height) refuted as canonical. Six independence theorems certify that no banked principle shortcuts any of it, and exactly one datum (the V-side quarter-turn) is derived beyond counting, conditional on a candidate action that is itself unpromoted. This is a wall stated honestly to the last coefficient; it selects nothing, promotes nothing, and moves no observable. Conditional / candidate tier throughout; KILL-NODE 2 remains fired.

8 The Rev28 boundary arc: obstruction, export, affine cover, AX-MG

This section folds the D1126–D1131 arc (assessments A1430–A1435; house verifiers `s909`–`s914`, every one ×2 byte-identical, plus one pre-registered prebuild). It supersedes the parts of this appendix that treated the D1125 construction as a selector-deriving parent action, and it closes the boundary question at this tier.

8.1 The transgression obstruction

Novelty label: new specialization proved here.

Theorem 8.1 ($\mathfrak{T}_{AM}$ has no universal property of the requested kind; A1430, `s909`)**.** $\mathrm{Hom}_{S_3}(4\cdot 1 \oplus 3E,\, E_A) \cong \mathbb{R}^3$ *(20 unknowns, rank* 17*, nullity* 3*; basis the three copy projections,* $T_v T_v^{\mathsf{T}} = (\Sigma v^2) I_2$ *exact). With the equal-copy metric the coisometric transports form* S^2 *(*$\mathbb{RP}^2$ *after target sign);* no *nonzero transport is natural under the copy-isometry group, and every point has* $O(2)$ *stabilizer — so uniqueness-up-to-unique-isomorphism fails twice over. The partial coimage* $\mathbb{R}^3 \to \mathbb{R}^3/\ker J_{\mathrm{occ}} \to E_A$ *is canonical* only *once* J_{occ} *is supplied, and* $\ker J_{\mathrm{occ}} = \mathbb{R}(5,-18,1)$ *— exactly the banked null axis.*

Novelty label: new specialization proved here.

Corollary 8.2 (AX-AM is a choice)**.** AX-AM is upgraded from "unconstructed" to *provably a choice*: price $[j_A] \in \mathbb{RP}^2$ plus one orientation/lift bit (plus AX-MET's 5 metric degrees of freedom after scale). AX-SPH $\Rightarrow$ naturality strictly; the converse is refuted by the weighted-trace countermodel τ_D, $D = \mathrm{diag}(2,1)$.

Consequence for D1125 (binding). The D1125 return's own falsifier #7 fires: the root-line minima do not extend beyond the single evaluated occupancy map. **D1125 is killed as a selector-deriving microscopic parent action.** It survives as a mathematically consistent *loaded finite effective action* once $\mathfrak{T}_{AM}$, the copy metric and the discrete marking are declared inputs — a demotion of provenance, not an algebraic inconsistency. [LIB2-260] KILL-NODE 2 is fired unconditionally; the earlier conditional rescue is withdrawn. Correction to the banked record: "all ten C fixed exactly" becomes *fixed up to one unselected root-marking torsor point and one orientation branch.*

[LIB2-260]

8.2 Support, export, and the collapse

Novelty label: new specialization proved here.

Theorem 8.3 (The annular projector and the categorical absence of $q = \pm 1$; A1431, `s910`)**.** $P_{\mathrm{ann}} = E_0 + E_2 + E_{-2} = \frac{1}{5}[3I - \varphi^{-1}(R + R^{-1}) + \varphi(R^2 + R^{-2})]$ *is idempotent, symmetric, of rank* 3*, Fourier-diagonal* $(1,0,1,1,0)$*. The* $q = \pm 1$ *modes are* ***absent from the finite*** C_5 ***image****: no simple sector supplies* $V_{\pm 1}$ *— categorical, not accidental. Since* P_{ann} *is a polynomial in* R*,* $[P_{\mathrm{ann}}, U] = [P_{\mathrm{ann}}, L] = 0$ *and the transfer restricts with no by-hand projection.*

Remark 8.4 (typing, must travel)**.** The theorem lives in the annular/tube representation *with* the banked C_5 rotation supplied; the bare planar $f_4 = 0$ quotient has no rotation and hence no q-support.

Novelty label: new specialization proved here.

Theorem 8.5 (Operator-ray collapse and non-factorization; A1431)**.** *On* $\operatorname{Im} P_{\mathrm{ann}}$ *the minimal polynomial of* L *is* $x(x-\lambda_s)$*, so* $L^2 = \lambda_s L$*,* $H_2 = \varphi^2 L$ *and* $L_{\mathrm{eff}} = 2\varphi^{-3}L$*: the span* $\{L, L^2, H_2\}$ *has dimension* 1 *on export. The basis ambiguity does not shrink on export — it* ***trivializes****, so no exported shape datum can distinguish the routes, ever. Consequently the* 0.0331562% *two-circulation evaluation does* not *factor through* P_{ann}*: were it to, it would be the certified baseline family at* $w' = 1.42120$*, yet it beats that family's certified global optimum by a factor* 3.954412*. Its gain lives in the unprojected complement.*

Novelty label: new specialization proved here.

Corollary 8.6 (AX-SEQ)**.** AX-SEQ is **killed as a distinct physical completion law**: on the exported carrier "sequential circulation" is the rescaling $2\varphi^{-3}L$, not a new shape. It survives only as that internal rescaling. The orientation bit is retired at 0 bits (the target-plane orientation torsor is canonically $\{+J, -J\}$).

8.3 The affine cover and the interface

Novelty label: new specialization proved here.

Theorem 8.7 (The ladder is the affine universal cover; A1432, `s911`)**.** *The ladder is* $\ell^2(\mathbb{Z})$ *with the affine-dihedral algebra* $\mathbb{R}[D_\infty]$ *(*$J_t = U^t J$*,* $J_t^2 = I$*,* $J_t U J_t = U^{-1}$*), lying* over *a pro-family of algebraic cyclic quotients.* $\mathbb{Z}$ *is neither the inverse limit nor a direct limit of the finite levels: the finite* C_5 *carrier is a* ***quotient level****, not an embedded submodule. Two corrections travel with this: the level data is a* family *until divisibility/compatibility arrows are specified, and periodization is algebraic-then-complete, not a bounded* ℓ^2 *map.*

Novelty label: new specialization proved here.

Theorem 8.8 (Interface obstruction with pointed rescue; A1432)**.** *There is no finite-to-ladder section (*$\mathbb{Z}$ *is torsion-free), no continuous section from the profinite side, and* $\operatorname{Hom}(\mathbf{1}, \ell^2(\mathbb{Z})) = 0$*; the marked evaluation* E_K*,* $K = \{3, 11, 17\}$*, is bounded but not translation-equivariant.* ***No unpointed affine-annular morphism creates the ladder source/evaluation map.*** *The rescue is* pointed*: in* $(H_{\mathbb{Z}}, [g_\rho], K)$ *with* $g_\rho(N) = \varphi^{-|N-\rho|}$ *the construction is legitimate at the price of one continuous registration coordinate* $\rho \in \mathbb{R}/20\mathbb{Z}$ *— but only inside the pre-declared golden ansatz; without it the boundary-state choice is infinite-dimensional.*

Novelty label: new specialization proved here.

Theorem 8.9 (The affine support theorem; A1432)**.** $\lambda(k) = -4\sin^2 k$*; the safe set is* $|\sin k| \le \varphi/2$ *with the golden boundary identity* $\varphi/2 = \sin(3\pi/10)$*;* $P_{\mathrm{aff}} = \mathbf{1}_{A_{\mathrm{safe}}}$ *in Borel functional calculus, with kernel* $p_0 = 3/5$*, vanishing odd coefficients and* $p_j = (1+(-1)^j)\sin(3\pi j/10)/(\pi j)$*; and* $P_{\mathrm{aff}}|_{C_5} = P_{\mathrm{ann}}$ *exactly — the finite theorem recovered from the continuum.*

Remark 8.10 (provenance, must travel)**.** Identifying P_{aff} as *the physical export* uses a maximal-positivity- domain clause: no continuous parameter, but a real physical attachment, stated as such.

Priced axiom 8.11 (AX-AFF-ONSET)**.** The affine-mirror class is $\omega_N(\mu_t) = [t] \in H^1(C_2, A_N^-) \cong A_N/2A_N$; fixed points exist iff $\omega_N = 0$ iff (N odd) or (t even). Fixed points push forward, so a commensurate level can never map onto an incommensurate lower one — and the banked pattern ($\omega_6 \neq 0$, $\omega_{n\geq 7} = 0$) is exactly where strict pro-functoriality breaks. **Price:** zero continuous dimensions, one $\mathbb{Z}_2$ cohomology defect, and one *integer marking* at $n^* = 7$ — not "one bit" until a finite candidate set is derived. **The level structure does not select seven** (onsets are placeable at 7, 8 or 9 by parity controls). This closes $r1$, standing since S229, as a priced input rather than a theorem.

The final kill of 0.0331562%**.** Legally exported and fully re-optimized over the pre-declared bands, the sequential family's minimax is 0.2744490308% (three-way equioscillation) — a factor 4.05454 better than the affine-safe baseline and a factor 2.74449 *above* the frozen 0.1% bar. At the old unprojected optimum, ≈36.8% of the output energy lies outside the safe carrier. The 0.0331562% row is dead as exportable physics: a second, final kill at the strictly larger affine tier, with located cause. The 0.2744490308% row stands in the miss column — it is a legal miss, not a rescue.

8.4 Non-selection, and AX-MG as a genuine axiom

Novelty label: new specialization proved here.

Theorem 8.12 (Non-selection; A1433, `s912` 33/33 plus a pre-registered prebuild 27/27)**.** *For every $0 < r < 1$, $(L - E(r))g_r = -\alpha(r)\delta_\rho$ with $E(r) = (1-r)^2/r$ and $\alpha(r) = (1-r^2)/r$: each r is the unique positive bound-state ray of a natural pointed rank-one attachment.* ***No banked spectral mechanism distinguishes*** *φ^{-1}. The three candidate mechanisms die individually: the window edge (a zero of f, not a fixed point), the transfer fixed point (only $\lambda = 0$), and the ray constant (an overall transfer scale, untouched by the α-family). Independently, the house pre-registered coupling collapse $c(\kappa) = 2\sinh\kappa$ is the same statement.*

Novelty label: new specialization proved here.

Proposition 8.13 (Positivity does not select)**.** *The family $h_a = |\sin(an)/(\pi n)|^2$ is positive, reflection-even and exactly safe-supported, with Gram rank 8/8 and unbounded a-family: the admissible cone is* ***infinite-dimensional*** *even under strict safe support. The positivity-uniqueness route is closed permanently. A typing correction travels with it: $\hat{g}_r(k) > 0$ everywhere, so the golden source itself is not safe-supported; the pointed architecture is necessarily $g_\rho \in \ell^2(\mathbb{Z})$ with P_{aff} acting* downstream.

Priced axiom 8.14 (AX-MG — normalized Markov-geodesic boundary coherence; A1433–A1435)**.** *One unit affine transfer ≡ one complete normalized Fibonacci cap-cup/Jones interface; boundary amplitudes multiply along the unique rooted geodesic.* [P-004] Provenance verified at primary source: the mounted results carry complete-interface weight $\sqrt{5}/2 - 1/2 = \varphi^{-1}$ and isolated cap $\varphi^{-1/2}$ exactly. From $d_\tau^2 = d_\tau + 1$ one gets $d_\tau = \varphi$ unique positive, composition forces $g_\rho(N) \propto \varphi^{-|N-\rho|}$, and the spectral point φ^{-3}, unit coupling and $\|g\|^2 = \sqrt{5}$ become corollaries. **Final Rev28 wording, two clauses:** COEFFICIENT-ONE (one positive real modulus c fixed to 1) + DEGREE-ONE (one integer $m \geq 1$ fixed to 1); zero new numerical constants; only $w = c\varphi^{-m}$ is scalar-visible.

Novelty label: new specialization proved here.

Theorem 8.15 (Both clauses are independent; A1434 `s913`, A1435 `s914`)**.** Degree. *The countermodel pair — A (one interface, $w = \varphi^{-1}$: $\varepsilon = \varphi^{-3}$, $\alpha = 1$, $\|g\|^2 = \sqrt{5}$) and B (two serial interfaces, $w = \varphi^{-2}$: $\varepsilon = 1$, $\alpha = \sqrt{5}$, $\|g\|^2 = 3\sqrt{5}/5$) — are both coefficient-one banked words, both positive, localized, covariant and support-compatible; no theorem formed from the banked rows selects A over B.* Coefficient. *The pair A ($m = 1$, $c = 1$) and B ($m = 1$, $c = \frac{1}{2}$) agree on normalized trace, quotient/support factorization, internal interface weight and the -1 acceptance; B fails only* edge *trace-preservation, which was never banked — requiring it inserts the clause itself. The general-c calculus is exact ($w = c\varphi^{-1}$, $\varepsilon = c/\varphi + \varphi/c - 2$, $\alpha = \varphi/c - c/\varphi$, $\|g_c\|^2 = (\varphi^2 + c^2)/(\varphi^2 - c^2)$, localization exactly $0 < c < \varphi$), and the golden rows are* equivalent *to $c = 1$, not selectors — so "banked values select c" is permanently blocked.*

Novelty label: new specialization proved here.

Theorem 8.16 (The reduction theorem; A1435)**.** *In the $M_2(\mathbb{R})$ model with the diagonal expectation, $T_c = c\mathsf{E}$ is unital $\iff$ trace-preserving $\iff$ idempotent $\iff$ $c = 1$ — and* none *of those edge properties is banked. They are restatements of the coefficient-one clause, not weaker derivable facts. Any future derivation must produce one of these edge properties from a genuinely new tier.*

Novelty label: new specialization proved here.

Theorem 8.17 (Lift degeneracy; A1435, `s914`-X10)**.** *$(m, c) = (1, \varphi^{-1})$ and $(2, 1)$ share the scalar character $w = \varphi^{-2}$: scalar-character data alone cannot separate degree from coefficient.* ***Any future boundary-measurement claim must state which lift it sees.*** *This is the first theorem of the observation-functor faithfulness wall (Appendix T, Appendix X).*

The boundary question, closed at this tier. The reclassification ran: derive g_ρ (done, conditionally) → derive AX-MG (closed by independence, both clauses) → *accept and extract.* Nothing derivation-shaped remains at this tier. The boundary ledger reads: **AX-MG (two independent clauses, priced, dated) + ρ (continuous) + AX-AFF-ONSET (integer marking)**. The scalar shadow is real but axiom-fenced: the coefficient-one clause is load-bearing *by declaration*, honestly typed.

8.5 The conditional operator law, and what it is not

Novelty label: new specialization proved here.

Proposition 8.18 (L_{eff}, correctly labelled)**.** *$f(\lambda) = \lambda + \varphi^{-2}\lambda^2$ is positivity-compatible exactly on $-\varphi^2 \le \lambda \le 0$; on the two-step C_5 carrier the violating modes are precisely the categorically absent $q = \pm 1$. On the affine-safe carrier* $\operatorname{spec}(L) = [-\varphi^2, 0]$ *is continuous, so there is* no *ray collapse and $L_{\text{eff}} = L + \varphi^{-2}L^2$ is nontrivial for the first time on an exported carrier. The completed source lives on the fixed three-site set $\{\rho - 1, \rho, \rho + 1\}$ and $[P_{\text{aff}}, L_{\text{eff}}] = 0$, so the identity descends.*

Label (binding, A1436 keep #4). L_{eff} is an *exact conditional operator law* and a *dynamics-tier candidate pending a semigroup/positivity test.* It must not be printed as "dynamics" unqualified: nothing banked tests positivity-preservation of $e^{tL_{\text{eff}}}$. The law is conditional on AX-MG, value-open, and behind the `s900` wall; the golden source/evaluation *shape* is dead. χ has no value.

8.6 Carried rows from the S248 queue

Remark 8.19 (merged axiom clause and the null axis)**.** S_{shape} and AXIOM-K1 merge into a single conditional clause, consequence-equivalent *on the positive cone only.* The evaluation normal $n = (5, -18, 1)$ spans $\ker J$; the kernel-axis candidate is dead twice (evaluation-null and rescaling), and $[j_A] \in \mathbb{RP}^2$ stands off shell — collapsing to the order line only at the evaluated tier (Appendix T, Theorem 8.1 notwithstanding: the collapse is an attachment, not a derivation).

Remark 8.20 (quarter-turn descent)**.** The V-side quarter-turn $z_- = \pm i z_+$ descends under D_5 parity (Hom = 1+1) and survives inside AX-DIH, whose $\mathbb{Z}_{20}$ compatibility is exact ($H^2 = I$, $HKH = K^{-1}$, $HMH = M^{-1}$, $H(-I)H = -I$): the D_5 extension is clock-legal and the one-central--1 bank is untouched. The D_5 reflection is a normalizer involution, typed distinct from the central $-I$ — not a fifth costume.

Remark 8.21 (shape diagnostics, unpromoted)**.** The 0.131113% near-bar two-parameter circulation family and the φ^5 no-fit obstruction remain value-open remarks behind the `s900` wall; nothing is scored. The banked-period-70 two-parameter harmonic remains a *watch* item, adoption refused.

Remark 8.22 (map erratum carried in)**.** The S248 map's "one gluing datum" line is superseded: there are two independent continuous data at that tier. See also Appendix T, where the gluing question itself is retyped — the sheets are not glued at the F_4 tier at all.

9 The selector production arc: eigenspaces, weights, and the priced action shape (Rev31; S278–S279, A1514–A1516)

The arc in one sentence (A1514→A1515→A1516). The selector Q_T — previously a loaded object — is now grammar-derived in stages: its *eigenspaces* are forced at zero price by the loaded plane's own transport-defect algebra (A1514, Tier-B unconditional; Appendix E §on the forced interior split); its eigenvalue *weights* are the Hessian of a rank-normalized carrier-character action, every constant a rank (A1515, the variational representation theorem below); and its nonlinear *action shape* is **not** grammar-selected — a four-dimensional moduli space survives, with an explicit, minimal, non-intrinsic selecting clause adopted at S279 as a *declared structural principle* (A1516). After this arc the selector sector's loaded content is exactly: the standing 10+1 census, and the adopted principle. Nothing else. Conditional tier throughout; no observable moves.

9.1 The rank-character variational theorem (A1515)

Novelty label: new specialization proved here.

Theorem 9.1 (carrier character and the selector Hessian; A1515, house `s1052`)**.** *Let $P_B = i_B i_B^{\mathsf{T}}$ be the loaded plane, $\{E_a\}$ the Frobenius-orthonormal $\mathfrak{p}$-basis, and $\Pi_1, \Pi_{32}, \Pi_{36}, \Xi = \Pi_{10} - \Pi_{26}$ the zero-price joint-defect projectors of A1514. Then the carrier character of the loaded plane is diagonal in the joint projectors,*

$$R_B^{\text{char}}(a,b) \;=\; \operatorname{tr}\big[P_B\,(E_aE_b + E_bE_a)\big] \;=\; \tfrac{1}{2}\,\Pi_{32} + \tfrac{2}{3}\,\Pi_{36} \qquad [\textit{exact}],$$

and the bar-legal action

$$F_{\mathrm{char}} \;=\; \tfrac{12}{54}\,\mathrm{tr}\big[P_B\,(M + M^{-1} - 2I)\big] \;+\; \tfrac{1}{54}\,\langle y, \Xi\, y\rangle \;+\; \tfrac{1}{2}\,\langle y, \Pi_1 y\rangle, \qquad M = e^{2Y},$$

has its critical point at the base with $\nabla^2 F_{\mathrm{char}}(I) = Q_T$ exactly *(house central finite differences of the actual nonlinear functional, maxdev* 7.6×10^{-8}*, the fd floor). [LIB2-026] Every constant is rank-traced:* $16/27 = (24/27)(2/3)$*,* $1/27 = 1/r_+$*,* $4/9 = (24/27)(1/2)$*, with* $16 = \mathrm{tr}\, P_B$ *entering as the* derived *consistency* $24 \cdot \frac{2}{3} = 16$*, not as an inserted numeral. [LIB2-026]*

The fence adopted with the theorem (A1515, verbatim posture): no microscopic principle selects F_{char}; the mathematics is Derived/Certified, the attachment Structural/Conditional. [LIB2-026] That fence is exactly what §9.4 prices.

9.2 The two tens are two, and the intertwiner numeral is derived (A1515; house `s1053`)

Novelty label: new specialization proved here.

Theorem 9.2 (declared-class negative; A1515 ASK-3)**.** *The boundary ten (the tangent module of the priced ten-continuous boundary object of Appendix E's closure section) and the selector ten (the* Π_{10} *sector) are* inequivalent K_{dyn}*-modules: their Casimirs are* $\frac{1}{4}I$ *and* $\frac{1}{6}I$ *exactly, so* $\mathrm{Hom}_{K_{\mathrm{dyn}}} = 0$ *by Schur (house* `s1052`*, deviations* $\le 7 \times 10^{-16}$*). [LIB2-017] The dimension match identifies* nothing*; any future boundary↔selector bridge must be nonlinear or symmetry-breaking,* ***and priced****. [LIB2-017]*

Novelty label: new specialization proved here.

Theorem 9.3 (the obstruction size is the selector's own Casimir; house `s1053`)**.** *The stacked intertwiner equation of Theorem 9.2 has Gram matrix equal to the* Casimir of the Hom module*; its singular-value spectrum is therefore exact,* $\mathrm{sv}^2 \in \{\frac{1}{6}^{(10)}, \frac{1}{4}^{(20)}, \frac{1}{2}^{(70)}\}$*. The boundary ten is the* adjoint *module of* K_{dyn} *(bracket closure* 4.6×10^{-15}*; equivalence intertwiner residual* 4.7×10^{-16}*, invertible), so* $\mathrm{Hom}(\mathrm{ad}, B)$ *canonically contains one selector-ten copy (*$k \mapsto \rho_B(k)$*), and the minimal singular value is*

$$\mathrm{sv}_{\min} \;=\; \sqrt{\mathrm{Cas}_{\mathrm{sel}}} \;=\; \tfrac{1}{\sqrt{6}} \qquad [\textit{derived}].$$

The failure of the bridge is by exactly the amount the selector ten turns under K_{dyn}*.* Flagged, not asserted, one level up: *the numeral equality* $\mathrm{Cas}_{\mathrm{sel}} = \frac{1}{6} = \kappa^2_{\mathrm{clock}}$ *is recorded untyped; no claim attaches.*

9.3 The adjoint wall and the quadratic-bridge census (Rev32; S283, A1523 V6/V7, D1210 return §4; two-host byte-certified)

The boundary ten's module structure is now censused exactly, and the census is a wall: the adjoint end-structure is **rigid** — $\mathrm{End}_{K_{\mathrm{dyn}}}(\mathrm{ad}) = \mathbb{R} \cdot I$ (Schur, End dimension 1), the invariant metric is unique up to scale, there is *no* invariant symplectic form on the adjoint, no invariant Sym^2 self-map beyond the identity class, and the bracket is the unique invariant antisymmetric product. The quadratic bridges between the physical/selector ten P and the boundary ten B are counted exactly: $\dim \mathrm{Hom}_{K_{\mathrm{dyn}}}(\mathrm{Sym}^2 P, B) = 1$ and $\dim \mathrm{Hom}_{K_{\mathrm{dyn}}}(\mathrm{Sym}^2 B, P) = 2$ — so quadratic (symmetry-respecting, nonlinear) bridges exist and are few, and any future use of one must name which of the counted copies it takes and price the choice. [LIB2-089, LIB2-090] Every ten-claim names its ten (the Rev32 carrier fence, Appendix X, clause-27 side).

9.4 The selection obstruction, and the adopted principle (A1516; house `s1054`–`s1055`)

Novelty label: new specialization proved here.

Theorem 9.4 (the action shape is not grammar-selected; A1516, WIN-OBSTRUCTED)**.** *In the declared rank-normalized character-completion class (declaration hashed before analysis; target quarantined until the verdict step), the solution set of "critical point at the base + positive-semidefinite Hessian" modulo equivalence is a four-dimensional moduli space,*

$$\mathcal{S}^\circ/\mathrm{Eq} \;\cong\; \mathbb{R} \times \operatorname{int}\Delta^3,$$

three Hessian-weight ratios plus one nonlinear modulus invisible to the complete two-jet. [LIB2-027] (Scope, Rev32.9, A1632: $\mathcal{S}^\circ$ is the regular stratum, where the Hessian is positive definite; the positive-semidefinite boundary strata, where a Hessian weight vanishes, are the faces of Δ^3 and are not included.) The two-jet map has rank 4 with exact kernel $k_{\mathrm{jet}} = (1, 0, -\frac{4}{9}, -\frac{16}{27}, 0)$; its integral $\mathcal{N} = \tau - \frac{1}{2}\langle y, H_\tau y\rangle$ has vanishing two-jet, quartic leading term $\frac{8}{27}\operatorname{tr}(P_B Y^4)$, and $\mathcal{N} \geq 0$ (one-line proof: $2\cosh x - 2 - x^2 \geq 0$ with P_B and $f(Y)$ both PSD). Hence $F_s = F_{\mathrm{char}} + s\,\mathcal{N}$ is a continuous family of inequivalent bar-legal actions all *with second variation exactly Q_T (house `s1054`, fd floor):* ***exact Hessian agreement does not select the nonlinear action****. [LIB2-027] Categorically, the two-jet functor has affine-line fibres and no natural section in the declared condition family; F_{char} is neither initial nor terminal and represents no grammar-defined functor.*

Priced axiom 9.5 (the adopted structural principle Φ_{diag}; PI adoption S279)**.** The clause Φ_{diag} : $\{v = 0,\ w = 0,\ u = t,\ 27z = t\}$ — four coefficient relations — isolates exactly the F_{char} orbit and is minimal in codimension (dropping any one relation restores a projective one-parameter family). [LIB2-028] It is *not intrinsic*: it identifies the units of three inequivalent primitive generator lines (inequivalent by Theorem 9.3's Casimir separation), and independent generator rescaling moves the selected action. [LIB2-028] It is adopted here as a **declared structural principle** — the named form of the previously implicit choice that nature's action is F_{char}-shaped. [P-007] *Sharpening (house `s1055`):* the principle is a principle *of the minimal-word class.* Under one-word enlargement ($\tau_2 = c\operatorname{tr}[P_B W^2]$, which has no quadratic jet at all) the moduli grow $4 \to 5$ and Φ_{diag} requires the further relation $t_2 = 0$; the obstruction of Theorem 9.4 is *monotone* under class enlargement. The full adopted principle is therefore: **word minimality, plus the four diagonal relations**. [LIB2-028] This is a typing change, not a census change: the alternative price is the four-dimensional moduli, and the standing 10+1 census is untouched either way. *Rev32 status:* the remaining question — whether a *refined* condition family could derive these relations rather than adopt them (F-SEL(c)) — is **closed negative, scoped** (D1220→A1533): the lead candidate, Casimir-naturality, clears the circularity bar and *excludes* Φ_{diag}; no banked refined family selects it non-circularly. The adoption therefore stands at exactly the printed price, and the closure's scope clause and reopen condition are stated in Appendix E (Rev32 addendum, "the selection frontier").

Bottom line of the production arc. Eigenspaces: forced, zero price (A1514). Weights: a variational representation theorem, every constant a rank (A1515). Nonlinear shape: obstructed — a 4-moduli space with an adopted four-relation principle, minimal and explicitly non-intrinsic, monotone under enlargement (A1516; `s1054`–`s1055`). The two tens do not identify (Casimirs $\frac{1}{4}$ vs $\frac{1}{6}$), and the obstruction between them is the selector's own Casimir, $1/\sqrt{6}$, derived. The selector sector's loaded content after this arc: the 10+1 census + the adopted principle. The forbidden sentence stays forbidden; nothing is promoted; no observable, tier, engine value, or clock claim moves.

Leibniz Quantum Beats Newton

Appendix T — The Cartan Carrier, the Observation Functor, and the Record (Rev33.2)

Rev28. The D1133–D1146 arc (assessments A1437–A1449; house verifiers `s915`–`s929`, every one ×2 byte-identical). Fourteen rounds in which the geometry spine was separated into two operators and one annulus, the transition cocycle was derived at zero continuous parameters, a four-dimensional Cartan carrier was closed inside the Albert signature sector, a reflection-positive state was constructed at finite regulator, and — in the closing five rounds — the physical observation functor was explicitly built, its price reduced to two zero-dimensional clauses, and the actuality row split into a derived record type and a realized value proved to be sample data. Nothing here promotes an observable; α and colour stay fenced; $V_{\mathrm{CKM}} = I$; the engine is unchanged; KILL-NODE 2 remains fired off shell and for the full microscopic loading.

Tom O'Sieg

August 2026

Posture (binding). This appendix is a *ledger of typed objects*, not a promotion. Every row below carries the tier its assessment gave it and no more: THEOREM rows are machine-exact and independently reproduced ×2; DERIVED-CONDITIONAL rows name the clause they hang on; CANDIDATE rows are named and priced but not adopted. Two general fences travel with everything here. (i) **The observation-functor faithfulness wall**: the question "which loaded choices can any admitted observable distinguish?" has two independent theorems on the books and no computed kernel; until that kernel exists, no scalar bit-or-dimension total is available for this sector, and none is printed (Appendix X). (ii) **The standing rule**: no χ-completion or χ-expression round runs against the `s900` ratio vector until a physical generator and response map exist; the route is generator → semigroup/two-point → normalized response → χ, or nothing. χ is VALUE-OPEN throughout this appendix, and every $\chi^2 = 1/4$ statement below lives strictly inside a named attachment.

Contents

1 Scope: what this arc did and did not do **1**

2 The geometry spine: two operators, one annulus **2**

3 The transition cocycle and the corrected object **2**

4 The Cartan carrier **4**

5 The fifth direction, the parent action, and the solder **4**

6 Residuals: real structures, the spin–charge no-go, placement **5**

7 The state and the measure **6**

8 The frontier round: AX-COT, positivity, envelope, actuality **8**

9 The three-price closure **11**

10 The observation functor $\mathcal{O}_{\mathrm{MOS}}$ **11**

11 The four residuals **13**

12 Actuality: the record type is derived, the value is data **14**

13 Price ledger and open frontier **15**

14 A Clifford isotropy law and a braid representation on V_{56} (Rev30, A1499) **15**

14.1 The universal isotropy law 16

14.2 The braid representation, and the fence around it 16

15 The carrier arc: from the Albert solder to the remaining physical attachment (Rev32.2 fold, A1553–A1561) **17**

15.1 The solder and transported pointing 18

15.2 Why the invariant odd covector failed, and what replaced it 19

15.3 The MOS channel, the score solder, and the normalization no-go 20

15.4 The declared amplitude premise (PI ruling S300) and the two alternatives not adopted 20

15.5 The intrinsic-basepoint placeholder (declared, no subgroup selected) 21

15.6 State after the fold 21

1 Scope: what this arc did and did not do

Appendix S closed the discrete half of the parent-action wall as a priced ledger. Appendices R and S left three things open that this arc addresses: the geometry spine (how many bulks, how many charts, what "W" is), the dynamics/measure floor (does a reflection-positive state exist at all), and the observable attachment (what functor takes a substrate object to a number a reader can see).

The arc did not derive a mass, did not move an observable, and did not un-fire KILL-NODE 2. What it did is narrower and, we think, more useful: it replaced a family of informal identifications with named operators, computed their relations exactly, and reduced the price of the observable

attachment to two zero-dimensional clauses plus one realized record — while proving, at each step, that the remaining freedom is not continuous.

2 The geometry spine: two operators, one annulus

Novelty label: new specialization proved here.

Theorem 2.1 (The two-W separation; A1437, `s915` $27/27 \times 2$)**.** *The programme's informal label "W" denoted two categorically different operators. [LIB2-255]*

$$W_{\text{sig}}\, D_{KK}\, W_{\text{sig}}^{-1} = +D_{KK}, \qquad S_{\text{shell}}\, D_{KK}\, S_{\text{shell}}^{-1} = -D_{KK}.$$

W_{sig} is the compact/split Wick chart *(order 4, Λ^2-induced, commuting with the grading, $\Sigma = W^2$ real ± 1); S_{shell} is the shell or grading-*orientation *reversal ($27_{+2} \leftrightarrow 27'_{-2}$, $1_{+6} \leftrightarrow 1_{-6}$), anti-symplectic, with $SWS = W^{-1}$.*

Remark 2.2 (terminology, binding)**.** The single label "W" is retired suite-wide in favour of the pair. [LIB2-255] This was a house nomenclature casualty, lane-caught (S251); it is recorded as such in the casualty ledger of Appendix X.

Novelty label: new specialization proved here.

Theorem 2.3 (No third cascade; A1437, three independent proofs)**.** *The τ-conjugates of D_{KK} are all equal. Triality organizes the $8_v/8_s/8_c$ fibre; it does not generate a second KK grading. The A_4 completion is **refuted** in both realizations: at the Wick tier τ fixes $\{\Sigma, S, \Sigma S\}$ pointwise, giving $V_4 \times C_3$ (order profile 1/3/2/6, not A_4's 1/3/8); at the Jordan tier τ_P permutes slot involutions instead. Keeping the order-4 chart, $\langle W, S\rangle$ closes at order 8, so the precise atlas home of the two charts is $D_8 \times C_3$; at the Jordan tier the group is $\mathbb{Z}_2 \times S_3$. These are different generator sets, not a contradiction.*

Remark 2.4 (the honest rider)**.** An A_4 completion is *constructible* only by adding a new cross-layer identification (triality $\leftrightarrow$ sheet flips) — a separately priced attachment, not a consequence of D_4 triality.

Novelty label: new specialization proved here.

Theorem 2.5 (One radius; A1437, `s915`-X5)**.** *There is a unique symmetry-fixed radius ray: multiple charts with one shared radius is the only symmetry-compatible reading. Literal multi-bulk readings are priced at up to $+5$ unforced continuous moduli (extra unforced moduli 1/2/5/3 for 2/3/6/4 copies).*

3 The transition cocycle and the corrected object

Novelty label: new specialization proved here.

Theorem 3.1 (The cocycle, derived at zero continuous parameters; A1438, `s916` $24/24 \times 2$, house prebuild $14/14 \times 2$ pre-registered)**.** *$g_{+-} = \Gamma_J = W^2$, the Cayley–Dickson grading involution $\Gamma(p, q) = (p, -q)$: involutive, an $\mathbb{O}_s$ algebra automorphism, a cubic-norm automorphism, and*

commuting with D_{KK}, with the shell reversal, and with Peirce triality. It is identically the house's pre-registered candidate g_{split} (trace fingerprint $\text{tr}_{27} = +3$ against slot class -5), named before the round ran.

Novelty label: standard theorem used.

Proposition 3.2 (The Albert splitting)**.** $27 = 15_+ \oplus 12_-$ *with* $J_{15}^{(+)} = J_3(\mathbb{H})$ *and* $J_{12}^{(-)} \cong (\mathbb{H}\ell)^3$*; per Peirce sector* $9 = 5_+ \oplus 4_-$*.*

Novelty label: new specialization proved here.

Proposition 3.3 (Čech verdict)**.** *All* 216 *six-chart cocycle equations pass and the cocycle is an explicit coboundary; the transition group is* $C_2 \times C_3 \cong C_6$*, and* Γ *lies in the identity component via a real unit-quaternion path, so* $[g] = 0$*: there is no automatic principal-F_4 nontriviality. A* reduced flat $\mathbb{Z}_2$ *(structure group cut to* $\{I, \Gamma\}$*) survives as one discrete clutching class — possible, but not locally selected, and priced separately. [LIB2-256]*

Novelty label: standard theorem used.

Theorem 3.4 (The non-gluing obstruction; A1438, `s916`-V7)**.** *The restricted norm inertias are* $\mathbb{H}_c : (4, 0)$ *and* $\mathbb{H}_s : (2, 2)$*, so by Sylvester no norm-preserving automorphism carries $\mathbb{H}_c$ to $\mathbb{H}_s$: the det-sheet and KV-sheet constructions* ***cannot*** *be glued by an ordinary real frame-preserving $F_{4(4)}$ transition. "Gluing the sheets" was a category error at the F_4 tier.*

Named datum 3.5 (The corrected object $\mathfrak{B}_W$)**.** $\mathfrak{B}_W = (J_3(\mathbb{O}_{\mathbb{C}}), \kappa_s, \kappa_c, W)$ with $\kappa_c = W\kappa_s W^{-1}$ and $W^2 = \Gamma_J$: a *real-form correspondence*, not a nontrivial bundle. The two signature presentations are two real structures in one complexified Albert fibre; the κ_c-fixed locus carries the positive-definite norm $|p|^2 + |r|^2$. [LIB2-256] **Bibundle, not bundle** — the earlier "nontrivial bundle" reading is superseded. [LIB2-256]

Novelty label: new specialization proved here.

Theorem 3.6 (The faithfulness wall, second derivation; A1438, `s916`-V9)**.** $\Pi_{\text{even}}\Gamma_J = \Pi_{\text{even}}$ *and* $P_-\Gamma_J = -P_-$ *exactly: the transition is* invisible *to any readout retaining only the* 15*-dimensional even sector. Distinguishability requires one of four named new structures — a moving physical projector, a* 15/12*-mixing connection, an odd-component boundary state, or a frame-moving E_6 map.*

Remark 3.7 (overdetermined)**.** Theorem 3.6 is the observation-functor faithfulness wall of Appendix X, derived a *second*, independent way: once as an architecture argument (from lift degeneracy, Appendix S) and once as an exact projector computation. The head-wall consensus is overdetermined.

Remark 3.8 (accounting correction)**.** Local overlap ≠ global holonomy ≠ Real-form correspondence ≠ physical observation map. [LIB2-257] Charging these as one "W bit" was a conflation; the row is retired and replaced by the four-structure accounting entry. [LIB2-257]

4 The Cartan carrier

Novelty label: new specialization proved here.

Theorem 4.1 (The Cartan closure; A1439, `s917` $28/28 \times 2$)**.** $\mathrm{tr}(X_a X_b) = -18\delta_{ab}$, $[X_a, X_b] = 3M_{ab}$, *with the exact* $\mathfrak{so}$*-relations for* $[M, X]$ *and* $[M, M]$*; the closure has dimension* 10 *and invariant trace inertia* $(0, 10)$*, hence* $\mathfrak{so}(5)$*: an* $S^4 = SO(5)/SO(4)$ *Cartan geometry inside the Albert signature sector. Consistent index-set rotations give closed real spans with inertias* $(4, 6) \Rightarrow \mathfrak{so}(4, 1)$ *and* $(6, 4) \Rightarrow \mathfrak{so}(3, 2)$*.*

Remark 4.2 (the AdS branch is priced)**.** The $\mathfrak{so}(3, 2)$ reading is a *conditional import* of the banked $\mathfrak{sp}(4, \mathbb{R}) \cong \mathfrak{so}(3, 2)$ seed (no new fit). Absent the import, dS-vs-AdS is one discrete branch, priced. [LIB2-338]

Novelty label: new specialization proved here.

Proposition 4.3 (Π_{sig}, the carrier projector; `s917`-C2)**.** $\Pi_{\mathrm{sig}} = \frac{1}{6}(I - \Gamma)(I + \tau + \tau^2)$ *is an exact projector of rank* 4 *with* $\Pi\Gamma = -\Pi$ *and* $\Pi\tau = \Pi$*: the unique rank-*4 *joint isotype, signature-odd and triality-neutral.*

Novelty label: new specialization proved here.

Proposition 4.4 (Carrier metric and connection family; `s917`-C1/C3)**.** B *has inertia* $(15, 12)$*;* $h_\Gamma = B \circ \Gamma$ *is symmetric with inertia* $(27, 0)$*;* $B(u_a, u_a) = -6$ *and* $h(u_a, u_a) = +6$*. For* $x \in J_-$ *the operator* L_x *is cubic-compatible,* Γ*-odd,* h_Γ*-skew and* τ*-covariant; the family has dimension exactly* 12 *with a* 4*-dimensional* τ*-neutral sector.*

Remark 4.5 (a guard against retrofit)**.** Ghost-safety *alone* leaves a 180-dimensional family. What selects the carrier metric is *character naturality*, not positivity. This is stated so that no reader reads the metric as chosen for its signature.

Novelty label: new specialization proved here.

Theorem 4.6 (The phase-only no-go; `s917`-C6)**.** *The Albert cubic splits into* 29 *odd-degree-*0 *and* 60 *odd-degree-*2 *monomials with no odd-parity terms, so a relative phase rescales the two parts differently: the phase-only bridge is not cubic-compatible without a priced weight-*(-2) *compensator field.*

5 The fifth direction, the parent action, and the solder

Novelty label: new specialization proved here.

Theorem 5.1 (Branching and the canonical fifth direction; A1440, `s918` $36/36 \times 2$, prebuild $38/38 \times 2$)**.** $27 \downarrow \mathfrak{so}(5) = 6 \cdot 1 \oplus 5 \oplus 16$ *with exact Casimir spectrum* $(0^6, 4^5, (5/2)^{16})$*. The spin-factor relations* $u_a \circ u_b = 0$*,* $u_a \circ u_a = n$*,* $n \circ n = -3n$ *give*

$$e_5 = -\tfrac{1}{3}n = -\frac{2(u \circ u)}{h_\Gamma(u, u)},$$

coordinate-free and u*-independent, and* $\mathcal{V}_5 = \mathbb{R}e_5 \oplus E_{\mathrm{sig}}$ *is the spin factor* $J(Q_4)$*:* ***one*** *canonical* $5D$ *carrier.*

Terminology fence (binding). e_5 is an *internal Cartan fifth direction.* It is **not** a derived spatial circle and not a numerical KK radius. No register-C (rest-mass ladder) semantics attach to it: the H5 bridge is not banked, and nothing in this appendix supplies it.

Novelty label: new specialization proved here.

Theorem 5.2 (Triality is the imaginary unit; signature is chirality; `s918`-C5)**.** *$J_\tau = (\tau - \tau^2)/\sqrt{3}$ satisfies $J_\tau^2 = -I$; with $\gamma_a = 2J_\tau P_a$ one has $\{\gamma_a, \gamma_b\} = 2\delta_{ab}$ and $\gamma_0\gamma_1\gamma_2\gamma_3 = \Gamma$ exactly, and $\frac{1}{4}[\gamma_a, \gamma_b] = -M_{ab}$. The signature involution* is *chirality. Both Lorentz conventions are internal.*

Novelty label: new specialization proved here.

Proposition 5.3 (Metric and orientation are trace-derived; `s918`-C4)**.** *$\mathrm{tr}(\gamma_a\gamma_b)/16 = \delta_{ab}$; $\mathrm{tr}(\Gamma\gamma\gamma\gamma\gamma)/16 = \epsilon_{abcd}$ over all 24 permutations with repeated-index controls; $\mathrm{tr}(\Gamma M_{ab}M_{cd}) = 4\epsilon_{abcd}$; $\mathrm{tr}(\Gamma P_a P_b) = 0$. No epsilon tensor is inserted by hand.*

Named datum 5.4 (The signature-sector parent action — CANDIDATE)**.** Given those traces, $S_{\mathrm{MM}} = \frac{1}{4g^2}\int \mathrm{tr}(\Gamma F \wedge F)$ is MacDowell–Mansouri, yielding Einstein–Hilbert plus a cosmological term after the Euler discard, with $\kappa = g^2\ell^2/2$ and $\Lambda = -3\epsilon/\ell^2$; the Einstein–Cartan equations follow, torsion algebraically sourced. [LIB2-168] **Scope fence (attested in-return):** this is a parent action for the signature/Cartan sector — *not* the full particle/generation/gauge/mass action. [LIB2-168] It does not un-fire KILL-NODE 2 and does not rehabilitate the D1125 parent action. The action assembly is form-calculus tier; the traces are machine-exact in both lanes. [LIB2-168]

Novelty label: new specialization proved here.

Proposition 5.5 (The scale is not derivable; `s918`-C8)**.** *$(e, \ell) \mapsto (\lambda e, \lambda\ell)$ fixes both $\mathcal{A}$ and $\mathcal{F}$: the dimensionless algebra **cannot** determine $|\ell|$. ℓ registers to the suite's one banked Bargmann/radion class — no second scale is introduced — and g is one new dimensionless coupling, priced.*

Named datum 5.6 (SOLDER-BRANCH — argument tier, premise flagged)**.** $e = 0$ is a stationary branch of every action in the source-free local Lorentz-invariant polynomial class (e-degrees 0/2/4 only; no invariant linear in e). **House audit, must travel:** the classification premise is encoded as a data table in the lane kernel, not machine-proved; it is standard invariant theory and the variational step given it is elementary, but the grade is *argument-tier*, not kernel-exact. Priced escape menu: a declared nondegenerate sector, a global/topological sector, a source, or a non-polynomial barrier.

Novelty label: new specialization proved here.

Theorem 5.7 (Faithfulness retypes the solder problem; A1441, `s919` $22/22 \times 2$)**.** $\ker \sigma_D = 0 \iff \mathrm{rank}\, e = 4$. *The polynomial no-go stands unchanged; what changes is the typing: $e = 0$ is stationary in the connection space but is **not an object of the faithful Dirac/observation category**. The honest residue — why the physical measure is supported on the faithful sector — is a dynamics/measure question, exactly where the programme's floor already sits. Zero new continuous price.*

6 Residuals: real structures, the spin–charge no-go, placement

Novelty label: new specialization proved here.

Proposition 6.1 (Branch real structures; `s919`-V1/V2)**.** *All three slot reflections restricted to S_{16} are exact Real structures ($R^2 = I$, fix-dimension 8, $RJ_\tau = -J_\tau R$, $R\tau R = \tau^{-1}$, $RP_aR = -P_a$,*

$RM_{ab}R = M_{ab}$*). For the active* c_3 *branch,* $\mathrm{Fix}(R_{c_3})$ *is the minimal* $\mathrm{Cl}(4,0)$ *module with commutant* $\mathbb{H}$ *(proved specifically* $\mathbb{H}$ *house-side: centre* 1 *plus traceless-squares-negative; a dimension count alone would admit* $M_2(\mathbb{R})$*). Hence the two-complex-Dirac reading is the complexification of* one *quaternionic real spinor.*

Priced axiom 6.2 (The branch Real clause)**.** $R_{c_3}\Psi = \Psi$. Discrete. It retires the $\mathbb{CP}^1$ *continuous* multiplicity price. **Pricing fence:** the clause inherits the loaded status of the B-map c_3 branch — without a branch the three conjugations are an S_3 torsor.

Novelty label: new specialization proved here.

Proposition 6.3 (The OS/Pin reflection exists in-substrate; `s919`-V3)**.** $\Theta = \gamma_0 R_{c_3}$, $\Theta^2 = I$, *signs* $(+,-,-,-)$*. Free-fermion reflection positivity remains a* conditional import *(a background and a positive mass operator are still required).*

Novelty label: new specialization proved here.

Theorem 6.4 (The spin–charge no-go; A1441, `s919`-V5/V6)**.** *The per-chirality complex-linear commutant is* $M_2(\mathbb{C})$ *on each side, so the multiplicity is* $\mathbb{C}^2$ *per Weyl half; a commuting nontrivial* $SU(3)$ *needs dimension* ≥ 3*.* ***Colour cannot act on the Cartan spinor.*** *The tempting single-carrier reading* $S_{16} \stackrel{?}{=} J_{13} \oplus J_{23}$ *is dead, and direct spin–charge locking is refuted.*

Remark 6.5 (this is colour-protective, not colour contact)**.** Theorem 6.4 *protects* the programme: spacetime is internal-charge-blind, now derived from the Lorentz side as well, rhyming with the banked colour-blindness exactness. No colour observable is touched; the α and colour fences are untouched.

Novelty label: physical interpretation not established here.

Proposition 6.6 (Placement, not emergence)**.** *Given Theorem 6.4, the fermion bundle must factorize* $\mathcal{E} = (S_+ \otimes F_L) \oplus (S_- \otimes F_R)$ *with connection* $\omega \otimes I + I \otimes A^{\mathrm{SM}}$*, and the existing B-map cubic lifts pointwise to the Yukawa form.* ***What this is not:*** *emergence. The compact* G_{SM} *boundary, its connection, the B-map selection, the hierarchies and the generation count all stay loaded. Zero new continuous price.*

Novelty label: standard theorem used.

Theorem 6.7 (The conformal-mode obstruction, machine-proved; `s919`-V9)**.** *For generic* $\sigma(x^0 \ldots x^3)$*,* $R = e^{-2\sigma}(-6\Box\sigma - 6|\nabla\sigma|^2)$ *and* $\sqrt{g}R = \mathrm{div} + 6e^{2\sigma}|\nabla\sigma|^2$ *exactly, giving the conformal kinetic term* $-(3/\kappa)e^{2\sigma}|\nabla\sigma|^2$*: the Euclidean MM/EH action is unbounded below in the conformal direction — the conformal-factor problem of Euclidean quantum gravity (G. W. Gibbons, S. W. Hawking and M. J. Perry, Nucl. Phys. B 138 (1978) 141), re-verified here symbolically; the machine proof is of the identity, not of a new result. [LIB2-169] The missing object is precisely a reflection-positive contour, regulator or discrete measure controlling the conformal mode together with the constraints. [LIB2-169]*

7 The state and the measure

Named datum 7.1 ($\mathfrak{M}^{\mathrm{Cartan}}_{\Lambda,t}$ — a reflection-positive state at finite regulator; A1442, `s920` 27/27×2)**.** A normalized reflection-positive state exists at the finite Cartan regulator tier: a heat-kernel × Haar

product on Spin(5) edges, with the discrete solder $e = \ell\pi_E(Ue_5)$ typed exactly and OS reflection positivity from nonnegative Peter–Weyl coefficients plus the crossing split. [LIB2-170] The truncated-kernel Grams are strictly positive-definite at $t = \frac{1}{2}$ and $t = 1$ with Casimir values $\{0, 4, 5/2\}$ built from the house 27-representation. **The OS Hilbert space and a positive transfer operator exist at finite regulator.** [LIB2-170] The standard lattice argument is argument-tier and accepted as typed; the machine controls are exact in both lanes. [LIB2-170]

Novelty label: standard theorem used.

Proposition 7.2 (Almost-sure faithful soldering)**.** $\det E = 0$ *is a nonzero polynomial condition with a smooth positive density, so rank* $e < 4$ *has measure zero at every finite regulator.* ***Caveats kept:*** *degenerate configurations remain in the topological support (a continuum limit could concentrate), and a symmetric finite-volume state has vanishing coframe expectation — the AdS coherent boundary state is a* choice*, typed as such. [LIB2-170]*

Novelty label: new specialization proved here.

Theorem 7.3 (The finite-algebraic gravitational sign problem; `s920`-W1/W2)**.** *On the* 16*-representation,* $B_C = -\mathrm{tr}(T_I T_J) = 4I_{10}$ *exactly, inertia* $(10, 0)$*;* $B_{\mathrm{MM}} = \mathrm{tr}(\Gamma T_I T_J)$ *has characteristic polynomial* $\lambda^4(\lambda - 4)^3(\lambda + 4)^3$*, hence inertia* $(3, 3, 4)$ *— self-dual positive, anti-self-dual negative, transvections null. [LIB2-171]* ***The pure MM/EH weight cannot be a positive coercive covariance.*** *[LIB2-171] This is the finite-algebraic twin of the conformal-mode obstruction (Theorem 6.7).*

Novelty label: new specialization proved here.

Proposition 7.4 (The positive completion cone)**.** $B_{\alpha,\beta} = \alpha B_C + \beta B_{\mathrm{MM}} > 0 \iff \alpha > 0,\ |\beta| < \alpha$*, with spectrum* $\{4\alpha^{\times 4}, 4(\alpha + \beta)^{\times 3}, 4(\alpha - \beta)^{\times 3}\}$*; the nonzero boundary* $\alpha = |\beta| > 0$ *is chiral-null (inertia* $(7, 0, 3)$*); the apex* $\alpha = \beta = 0$ *is the zero form (Rev32.9, A1674 F02). [LIB2-172]* ***Not yet established:*** *nonlinear RP at* $\beta \neq 0$ *was closed only for the anisotropic heat-kernel family (below); a simplicity/constraint mechanism giving GR rather than Cartan Yang–Mills; and the continuum RG limit. [LIB2-172, LIB2-189, LIB2-301]*

Novelty label: new specialization proved here.

Proposition 7.5 (The free quaternionic fermion measure is positive; `s920`-W6)**.** *On the h-orthonormalized branch module the Euclidean gammas are real symmetric, the quaternion commutant units commute with* D_{free}*, and* $\det(mI + D) = m^8(m^2 + 3)^8$ *exactly, so on the regulated branch module* D *is a* 24*-dimensional operator (eigenvalue* 0 *with multiplicity* 8*, and* $\pm i\sqrt{3}$ *with multiplicity* 8 *each; Rev32.9, A1677 F04). [LIB2-188] Kramers positivity is confirmed.* ***Fence:*** *positivity is not generic (break controls in both lanes); the free sector is closed, the interacting chiral sector is open. [LIB2-188, LIB2-307] The named missing object is* $\mathfrak{t}_{\mathrm{det}}$*, a gauge-invariant, reflection-compatible, globally phase-consistent trivialization of the chiral determinant line — anomaly cancellation is necessary but not sufficient. [LIB2-188, LIB2-307]*

Novelty label: physical interpretation not established here.

Proposition 7.6 (The global $\mathbb{Z}_2$, retyped; `s920`-W3)**.** $C_5 = \Gamma|_5$ *has determinant* $+1$*, fixes* e_5 *and is* $-I_4$ *on* E_{sig}*; conjugation fixes* M *and flips* P*, so* $e \to -e$ *is an ordinary* $SO(4)$*-stabilizer* gauge move *in the compact Euclidean regulator. After continuation,* $-I_4$ *is determinant* $+1$ *but non-orthochronous, so the global bit retypes as* $SO(3,1)/SO^+(3,1)$ ***time-orientation*** *data selected by boundary condition. The identification with the banked FTS shell reversal remains structural, not derived.*

On the live 56, Γ is the compact gauge rotation $\exp(\frac{\pi}{2}\rho(\mathrm{diag}(j,j,j,-j)))$ at the origin, which preserves the end, the cubic sign and χ (`s1254`, A1771); the time-orientation retyping above is a statement about the continued carrier and its boundary choice, not about this action.

Remark 7.7 (the measure does not select the branch)**.** Under any S_3-invariant finite-volume state the S_3 orbit of the ordered golden vacuum has equal weight: the active c_3 branch is extremal/boundary-state data (or a future SSB limit), exactly consistent with the S_3-torsor pricing of Axiom 6.2. No silent upgrade occurred.

8 The frontier round: AX-COT, positivity, envelope, actuality

Priced axiom 8.1 (AX-COT — the Cartan–occupancy transgression; A1443, `s921` $30/30 \times 2$)**.** selector/order/depth ≡ transvection/polar/oriented-axial. [P-003] The S_3 relabelings are Lie-algebra automorphisms of the banked $\mathfrak{so}(5)$, and $\mathfrak{so}(5) = \mathfrak{p}_4 \oplus A_3 \oplus B_3$ with $\mathfrak{p}_4 = 2\cdot 1 \oplus E$, $A_3 = 1 \oplus E$, $B_3 = \mathrm{sgn} \oplus E$, so $B_3 \otimes \mathrm{sgn} = 1 \oplus E$: the $(4+3+3)$ match against the banked source-side occupancy carrier $H^{\mathrm{src}}_{\mathrm{occ}} \cong 4\cdot 1 \oplus 3E$ is *real*. **The load-bearing clause:** the *identification* of the two $(4+3+3)$ systems is a cross-layer physical typing that representation theory does not force. Zero continuous moduli.

*Naming and the exact size of the clause (*Rev29*).* The two objects are **different** and are never both to be called "H_{occ}": the source-side carrier is $H^{\mathrm{src}}_{\mathrm{occ}} \cong 4\cdot\mathbf{1} \oplus 3E$ with S_3-character $\chi = (10,4,1)$ (Appendix S, Thm. "Arrow space and the evaluation relation"), while the object named here is the Cartan carrier $H_{\mathrm{Cartan}} = \mathfrak{so}(5) \cong 3\cdot\mathbf{1} \oplus \mathrm{sgn} \oplus 3E$ with $\chi = (10,2,1)$. Both are 10-dimensional and both split $4+3+3$, but they are not isomorphic as S_3-modules; the virtual difference is $H^{\mathrm{src}}_{\mathrm{occ}} \ominus H_{\mathrm{Cartan}} = \mathbf{1} \ominus \mathrm{sgn}$, with character $(10,4,1) - (10,2,1) = (0,2,0)$, which is exactly the character of $\mathrm{triv} - \mathrm{sgn}$. **That difference is exactly AX-COT**: the match above is obtained only through $B_3 \otimes \mathrm{sgn} = \mathbf{1} \oplus E$, so the sign twist absorbing $\mathbf{1} \ominus \mathrm{sgn}$ *is* the axiom's content, and AX-COT is precisely the price of turning $(10,2,1)$ into $(10,4,1)$.

Novelty label: new specialization proved here.

Theorem 8.2 (AX-COT independence; A1444, `s922` $45/45 \times 2$)**.** *Every proposed derivation route fails exactly: S_3 matching (endomorphism dimension 25), positive metrics (an $O(4)\times O(3)$ residue), semantic blocks (an $O(2)$ survives), the kernel line (exact rational countermodel), the order↔depth swap (equivariant orthogonal, but it moves the arrow), and the Gram/eigenline rescue (copy-unit dependent, hence metric-circular). Ordinary equivariance is provably insufficient.* ***Price:*** *one independent cross-layer typing row with zero continuous moduli — the same epistemic shape as AX-MG after its two independence theorems.*

Novelty label: new specialization proved here.

Theorem 8.3 (The AX-COT continuous-stabilizer obstruction is a theorem; A1445, `s923` $71/71 \times 2$)**.** *Solving $JQ = J$ for fully general Q gives exactly $Q = I + nw^{\mathsf{T}}$ (solution space of dimension 3); the $w^{\mathsf{T}}n = 0$ subgroup is 2-dimensional, determinant-one, fixes n and moves $(0:1:0)$; the Euclidean metric cuts the orthogonal stabilizer to exactly $\{I, H_n\}$, which* still *moves the order axis. The obstruction is now a theorem, not an example, and AX-COT stays independent.*

Novelty label: new specialization proved here.

Proposition 8.4 (The selected arrow, conditional; A1443, `s921`-U2)**.** ad_{P_0} *annihilates* B_3*, maps* A_3 *isomorphically onto the spatial solder, and* $\mathfrak{p}$ *is the solder target, so* $[j_A] = (0:1:0)$ *projectively: the OS time direction conditionally selects the arrow.* ***The*** $\mathbb{RP}^2$ ***wall is removed only at a new tier*** *— the six-tier finite-category independence theorem of Appendix S stands untouched; this is a bypass from above, not a boundary derivation. KILL-NODE 2 remains fired: the ten C-coefficients follow only under the triple conditional load AX-COT + the* D_5 *mixed dictionary + the Stiefel/radial branch, and the Stiefel/radial domain is underived.*

> **The ARROW tier is not gauge (*Rev29*, tier correction).** The arrow tier carried by $[j_A]$ — here and at Theorem 9.1 — is to be read exactly as *an unattached pre-observation loading/faithfulness datum whose present export is blind.* It is **not** a gauge statement: no redundancy is quotiented, no frame is fixed, and no equivalence between distinct arrows is asserted. The conditional selection above is a bypass from a new tier, which is why the $\mathbb{RP}^2$ datum survives it as a loading, not as gauge freedom; its present export is blind, and the Appendix S six-tier independence theorem stands untouched.

Novelty label: new specialization proved here.

Proposition 8.5 (Nonlinear OS positivity and the path no-go; `s921`-U3)**.** *The chirality spectrum* $\{-1^3, 0^4, +1^3\}$ *relative to* $B_C = 4I$ *is exact;* $\mathcal{L}_{\alpha,\beta}$ *is a positive elliptic invariant on the whole cone* $\alpha > |\beta|$*, the heat semigroup is a positive Markov convolution, and the reflected-kernel factorization gives OS positivity.* *[LIB2-189]* *But for* $\beta \neq 0$*,* $\alpha = 0$ *lies outside even the closed cone (the apex* $(0,0)$ *is in its closure; the no-path result concerns the nonzero pure-MM weight — Rev32.9, A1677 F03):* ***no continuous positive path in this family reaches pure MM/EH****.* *[LIB2-190]* *The named route is a simplicity/Plebanski-type constraint sector projecting the positive Cartan measure onto a gravitational subspace.* *[LIB2-190]*

Novelty label: standard theorem used.

Theorem 8.6 (No linear projector for simplicity; A1444, `s922`-T6)**.** *Some sums of simple bivectors are nonsimple (*$e_1 \wedge e_2 + e_3 \wedge e_4$ *has rank* 4*, while* $e_1 \wedge e_2 + e_1 \wedge e_3$ *is simple), so no linear projector retracts onto the full simple-bivector variety: keeping the full condition needs a nonlinear construction (Rev32.9, A1677 F02).* *[LIB2-191]* *A positive nonlinear simplicity* weight *exists at finite regulator: the sum-of-squares penalty* $\exp(-\lambda \sum_i p_i(B)^2)$ *in the five Pfaffian minors* p_i*, on the fixed-norm bivector domain (compact, so* $0 < Z < \infty$ *for every* $\lambda \geq 0$*), with reflection positivity by the standard paired-multiplication argument (argument-tier;* ***`s922`*** *T6 notes). The reflection and the reference measure are those of the finite-regulator state of Named Datum 7.1; the admissible multipliers are the reflection-paired products, whose pairing for this weight is not written out here. The weight is soft: its support is the whole fixed-norm domain, and the Pfaffian variety is reached only in the hard-constraint limit* $\lambda \to \infty$*, which is not taken (Rev32.9, A1677 F06; through Rev32.8 this read "a positive nonlinear simplicity measure (fixed-norm Pfaffian variety . . .)* does *exist" and called gravity-measure existence closed). Gravity-measure existence is argument-tier at that tier;* ***Einstein universality*** *— the AdS linearized spectrum of the sourced heat-kernel action — is the remaining named object.* *[LIB2-192, LIB2-301]*

Novelty label: new specialization proved here.

Proposition 8.7 (The associative envelope; `s921`-U4/U5)**.** $\mathrm{odd} \circ \mathrm{odd} \subset J_+$ *(all* 78 *products* Γ*-even),* $\operatorname{rank} Q_\Gamma = 15$ *exactly, the solder* 4*-plane alone squares to the* e_5 *ray, and* $\mathrm{Sym}^2(J_-) \twoheadrightarrow J_+ =$

$J_3(\mathbb{H}) \hookrightarrow M_6(\mathbb{C})_{\mathrm{sa}}$ *with* ι *a Jordan homomorphism on all* 225 *basis pairs.* ***Scope:*** *this advances the open associative-projection condition at the quadratic* Γ*-even graded tier only — not the full exceptional parent, not arbitrary composites.*

Novelty label: standard theorem used.

Proposition 8.8 (Born/Tsirelson at regulator tier, and the actuality no-go; `s921`-U7)**.** *The CHSH spectrum is* $\{\pm 2\sqrt{2}, 0, 0\}$ *exactly and the Born probabilities are operator-algebra theorems inside the regulator image. The symmetric two-outcome state is swap-invariant while no character is, so a probability law does not imply one realized outcome — generalized in A1444 to* any *nontrivial transitive G-set. The Appendix L firewall between conditional Born/Tsirelson structure and the single-outcome problem is respected and now theorem-backed.*

Novelty label: physical interpretation not established here.

Proposition 8.9 (Protective negatives)**.** *Pure Cartan/gravity operators act as* $X_S \otimes I_F$*, scalar on the internal B-map factor, so they cannot distinguish* y_u, y_d, y_e, y_ν*: the scoped within-generation hierarchy no-go is untouchable from the spacetime side. Compactness is derived; it is* ***not*** *group selection — the SM group stays loaded.*

Novelty label: standard theorem used.

Proposition 8.10 (Global anomalies and $\mathbb{Z}_6$; A1444, `s922`-T8; doc-gate arXiv:1910.11277 fetched and confirmed)**.** *The bordism table is known-literature; the matter side is kernel-exact in both lanes (*12 *doublets* $\equiv 0 \bmod 2$*;* $\Sigma Y = \Sigma Y^3 = 0$*), so there is no global gauge anomaly for any* $q \in \{1, 2, 3, 6\}$*. Under the* local-faithfulness principle *— one named discrete clause —* $\ker \rho_{\mathrm{local}} = \mathbb{Z}_6$ *by exact* 36*-element enumeration and* $G_{\mathrm{eff}} = (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$ *with no continuous parameter. The global bundle/line-operator tier stays open, and gauge-group* emergence *is not claimed.*

Novelty label: new specialization proved here.

Theorem 8.11 (The χ source–response wall; `s922`-T5)**.** *The* Spin(4)*-invariant symmetric response space is exactly* 2*-dimensional and* $(5)^{\mathrm{Spin}(4)} = \mathbb{R}e_5$ *exactly, so a linear Lorentz-scalar source is uniquely the normal line, with conditional channel response* 1/4*.* ***The state and the generator cannot fix*** χ***.*** *The named missing object is* $\mathfrak{N}_{\mathrm{mass}}$*: a normalized physical response map into the mass quadrature, with LSZ/self-energy attachment and dimensional registration. [LIB2-302] The* 1/4 *here is a typed structural channel constant behind an unproved coupling clause; no* χ *value moves.*

ERRATUM (issued S253, binding). An earlier row of this arc stated: "D_5-representation commutant = 3, hence lift ambiguity = 8 orthogonal / 4 orientation-preserving, zero continuous." **The commutant-3 count was computed on the wrong carrier** — the 5-point *permutation* module ($\mathbf{1} \oplus V_1 \oplus V_2$, multiplicity-free, reflection inertia $(3_+, 2_-)$). [LIB2-258] The actual chamber carrier $\langle U_0, S \rangle$ is $V_2 \oplus \mathbf{1} \oplus 2\epsilon$ (reflection inertia $(2_+, 3_-)$) with a 6-dimensional commutant containing an $O(2)$ complement-frame *gauge* — not a finite sign set. The finite-sign price row is **retired** and replaced by the minimal-dilation theorem (Theorem 9.3). "Zero continuous" stands; the discrete residue did not. [LIB2-258] Machine proof `s924`-W2/W3 ($47/47 \times 2$). This correction was caught by the review lane against a house-signed row and is recorded here rather than absorbed silently.

9 The three-price closure

Novelty label: new specialization proved here.

Theorem 9.1 (The order line; A1446, `s924` $47/47 \times 2$)**.** $\log(J_{\text{vac}})/\log\varphi = \operatorname{diag}(1,0,-1)$ *is the unique trace-free normalized logarithm, so* $o = -D_\varphi$ *on the nose; the centered* Δ_2*-harmonic line is uniquely* $\mathbb{R}(-1,0,1)$*,* $\Delta_2 J = (3,0,-15)$ *exactly, and* $\ker(\Delta_2 J)/\ker J \cong \mathbb{R}[e_{\text{order}}]$*.* ***The continuous*** $\mathbb{RP}^2$ ***source-arrow freedom collapses to the order line at the evaluated tier.***

Remark 9.2 (what selects it, and what does not)**.** The selecting premise is *log-Hodge naturality*: export the ordered golden spectrum through its unique log/harmonic generator — one zero-dimensional cross-layer functoriality clause. The *off-shell* arrow (arrows differing by $\ker J$) is not forced, and the independence theorem (Theorem 8.2) is not contradicted: this is a new named attachment, not a derivation from the untyped source. As of A1447 the log leg of this clause is itself a theorem (Theorem 10.1) and the clause is absorbed into AX-MOS. [P-005]

Novelty label: new specialization proved here.

Theorem 9.3 (The canonical minimal dilation; A1446)**.** $U_0 = C^{\mathsf{T}}RC + (I - C^{\mathsf{T}}C) \in SO(5)$ *has order* 5*, satisfies* $C_*U_0 = RC_*$ *and* $SU_0S = U_0^{-1}$*, and is the identity on* $\ker C_*$*: it is the* ***unique*** *minimal orthogonal dilation with* $V_{\ker} = I$*. Frobenius block additivity gives the exact branch distances* $5+\sqrt{5}$ *(canonical)* < 10 *(*V_1*-inserting)* $< 10 + 2\sqrt{5}$ *(*V_2*-duplicating). The residual* $O(2)$ *is complement-frame gauge and changes no exported quantity. [LIB2-258]*

Remark 9.4 (a coherence hit worth recording)**.** The rejected V_1-inserting branch is exactly the harmonic that the banked annular support theorems (Appendix S) exclude. Two independent constructions agree on which branch is unphysical.

Novelty label: new specialization proved here.

Theorem 9.5 (g_χ is derived away at the Riesz–Clifford tier; A1446)**.** *On a freshly built real Clifford module the* Spin(5) *commutant is* 4*-dimensional (quaternionic) but its* self-adjoint *part is* 1*-dimensional, so the space of self-adjoint* Spin(5)*-equivariant maps* $V_5 \to \operatorname{End}(S_{\mathbb{R}})$ *is exactly* 1*-dimensional (*180*-unknown nullspace computation, residual* $< 10^{-9}$*). Clifford isometry forces* ± 1 *and orientation picks* $+1$*; with the Riesz whitening* $H^{-1/2}e_5 = e_5/2$ *one gets* $\Sigma_\chi = \Gamma/2$ *and* $D^2 = \Lambda^2(\varphi^{2N} + 1/4)I$ *exactly.*

> χ **fence (binding, travels with every occurrence).** $\chi^2 = 1/4$ holds *inside* the attachment only. It is not a mass prediction. [LIB2-193] The banked exact single-χ ratio no-go (Rev27) is untouched. The Spin(4)-tier independence result (A1445: three same-symmetry countermodels with $\chi^2 \in \{1/16, 1/4, 1\}$) and the Riesz–Clifford-tier collapse here are *not* in conflict: the coefficient dies at the higher tier, where self-adjointness, equivariance, isometry and a whitened source are demanded simultaneously. No value is promoted.

10 The observation functor $\mathcal{O}_{\text{MOS}}$

Novelty label: new specialization proved here.

Theorem 10.1 (The relative modular score; A1447, `s926` $30/30 \times 2$, adjudicated against pre-registered gates `s925` 15/15)**.** $\operatorname{tr} J_{\mathrm{vac}} = \operatorname{tr} J_{\mathrm{vac}}^{-1} = 2\varphi$ *(normalization cancels exactly),* $\rho_+\rho_-^{-1} = J_{\mathrm{vac}}^2$*, and the half-log score is* $\operatorname{diag}(1, 0, -1)$ *on the golden lattice:* $K_{\mathrm{rel}} = D_\varphi$*.* [LIB2-173] ***The log leg is a theorem given additivity*** *— the countermodel* $f_{\mathrm{alt}}(x) = (x - x^{-1})/(\varphi - \varphi^{-1})$ *is normalized but non-additive (*$f_{\mathrm{alt}}(\varphi^2) = \sqrt{5} \neq 2$*).* [LIB2-173] D_φ *therefore has operator-algebraic meaning: it is the canonical score of the ordered state pair.*

Remark 10.2. This does not contradict the transgression obstruction of Appendix S: the bare module still has no natural point. The point comes from the *state pair.* [LIB2-173]

Novelty label: new specialization proved here.

Theorem 10.3 (Unital complete positivity fixes the scale)**.** *Total-probability preservation forces* $r = \sqrt{5}/2$ *uniquely (Gram =* $(4r^2/5)I_2$*), recovering the banked* C_* *exactly and upgrading its provenance from partial-isometry normalization to a* physical *premise. The residual freedom is* ± 1 *signs on the standing orientation torsor. Zero continuous coefficients.*

Novelty label: new specialization proved here.

Proposition 10.4 (The induced POVM, the minimal dilation, the whitening)**.** *The effects follow the exact pattern* $(1/4, 3/4, 1/2, 0, 1/2)$*, sum to* I_2 *and are all positive, with a* zero *effect on the absent support mode: the functor does not regenerate excluded modes. Channel covariance* $\Phi(U_0 A U_0^{\mathsf{T}}) = R\Phi(A)R^{\mathsf{T}}$ *is verified on a fully symbolic* 5×5 *matrix; the Choi matrix has rank* 1*, so the minimal Stinespring environment is unique and dark environments are representation artifacts.* $A = \operatorname{diag}(a, a, a, a, b)$ *with* $AH_hA = I$ *has the unique positive solution* $a = 1$*,* $b = 1/2$ *— the* $1/2$ *is an eigenvalue of the unique whitening, not a chosen coefficient.*

Novelty label: new specialization proved here.

Theorem 10.5 (OS evenization; A1447)**.** $\gamma_0\Gamma\gamma_0 = -\Gamma$ *matches* $e_5 \to -e_5$ *under OS reflection while* $\gamma_0 I \gamma_0 = I$ *does not, so full Pin/OS typing kills the* $e_5 \mapsto I_8$ *loophole that made* g_χ *free at the* Spin(4) *tier. The Dirac seed is reflection-*odd *(*$\Theta D \Theta^{-1} \neq D$*) while* D^2 *and* $|D|$ *are reflection-even and scalar:* ***the physical observable is the even positive spectral calculus, not the raw Clifford seed.***

Remark 10.6 (what this fences)**.** Theorem 10.5 further fences χ: the gap is an *observation spectrum*, not yet a rest mass.

Named datum 10.7 ($\mathcal{O}_{\mathrm{MOS}}$ — the observation functor)**.** The composite — ordered state pair $\to$ modular score $\to$ unital-CP normalized readout $\to$ minimal dilation $\to$ Pin/OS-even spectral calculus — is unique under seven premises, each with a verified countermodel showing it has teeth: additivity, unitality, minimality, Pin typing, OS typing, isometry, and the whitened source. [LIB2-174] **It contains no continuous modulus.** What the microscopic theory has not done is *select* it. [LIB2-174] *Level fence (Rev32.3):* $\mathcal{O}_{\mathrm{MOS}}$ is the ordered-state / record-quotient observation functor and nothing more; the microscopic mass/generation observation map $\mathcal{O}_{\mathrm{gen}}$ — the map that would read the noncommuting pair (Y_u, Y_d) and a relative left frame off the parent — is a distinct object and is *not built* (§15, face 6). [LIB2-005, LIB2-174] Statements that "the observation functor is built" refer to $\mathcal{O}_{\mathrm{MOS}}$ only.

Rev32.2 pointer. The record-quotient channel $W_{\lambda,\tau}$, the score solder Ξ, the closed normalization ledger and the declared amplitude premise are folded in §15; that fold does not select $\mathcal{O}_{\mathrm{MOS}}$ either.

The uniqueness above is uniqueness *within* the fixed ordered-state / readout / dilation construction of this section, and $|\lambda\mu| = 1$ in the response section is a declared response premise, not forced by the seven premises (Rev32.9, A1674 F05).

11 The four residuals

Priced axiom 11.1 (AX-MOS, compressed; A1448, `s927` $46/46 \times 2$)**.** Given the active c_3/c_2 branch, $(1, 0, -1)$ is the unique assignment by enumeration; the shell reversal gives $SJ_+S = J_-$ and $SD_\varphi S = -D_\varphi$ exactly, so the arrow *is* the standing OS/time $\mathbb{Z}_2$; the relative entropies are exactly equal both ways ($\log \varphi/\varphi$, via $1 - \varphi^{-2} = \varphi^{-1}$) and the spectra coincide, so no inversion-symmetric scalar orients. **AX-MOS carries no new bit inside the loaded branch — honest compression, not unconditional derivation.** Without the branch, an S_3 marking orbit remains.

Novelty label: new specialization proved here.

Theorem 11.2 ($\Pi^{\mathrm{MOS}}_{5\leftarrow 6}$, the supported observation quotient; A1448)**.** *P_{pair} is a coisometry with $P_{V_4} = P_{\mathrm{pair}}^{\mathsf{T}} P_{\mathrm{pair}}$; the Hodge frame is orthogonal; the exact $q = 0, \pm 2$ Fourier frame satisfies $QQ^{\mathsf{T}} = P_{\mathrm{ann}}$ with the $q = \pm 1$ modes annihilated (categorical support respected); $\tilde{\Pi} = QBP_{\mathrm{pair}}$ is an exact partial isometry ($\tilde{\Pi}\tilde{\Pi}^{\mathsf{T}} = P_{\mathrm{ann}}$, $\tilde{\Pi}^{\mathsf{T}}\tilde{\Pi} = P_{V_4}$) killing all pair-difference modes; K_S is exactly column-stochastic and uniform-to-uniform, with the $[1 + 2\cos(\theta_p - \theta_k)]^2/15$ conditional law verified entry by entry; and the uniqueness chain $O(3) \to O(2) \to \pm 1 \to 1$ has zero continuous moduli.* *[LIB2-175]*

> **Fence (machine-verified, binding).** Π^{MOS} has rank 3 and *relative* rank $2 < 5$, hence $\Pi^{\mathrm{MOS}} \neq \Pi^{\mathrm{microscopic}}$. [LIB2-175] The banked no-equivariant-transfer theorem and the 25-dimensional pricing row are *respected, not overturned*: this is the observable quotient of a still unconstructed microscopic loading. [LIB2-175] $\Pi^{\mathrm{full}}_{5\leftarrow 6}$ is open.

Novelty label: physical interpretation not established here.

Proposition 11.3 (The one-particle bare gap; A1448)**.** *$H_N(p)^2 = (|p|^2 + \Lambda^2(\varphi^{2N} + \frac{1}{4}))I_8$ fully symbolically, so $M_N = \Lambda\sqrt{\varphi^{2N} + \frac{1}{4}}$, with $\chi^2_{\mathrm{local}} = \frac{1}{4}$ as an orthogonal squared component and the OS rank-one reflection kernel positive semi-definite.* ***AX-MASS-LABEL****, sharpened:* the MOS zero-momentum seed is the mass part of the derived Clifford Hamiltonian — *a one-particle attachment, no coefficient chosen.* *[LIB2-193] The interacting pole, LSZ and occupancy are all open, and the single-χ ratio no-go stands.* *[LIB2-193, LIB2-302]*

Novelty label: new specialization proved here.

Proposition 11.4 (The record process; A1448)**.** *$T_5 = |U_0|^{\circ 2}$ is doubly stochastic with placement 3 exactly decoupled; T_4 is strictly positive, symmetric, doubly stochastic, reversible and primitive, with exact spectrum including $\rho^* = \varphi/2$ and golden mixing gap $1 - \varphi/2 = (3 - \sqrt{5})/4 = \varphi^{-2}/2$; all-pairs Kolmogorov consistency holds and the transitive fixed-point no-go is verified on three control actions. The placement-3 decoupling matches the zero POVM effect of §10 — an independent cross-check.*

12 Actuality: the record type is derived, the value is data

Novelty label: new specialization proved here.

Theorem 12.1 (History $\iff$ character $\iff$ boundary record; A1449, `s929` $27/27\times 2$, adjudicated against pre-registered gates `s928` 12/12)**.** *The surd-form T_4 equals the house T_4 entry by entry (16 symbolic identities); U_4 is orthogonal and exactly unistochastic; the record isometry satisfies $V^{\mathsf{T}}V = I$ exactly; squared branch norms equal cylinder probabilities (Rev32.9, A1677 F01); and distinct histories are orthogonal — a strongly decoherent classical record process derived from $\mathcal{O}_{\mathrm{MOS}}$. [LIB2-176] Finite-stage characters are exactly point evaluations and compatibility gives* $\operatorname{Spec} C(\Omega_T) \cong \Omega_T$*. [LIB2-176]* ***The three ledger descriptions are one object; the "kind" question is closed.*** *[LIB2-176]*

Novelty label: standard theorem used.

Proposition 12.2 (Stochastic actualization)**.** *In the Markov-kernel category the morphism $1 \rightsquigarrow \Omega_T$ is derived. Demanding a* deterministic natural section *was a category error for a stochastic law: probability theory returns a random element, not a deterministic function of its own law. [LIB2-177]*

Novelty label: standard theorem used.

Proposition 12.3 (The tail/Poisson rescue is refuted)**.** *Bounded harmonics are constants (*$\ker(T_4 - I) = \mathbb{R}\mathbf{1}$*), so "the actual history as an asymptotic boundary point" is dead for this primitive chain — the exact complement of the house theorem $\|T_4^n - U\| = (\varphi/2)^n$. Two different proofs, one wall.*

Novelty label: new specialization proved here.

Proposition 12.4 (The law is atomless; the record is extensive)**.** $p^* = (5+\sqrt{5})^2/64 = 5\varphi^2/16 < 1$*, so every point history has measure zero; the arithmetic sampler's finite-depth intervals form an exact partition of unity; and the entropy rate is* 0.8097355 *nats* $= 1.1682014$ *bits per step.* ***A realized history is extensive record data, not "one selector bit."***

Novelty label: physical interpretation not established here.

Proposition 12.5 (The conserved-$\mathbb{Z}_2$ sector record — conditional)**.** *$T_4 \oplus T_4$ has invariant centre $\mathbb{C}^2$ (two extremal sector characters) and exact mixing kills it. Price: no continuous parameter, at most the standing WH/BH-or-time bit — but its exact conservation and its identification with the shell operation must be proved. That is the named next object for anyone who wants the sector record unconditionally.*

Named datum 12.6 (AX-MAP-HISTORY — CONDITIONAL CANDIDATE, named, not adopted)**.** By exhaustive enumeration over all simple cycles, $2 \leftrightarrow 4$ is the unique per-step-maximal orbit, with exact identities $p^* = 5\varphi^2/16$, edge ratio $4/3$, per-period ratio $16/9$, and second cycle $15\varphi^2/64$. This orbit is *exactly* the $\mathrm{Aut}(T_4)$ $\mathbb{Z}_2$ orbit banked house-side in advance, by an independent computation. **Honestly typed: adopting it changes the question** — the orbit is atypical, of μ_T-measure zero. Named, not adopted.

Actuality / record status — the X_{actual} row splits. $X_{\text{record-law/type}}$ = DERIVED. $X_{\text{realized-value}}$ = NOT DETERMINED BY THE LAW — it is sample/record data, by theorem. [LIB2-177] The Appendix L firewall stays in force for the specific realized outcome; what narrows is the blanket claim that no actuality object exists. [LIB2-177] The remaining objects are: one realized record (data, by theorem), the conditional conserved-$\mathbb{Z}_2$ sector identification, and AX-MAP-HISTORY as a candidate only.

13 Price ledger and open frontier

Row	Price	Status
AX-COT	one cross-layer typing clause, zero continuous moduli	independent (Thm 8.2); obstruction a theorem (Thm 8.3); discharged on shell by log-Hodge naturality, itself absorbed into AX-MOS
AX-MOS	the active c_3/c_2 branch + one time-orientation bit	**no new independent clause** — compressed into standing loading (Ax. 11.1)
AX-MASS-LABEL	gap $\leftrightarrow$ rest mass, one-particle	interacting pole/LSZ/occupancy open
Branch Real clause	discrete; inherits the loaded c_3 branch	retires the $\mathbb{CP}^1$ continuous price
Local-faithfulness principle	one discrete clause	buys $G_{\text{eff}} = \text{SM}/\mathbb{Z}_6$, no continuous parameter
Signature-oddness	the residue of the typing clause	two independent uniqueness theorems, one per lane
Chamber lift	**zero** continuous; residual $O(2)$ is unphysical gauge	Thm 9.3; the earlier finite-sign row is retired by erratum
g, ℓ	one dimensionless coupling; registration to the one banked scale class	$\lvert\ell\rvert$ refuted as derivable
AX-MAP-HISTORY	—	candidate only; changes the question

Bottom line. Across fourteen rounds the price of the observable attachment went from three loosely typed rows to *two zero-dimensional named clauses plus one realized sample*, with no continuous modulus anywhere in this sector. The observation functor exists, is explicitly constructed, and is unique under seven premises with verified teeth. What the microscopic theory has not done is select it — and that is a selection statement, not a calculation. The named frontier is exactly: $\Pi^{\text{full}}_{5\leftarrow 6}$ · Einstein universality · $\mathfrak{N}_{\text{mass}}/\mathfrak{K}_\chi$ · the interacting mass attachment · the conserved-$\mathbb{Z}_2$ sector identification · one realized record. KILL-NODE 2 remains fired off shell and for the full microscopic loading; χ has no value; nothing here is promoted.

14 A Clifford isotropy law and a braid representation on V_{56} (Rev30, A1499)

Two exact structures on the frozen carrier were established at the close of the freeze-first arc (`s1026`, `s1032`). Both are statements about $\text{Rep}_{\mathbb{R}}(\mathfrak{so}(2,1))$ acting on V_{56}; neither is a physics result, and the second carries a fence that matters more than the structure does.

14.1 The universal isotropy law

The operator $P_{\text{act}} = 24W_0^2$ is a rank-24 projector, and $24W_0^2 = 24W_1^2 = P_{\text{act}}$, $24K_0^2 = -P_{\text{act}}$, with *all* pairwise anticommutators vanishing. Thus $\sqrt{24}\,(W_0, W_1, K_0)$ is an exact $\mathrm{Cl}(2,1)$ action on the active 24 — the spinor sector — and is zero on the 32-dimensional complement. [LIB2-014] The immediate consequence is a single law covering every carrier map T:

$$G_T = \tfrac{1}{24}\,\|P_{\text{act}}\,T\|_F^2 \;\cdot\; I_2.$$

This unifies two constants that had stood separately. Ten unit-norm active columns and one ($\|P_{\text{act}}T\|_F^2 = 10$ and 1; Rev32.9, A1677 F05) give $10/24 = 5/12$ and $1/24$: the two constants of record are dimension-weighted restrictions of one global Clifford identity, not independent facts. [LIB2-194] The mechanism polarity is worth stating because it was tested both ways — the same-carrier Clifford mechanism holds; a normal invariance-group enlargement does *not*, since the beat $\mathfrak{so}(2)$ is not normal in the $\mathfrak{so}(2,1)$, so an earlier proposed cure does not apply literally here.

14.2 The braid representation, and the fence around it

The operators $B_1 = I + \sqrt{6}L_+$ and $B_2 = I - \sqrt{6}L_-$ satisfy the braid relation $B_1B_2B_1 = B_2B_1B_2$ *exactly* and are symplectic: a genuine representation $B_3 \to \mathrm{Sp}(V_{56})$. The centre is computed and is **not** scalar — $(B_1B_2)^3 = I - 2P_{\text{act}}$ — so there is *no* global $\mathrm{PSL}(2,\mathbb{Z})$; only the active sector projectivizes. [LIB2-195] Independently, that central involution coincides exactly with a page-flip element obtained by a different route, the two arriving at the same object without either being tuned to the other. The graded structure is the exact $|1|$-graded parabolic: $[L_+, L_+] = [L_-, L_-] = 0$ with $[L_+, L_-]$ spanning $\mathfrak{g}_0$, so $(L_+ \oplus L_-, \omega) \cong T^*L_+$.

The fence, and why it is printed rather than assumed. The hyperbolic word satisfies $M^2 - 3M + I = 0$ on the active sector. Appendix I records that the figure-8 lepton neck has Alexander polynomial $t^2 - 3t + 1$. **They share a trace-3 shape; they are not one object, and the interface between them is now typed (Rev33.2).** At the level of the stated three-strand actions, the homology factor and the Fibonacci fusion block are explicitly intertwined with character-adjusted Burau/Temperley–Lieb specializations at $t = -1$ and $t = e^{\pm 2\pi i/5}$; the specialized linear representations remain inequivalent, although a further character twist gives a homomorphism from the homology image to $SU(2)$ (`s1280`, A1776). The trace-three identity is specific to the homology specialization, and the figure-8 fibre monodromy must be distinguished from closing the same three-strand word. At the fibre level linear, monoidal and braided arrows are obstructed in both directions, and the K_0 action (H_1 free of rank one over $K_0(\mathrm{Fib}) = \mathbb{Z}[\phi]$) exists on H_1 only, not on the 56 (`s1307`, A1785). One leg of the interface is now a map: a functor $\mathrm{Mod}^{\pm}(\Sigma_{1,1}) \to GL(V_{56})$ and a natural isomorphism $V_{56} \cong H_1(\Sigma_{1,1}) \otimes \mathbb{R}^{12} \oplus \mathbb{R}^{32}$ are exhibited on the registered arrays; the hyperbolic word acts as $\left(\begin{smallmatrix} 3 & -1 \\ 1 & 0 \end{smallmatrix}\right) \otimes I$, $SL(2,\mathbb{Z})$-conjugate to the figure-8 fibre monodromy, so its characteristic polynomial $(t^2 - 3t + 1)^{12}$ is the twelfth power of the figure-8 Alexander polynomial under that functor, and O, C_{56} are the orientation-reversing classes (`s1258`). This identifies the word with the monodromy *class*; it does not identify it with the lepton neck of Appendix I, and the leg into the winding / F_4 category stays absent. The braid result still lives in $\mathrm{Rep}_{\mathbb{R}}(\mathfrak{so}(2,1))$ on V_{56}: what exists is a common-parent interface, not a braided monoidal functor identifying the winding category (registered at Rev33.2 as $\mathrm{Wind}_{\mathrm{ev}} := T_\delta$, `s1334`) or the corpus F_4 category with the real-56 representation, and it licenses no lepton-neck identification, generation count or F_4 statement. The resemblance is exactly the kind that has produced a category error in this programme before: an object built in one category, carried across an absent interface, and renamed in another. It is recorded here *with* its fence so that a reader who notices the coincidence — and a reader will — finds the reason it is not being cashed, rather than an argument that quietly cashes it. Nothing in this section promotes an observable, moves a tier, or touches the engine; the constants 5/12 and 1/24 are constants of record internal to the frozen frame, not measurements.

[LIB2-194]

15 The carrier arc: from the Albert solder to the remaining physical attachment (Rev32.2 fold, A1553–A1561)

Reading rule and scope. This section folds adjudicated carrier and observation-map results into the suite; it does not promote a public observable. "Closed" below means closed at the explicitly named carrier, transported-point, or record-quotient tier. Tensor equivalence is not intrinsic basepoint selection; a quotient-tier channel is not the microscopic observation map; and the final spectral-pair row remains an output gate. [P-009] One amplitude premise is *declared* below by PI ruling (S300) and printed as a premise, never as derived; no torsor side is selected. House disjoint-path verifications of the lane results cited here: `s1148` (channel, 26/26 exact), `s1150` (solder forcing, 13/13 exact), `s1152`/`s1153` (basepoint census, 23/23 and 12/12).

15.1 The solder and transported pointing

The carrier arc begins with the exact rank-27 solder

$$B : J_3(\mathbb{O}_s) \longrightarrow V_{-2}, \qquad C_{\mathrm{shell}}(Bx, By, Bz) = N_{\mathrm{pol}}(x, y, z),$$

together with its trace-dual map $B^\vee$ into the opposite shell. The complete 27^3 polarized-cubic identity vanishes at exact basis precision, and the transported primitive frame has Peirce dimensions $1 + 1 + 1 + 8 + 8 + 8$ [A1553]. The transported unit has a 52-dimensional $F_{4(4)}$ stabilizer, while the ordered primitive frame has a 28-dimensional $\mathrm{Spin}(4, 4)$ stabilizer [A1553]. These are different objects. The result identifies the Albert and shell carriers; it does not make the unpointed shell choose the transported unit. [LIB2-012, LIB2-178]

Table 1: Canonical six-face ledger, generated from the A1555 JSON. Later amendments are printed immediately below and do not alter the source table.

Face	Name	A1555 status	Closing or blocking statement
1	Albert-to-shell tensor equivalence	CLOSED-as-carrier-machinery	An invertible real B preserves the full polarized Albert cubic on the grade -2 shell, and a trace-dual $B^\vee$ preserves the Jordan-trace/symplectic pairing. Object: D1234 rank-27 B, $B^\vee$, and exact shell cubic. [A1555]
2	Intrinsic E6/F4 basepoint and Peirce-frame selection	OPEN	The unpointed shell theory itself must select an Albert unit and ordered primitive frame, uniquely up to the appropriate stabilizer, without importing the already-pointed Albert source. Object: A transported point exists; an intrinsic selector does not. [A1555]
3	Positive complex generation structure	OPEN	A registered positive complex polarization must descend from the paired 56-shell carrier to a selected three-complex-dimensional generation quotient compatible with the pointed frame. Object: The transported full-56 candidate descends as $R^2 = +I$, not as a complex generation structure. [A1555]
4	Visible noncentral image of the odd datum	OPEN	A registered source-character solder must carry microscopic odd data into the unique noncentral Hermitian S_3-sign line H_{sgn}, without identifying the A689 multiplicative grading with that character by fiat. Object: The A689 grading is registered only as multiplicative Z2 data; D1236 finds no registered full transported readout that could turn it into a physical sign response. [A1555]

Face	Name	A1555 status	Closing or blocking statement
5	Selected faithful observation door	OPEN	One Appendix-T door must be executable as a transported microscopic readout, nonzero on the relevant odd transition and well-defined modulo the ordered-frame stabilizer. Object: Appendix T names four doors and constructs a lower-rank MOS quotient, but Π^{full} and a transported reference/readout remain open; D1236 stops at gate 7. [A1555]
6	Physical spectral pair and relative left frame	OUTPUT GATE	The completed observation map must yield two normalized, normal, nondegenerate generation operators and a unique relative left frame modulo phases and common ordering. Object: This is the downstream success test, not an independent upstream datum; no physical pair is formed in D1236. [A1555]

Dated amendments to Table 1. The table above is reproduced from the A1555 JSON as required. The preceding box-top audit established the separation between transported pointing and intrinsic basepoint selection and required that distinction to govern the fold [A1554]. The subsequent adjudications sharpen the rows as follows, without rewriting that source record. Face 2 is now DEFINITIONAL AT EVERY SHIPPED REGISTERED TIER: the locked census found 30 candidate rows, 5 intrinsic rows and no intrinsic datum with a nonzero stabilizer-fixed vector. [LIB2-178] Its missing object is a theory-selected subgroup $H_{\mathrm{bp}} \subset E_{6(6)}$ with one non-null fixed line [A1561]. [LIB2-178] Face 3 closes locally at the transported point up to the shell-exchange torsor: the trace-dual polarization gives a positive three-complex-dimensional paired shell space, while intrinsic unpointed selection remains open [A1557]. [LIB2-013] Face 4 has a covariant local representative, unique as an equivalence class but still carrying the unspent torsor side; the direct A689 grading route is closed negative because its transported S_3 character is trivial [A1556,A1557]. [LIB2-180] Face 5 has a rank-3, relative-rank-2 record-quotient channel, but the microscopic support-restoring map and its cross-object normalization are not selected [A1558–A1560]. Face 6 remains an output gate and is not entered.

15.2 Why the invariant odd covector failed, and what replaced it

The first attempt demanded an invariant scalar row $\ell_{\mathrm{odd}} : J_- \to \mathbb{R}$. The largest stabilizer subgroup preserving J_- has no nonzero invariant covector there; the constraint stack has full rank with smallest nonzero singular value $1/\sqrt{2}$ [A1556]. [LIB2-179] The obstruction is representation-theoretic, not numerical. In the same round the A689 split-norm $\mathbb{Z}_2$ was computed to carry the trivial, not the alternating, S_3 character [A1556]. [LIB2-180] Those two statements retire the direct A689$\to H_{\mathrm{sgn}}$ shortcut.

The well-typed successor is state-dependent and covariant. [LIB2-179] On the transported paired-shell generation space G_6, the source intertwiner space $\mathrm{Hom}_{S_3}(J_-, \mathrm{sgn})$ has real dimension four, while its normalized sphere is a single orbit of the connected covariance group. [LIB2-181] The local observation

leg therefore has one continuous equivalence class,

$$\Pi_{\tau,[\ell]}(x) = \tau\,\ell(x) H_{\mathrm{sgn}},$$

with no surviving continuous source-line choice and one unspent endpoint-exchange torsor $\tau = \pm 1$ [A1557]. [LIB2-181] This is a transported-point construction; it is not yet the full microscopic observation map. [LIB2-181]

15.3 The MOS channel, the score solder, and the normalization no-go

At record-quotient tier a unital completely positive channel exists,

$$W_{\lambda,\tau}(A) = \frac{\operatorname{Tr} A}{2} I_3 + \frac{\lambda}{2} \operatorname{Tr}(QA)\,\tau H_{\mathrm{sgn}}, \qquad |\lambda| \le \sqrt{2},$$

and reproduces the local odd leg at the endpoint $|\lambda| = \sqrt{2}$ [A1558]. [LIB2-182] Appendix-T whitening does not force that endpoint: channels with distinct λ share the registered source whitening, covariance, zero-effect and torsor properties [A1558]. [LIB2-182]

The reversal-covariant score solder

$$\Xi : C^*(D_\varphi) \cong \mathbb{C}^3 \longrightarrow M_2(\mathbb{C})$$

also exists. Its odd amplitude obeys $|\mu| \le 1$, so the actual response is the product $|\lambda\mu| \le \sqrt{2}$ [A1559]. [LIB2-183] The M_5 lift remains an 11-dimensional quotient family, and the complete registered normalization census grew from 28 to 40 typed rows with zero forcing rows [A1559,A1560]. [LIB2-184] A rank-5 extension exists as ambient linear algebra, but the record-support condition forbids it at the printed MOS tier by an exact rank-two support certificate [A1560]. [LIB2-185] Thus the normalization ledger is closed at every tier reached by the supplied package; reopening it requires a genuinely microscopic support-restoring operator-system map. [LIB2-184, LIB2-185]

The companion wall object $\alpha : S_3 \to \mathrm{Aut}(A_{\mathrm{MOS}})$ is *absent* in the permutation class (the P_α census, A1559: no order-3 source permutation is compatible with the record projection); a non-permutation microscopic attachment is not excluded. Independently, any S_3 marking, even if selected, places D_φ in the 2-isotype and never in sgn (house `s1149`, $18 + 1$ gates), so the sign must come from the carrier side ($J_- = 4 \cdot \mathrm{sgn} + 4 \cdot 2$), not from the state pair. With A1559 the α row of the wall is closed for this arc.

15.4 The declared amplitude premise (PI ruling S300) and the two alternatives not adopted

PREMISE-GRADE-4-RESPONSE [A1560; declared S300]. On the transported record-quotient response, identify the normalized score response with the registered unit grade-four carrier response, fixing

$$|\lambda\mu| = 1.$$

This removes the one positive response-amplitude modulus and leaves the extreme-pair torsor side $\tau = \pm 1$ unselected. [LIB2-186] It does *not* construct Π^{full}, does not prove that the grade-four carrier is the microscopic readout, and derives no CKM quantity. [LIB2-186] **Status: PREMISE, declared by the PI over the A1560 three-box menu; not derived.** Retire it if a registered microscopic pullback supplies a different normalization or no normalization.

[LIB2-186]

The two alternatives on the same menu, recorded so that the choice is visible: *PREMISE-UNIT-RESPONSE* (the cross-object isometry $\|D_\varphi\|_{\mathrm{HS}} = |\lambda\mu|\,\|H_{\mathrm{sgn}}\|_{\mathrm{HS}}$, hence $|\lambda\mu| = \sqrt{2}$, the D1238 leg at the CP boundary) and *DECLINED* (no quotient-tier response-isometry premise; the registered result stays $|\lambda\mu| \le \sqrt{2}$ with one continuous magnitude). [LIB2-187] Neither is adopted. [LIB2-183, LIB2-187] Under every one of the three the physical spectral-pair gate is not entered, and the 40-row normalization census has no forcing row [A1559,A1560]: the value printed above is chosen, and the ledger records that it could not be forced. [LIB2-187]

15.5 The intrinsic-basepoint placeholder (declared, no subgroup selected)

DECLARED-BASEPOINT-PLACEHOLDER [A1561]. A future intrinsic selector must register, before naming any vector or frame, a subgroup $H_{\mathrm{bp}} \subset E_{6(6)}$ whose action on the unpointed 27 has a single non-null fixed line. The cubic normalization $N(x_0) = 1$ would then select one real point on that line, subject to the residual normalizer acting trivially. [LIB2-005] No subgroup is selected by this placeholder; without one, the transported point remains declared rather than intrinsically derived.

[LIB2-005, LIB2-325]

15.6 State after the fold

The arc has closed the Albert-to-shell carrier identification, locally supplied a positive paired-shell polarization and a covariant odd response class, and constructed the record-quotient UCP channel and reversal-covariant score solder. It has also proved two separate walls: the intrinsic basepoint is absent over the complete shipped candidate universe, and no registered normalization fixes $|\lambda\mu|$ (the printed $|\lambda\mu| = 1$ is a declared premise). [LIB2-184] The remaining route is therefore not another coefficient search. It is the joint construction of a theory-selected basepoint subgroup and a microscopic support-restoring observation map; only after both exist may the spectral-pair/output gate be opened [A1553–A1561]. [LIB2-005]

Leibniz Quantum Beats Newton

Appendix X — Input Ledger (Rev33.2)

Rev25 adds the registration of the framework's one dimensionful input (the depth-cylinder Bargmann class, §"The registered input"), supersedes the two-scale calibration paragraph, and banks the S209–S226 clean negatives.

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August 2026

Abstract

This appendix lists the algebraic inputs used by the fit-free PMNS+CKM sector of the Rev29 suite and separates them from measured anchors. Its purpose is referee-facing: the mixing sector carries no fitted *coefficients* within, and relative to, the loaded correspondence map — which after Rev29 is not the same as being zero-parameter (see the supersession box below) — the working vacuum J_{vac}, the Route B selection, and the gauge/representation embedding are declared *inputs*, loaded not derived frame-free (the frame-free selection is closed *negative*: Appendix F, Thm 7.1). "Zero-fit" is therefore a statement about the *interior* of the correspondence, not a claim that the boundary itself is derived; that claim is only useful if every surviving algebraic ingredient is named explicitly and its provenance is stated in one place.

Rev29 supersession (binding on every row below). The row-by-row re-tiering (Rev29 re-tiering, Paper 3 §"Row-by-row re-tiering"; kernel `s959`) moved six of eleven Paper 3 ledger rows. After propagation **two** of the eight mixing observables sit at Derived-conditional or above ($\theta_{12}^{\text{PMNS}}$, δ_{CP}, premises printed). The exact Jordan mismatch identity 7/16 remains theorem-grade, but the physical θ_{23} octant attachment is Loaded-correspondence (A1566/D1244, Rev32.6); the CKM *first row* is Loaded, because Paper 3 selected Route B over Route A by comparing both to kaon data; δ_{CKM} is Coincidence-class while the motivation for G_7 is missing; θ_{13} is structural and $|V_{cb}|$ is Loaded-correspondence (physical) / Structural formula. **No row in this ledger, and no row in Paper 3 Sections 2–5, may be cited as zero-parameter.** Every "six of eight" tally below is a pre-Rev29 marking and is superseded. No number changes.

Provenance convention (Rev27, two-axis; see Paper 0). The **Status** column below is the *mathematical* axis (the Paper 0 five tiers); each entry's *physical attachment* — derived-attachment / conditional-readout / loaded-correspondence / fitted / external-input — is carried in the evidence column and, where load-bearing, stated inline. In these terms "zero-fit" is a Loaded-correspondence statement (fit-free interior, declared-input boundary), not a Derived-attachment one.

Table 1: Fit-free-interior ledger for the loaded mixing correspondence. **Rev32.1 reading rule: tiers name mathematical provenance; physical attachments are stated separately; no row may be cited as zero-parameter; displayed σ values are named comparator distances, not profile likelihoods.**

Input or relation	Explicit form	Status	Algebraic source / evidence
Golden ratio	$\phi = (1+\sqrt{5})/2$	Mathematical constant	AX1; no fit to particle data.
Working vacuum	$J_{\text{vac}} = \text{diag}(\phi, 1, \phi^{-1})$	CONDITIONAL SELECTOR THEOREM (Appendix E; DET-7 as structural postulate)	Vacuum-selector theorem: uniquely forced by unit determinant + inversion symmetry + DET-7 (field-free), and independently as the golden-field unit triple; see Appendix E [closes A641/Q5A].
DET-7 invariant	$\det(\text{Gram}(J_{\text{vac}}, J_{\text{vac}}^{\#})) = 7$	STRUCTURAL (derive-target)	Fixes $n_{23} = 7$ and the mismatch normalization; algebraically $\det G = 7 \Leftrightarrow \sqrt{5} \Leftrightarrow \phi$ (Appendix I, §2, the $\det G$ lemma). First-principles origin of the integer 7 remains open (Appendix I, Q4) — carried as a named structural input, not promoted to an axiom.
C1 coefficient	$T_{\text{half}} = T_{\text{scalar}} + \frac{1}{2}T_{\text{oct}}$	Theorem	Exact $c = 1/2$ from DET-7, Frobenius norm, and Peirce orthogonality.
V_3 rotation frequency	$\Omega = \sqrt{5}/2$	Theorem	Equal to the $(1,3)$-sector eigenvalue of $L_{J_{\text{vac}}}$.
Canonical PMNS time	$t^* = 2\pi/\sqrt{5}$	Theorem	From $\exp(t^* T_{\text{half}}) J_{\text{vac}} = J_{\text{vac}}^{\#}$.
Solar angle	$\tan\theta_{12}^{\text{PMNS}} = \sqrt{3}/\phi^2$	DERIVED-COND. (Rev29 re-tier; ordering ladder- and interface-conditional)	Peirce spectral ratio with $\phi^2 + \phi^{-2} = 3$; identity exact.
Atmospheric mismatch / physical θ_{23} angle	$\sin^2\theta_{\text{mismatch}} = 7/16$	internal theorem; physical LOADED-CORRESPONDENCE	DET-7 mismatch geometry exact; lower-octant registration selected against data; `s501` does not supply a unique selector (A1566/D1244).

Input or relation	Explicit form	Status	Algebraic source / evidence
Reactor angle	$\sin^2\theta_{13} = \sin^4(\pi/8)$	Structural $\star\star\star$	D_4 triality plus half-angle identity; round-trip T^2 squaring identified (exponent-2, s502/A1275), forcing pending; *Rev29*: the $\pi/8$ half-angle is **not** canonicalized by the $T^2 = -1$ moment structure (that reading re-killed S261) — it is a declared readout normalization; numerical agreement at 1.8σ (NuFIT 6.1: $\sin^2\theta_{13} = 0.02248^{+0.00055}_{-0.00059}$).
Leptonic Dirac phase	$\delta_{CP} = -2\pi/\sqrt{5}$	DERIVED-COND. ($(1,3)$ framing, ordering)	V_3 rotation theorem; identity exact.
Cabibbo entry	$\lvert V_{us}\rvert = \sin\theta_{12}^{\mathrm{PMNS}}/\sqrt{6}$	LOADED (route selected against kaon data; Rev29 first-row tier)	Route B / Peirce J_{12} weight; identity exact given the route.
Quark 2–3 entry	$\lvert V_{cb}\rvert = 1/(9\sqrt{7})$	LOADED-CORRESPONDENCE (physical) / STRUCTURAL mathematics	DET-7 "7 unification"; inherits the loaded atmospheric attachment; integer 9 unforced and recorded as data-selected (A1566).
Quark 1–3 entry	$\lvert V_{ub}\rvert = \lvert V_{us}\rvert\lvert V_{cb}\rvert/\sqrt{6}$	LOADED (inherits the first row)	Hierarchy relation after fixing $\lvert V_{us}\rvert$ and $\lvert V_{cb}\rvert$.

Input or relation	Explicit form	Status	Algebraic source / evidence
CKM Dirac phase	$\delta_{\mathrm{CKM}} = \arctan\sqrt{3(\phi + 5\phi^{-5})} = 68.13°$	COINCIDENCE-CLASS; PDG 2026 comparator distances (source ledger, shipped engine and resolver scope: the note after this table): $+1.40\sigma$ from the global-fit phase 1.154 ± 0.025 rad (its published representation, which is primary — R-29; the degree cells $66.12° \pm 1.43°$ give $+1.41\sigma$) and $+0.64\sigma$ from the direct $\gamma = 66.4^{+2.7}_{-2.8}{}^{\circ}$	The first-principles origin of the $5\phi^{-5}$ correction remains open; the arithmetic identity does not supply the missing motivation. The N-1 blind formula census prices this form at its complexity ceiling (census note after the source-ledger note below).
Resolvent family	$R_d = 1/(\phi^{2d} - 1)$	Structural exact family	Used in the mass sector; not fitted to CKM/PMNS data. Superseded for m_μ/m_τ by the compact ratio, which is itself STRUCTURAL / LOADED-CORRESPONDENCE (*re-tiered Rev29*; the "proved, zero free parameters" typing is withdrawn).
Measured anchors	$\alpha,\ v_H,\ M_{\mathrm{Pl}}$	Measured	External anchors; not tuned to close the mixing sector.
Mass anchor	$m_\tau = 1776.93(9)\,\mathrm{MeV}$ (PDG; CODATA 2022 lists 1776.86(12))	Measured	Single external anchor; gives $m_\mu = 106.05\,\mathrm{MeV}$ via $m_\mu/m_\tau = (\phi/\sqrt{5})^{8/3} \times \sqrt{2}/10$, re-tiered STRUCTURAL / LOADED-CORRESPONDENCE / REPRODUCED (*Rev29*); the "zero free parameters" citation is **withdrawn** — see "Provenance of the charged-lepton ratio" below.

Input or relation	Explicit form	Status	Algebraic source / evidence
Koide selector	$K = 2/3$	External selection rule	Integral to the m_e chain (KOIDE-CONSISTENT ⋆⋆⋆⋆ under this external rule): J_3 UV BC at Λ_{G_2} forces Koide to act on $m_{\mu,\mathrm{phys}} = 105.72\,\mathrm{MeV}$ [A636, A637].
Mass exponent	$p = 8/3 = \dim(\mathbb{O}_s)/\mathrm{rank}(J_3)$	Exact dimension ratio [A616]; LOADED-CORRESPONDENCE (*Rev29*)	The dimension ratio is an exact algebraic fact; that it is *the* mass exponent is not, alone, established by it (two-axis rule; Paper 0 prints the same row as "exact dimension ratio; LOADED-CORRESPONDENCE: not, alone, a mass exponent"). *Rev29*: the PROVED typing is withdrawn.
Normalisation constant	$C = \sqrt{2}/10 = \sqrt{2}/\dim(J_2(\mathbb{O}_s))$	Exact dimension count [A618]; LOADED-CORRESPONDENCE (*Rev29*)	$J_2(\mathbb{O}_s) = \mathbb{R}f_1 \oplus \mathbb{R}f_2 \oplus P_{12}$, dim = 10; off-diagonal trace norm = $\sqrt{2}$. The *divide-by-dimension* step is argued by analogy (A618:134–140), and A618:147 lists the formal derivation as BLOCKING — closed nowhere in the corpus. *Rev29*: the PROVED typing is withdrawn.
Tau-sector dynamical scale	$\Lambda_{G_2} = 260 \pm 0.5\,\mathrm{MeV}$ ($\pm 0.19\%$, the rounding rule, R190)	Calibration: the value is taken from hadronic phenomenology; it is not a measured Λ_{QCD} in any scheme (scheme table; Λ_{G_2} note below the input-count box) [A618]	Analogous to Λ_{QCD}; one physical dimensional-transmutation anchor for the heavy-lepton sector; not algebraically derivable.
Weak-angle selector	$\mathrm{AX6}_{\mathrm{pol}}$: $\Lambda_\star = 3P/Q(Y) = 6\phi - 1$ on the exact-colour orbit; readout $\sin^2\theta_W = 3/(4+\Lambda_\star) = \phi^{-3}$ exactly. The Det^2 object $\cos(6\psi) = \frac{1}{2}$ (50°) is a distinct object, retired as a selector of $\mathrm{AX6}_{\mathrm{pol}}$ (Rev32.2; `s1135`/A1549)	Independent orbit-selector postulate	Two objects carried one name and one symbol $\psi_\star$ in print; separated in Paper 4 (Rev32.2). A667's degree argument applies to the Det^2 object only. ϕ^{-3} is a constant algebraic identity, not a function of the orbit angle; $\mathrm{AX6}_{\mathrm{pol}}$ fixes the orbit branch and remains independent of the present framework.

Input or relation	Explicit form	Status	Algebraic source / evidence
Quark Gauge-Protection	$J_{\text{vac}} \in \text{singlet}(G_2) \Rightarrow \partial^2 V_{\text{CW}}/\partial P_{ij}^{\text{triplet}} = 0$	block-diagonality PROVED; the six zero triplet modes a CERTIFIED computation (A630); the Schur inference to a zero triplet Hessian at every loop order is *withdrawn* (S290.4; App. A §A.7.5 — this row kept the pre-ruling stamp through Rev32.8)	A630
Roman surface model	$(y'z', z'x', x'y') =$ Peirce product	STRUCTURAL ⋆⋆ ⋆⋆	A630, A632
Top Yukawa	$y_t = 1$; $m_t = v_{\text{EW}}/\sqrt{2} =$ 174,104 MeV (+0.87% above the PDG 2026 direct-measurement (MC) mass 172.60 ± 0.27 GeV; tree-level vs. pole-proxy, cross-scheme, declared at the scheme table below; naive comparator distance $d_{\text{cmp}} = +5.57$, experimental-error units only)	STRUCTURAL ⋆⋆ ⋆⋆ (*re-tiered Rev29*); d_{cmp} is *not* a tension and sets no status (Rev32 retype per the A1531/T2 cross-scheme adjudication; only a scheme-matched Z_{matched} may set BLACK/RED, and none is yet computable for this row)	A632
Light quark constituent	$m_{u,d}^{\text{const}} = \Lambda_{G_2}\varphi^{1/2} \approx$ 331 MeV	STRUCTURAL ⋆⋆ ⋆⋆	A631

Input or relation	Explicit form	Status	Algebraic source / evidence
Artin bridge	2-pt/3-pt QFT loops Artin-protected; std. RG valid	Artin's theorem PROVED (mathematics); its use as loop protection STRUCTURAL, scoped to the outstanding generator census (Appendix A §A.7; Rev32.9, A1642 O3)	A633
7/3 unification ratio	$n_{23}/\mathrm{rank}(J_3) = 7/3$; $m_b = (7/3)m_\tau = 4146\,\mathrm{MeV}$ (-0.95% vs. PDG 2026 $m_b(m_b) = 4186 \pm 6\,\mathrm{MeV}$; tau-pole-anchored bridge vs. $\overline{\mathrm{MS}}$, cross-scheme, declared; $d_{\mathrm{cmp}} = -6.64$; percentages and distances in this ledger are computed at full framework precision, $m_b = 4146.17$, Rev32.9); $m_c = (7/3)m_s^{\mathrm{const}} = 1291.9\,\mathrm{MeV}$ ($+1.49\%$ vs. PDG 2026 $m_c(m_c) = 1272.9 \pm 4.5\,\mathrm{MeV}$; constituent-anchored vs. $\overline{\mathrm{MS}}$, cross-scheme, declared; $d_{\mathrm{cmp}} = +4.22$; $+3.69$ with the Λ_{G_2} rounding band folded in, `s1211`)	STRUCTURAL ⋆⋆ ⋆⋆; the d_{cmp} values are naive cross-scheme comparator distances (experimental-error units only), not tensions, and set no status (Rev32 retype, A1531/T2)	A635, A637, A641. DET-7 Gram-ratio bridge. *Independent route*: $\dim(\mathrm{Im}(\mathbb{O}_s))/\mathrm{rank}(J_3) = 7/3$ (vacuum-free, A641/Q2A). Golden vacuum unique (algebraic-integer + palindromic char. poly. conditions) [A641/Q5A]. Generation-independence PROVED [A637/Q1B]. Formal S-matrix proof open [A640/Q1D].

Input or relation	Explicit form	Status	Algebraic source / evidence
QED correction at Λ_{G_2}	$\delta_{\rm QED} = \frac{3\alpha}{4\pi}\ln\frac{\Lambda_{G_2}^2}{m_{\mu,\rm phys}^2} = +0.3125\%$	Standard leading-log QED (imported physics); attachment STRUCTURAL: the cutoff at Λ_{G_2} is the framework's declared calculation input (N17; Rev32.9, A1642 O3)	A637/Q2A–Q2D. Artin-protected (2-gen associative). Direction: mass grows UV, so $m_{\mu,\rm phys} = m_\mu^{\rm tree}/(1+\delta)$.
m_e QED-Koide chain	$m_e = 0.5076\,$MeV (-0.66% PDG) via tree→QED→Koide	KOIDE-CONSISTENT ⋆⋆⋆⋆	A636, A637. Chain: $m_\mu^{\rm tree} = 106.05 \to \delta_{\rm QED} \to m_{\mu,\rm phys} = 105.72 \to K = 2/3 \to m_e = 0.508\,$MeV. The electron closure depends on the external empirical Koide rule $K = 2/3$.

Source ledger, shipped engine and resolver scope (moved out of the $\delta_{\rm CKM}$ row at Rev32.9: the table cell had grown past a page and lost its end in the PDF). Every printed comparator in this suite resolves to one row of the source ledger shipped *in the release bundle*, not embedded in this PDF: `ledgers/SOURCE_LEDGER_REV32.csv`, 61 rows with stable ids SL-01–SL-61, columns id / quantity / central / plus / minus / unit / scheme-scale / edition / status / source / url / value_kind, sha256 1891408410f5ffaf… (Rev32.6: six minus-exponent fields re-signed and the id column added, A1569; Rev32.9: the value_kind column added — SL-04/05 are 95% upper bounds, SL-27 a derived diagnostic, every other row a central value, S304.3; Rev33.0: SL-31 added, the house $J_{\rm PMNS}$ diagnostic, and the SL-12/13 scheme labels corrected from "PDG global fit" to the PDG 2026 direct averages, R174-c; Rev33.1: SL-32–SL-36 added — the five scorecard comparators that had lived only in the engine's INPUTS sheet, NuFIT 6.1 $\sin^2\theta_{12}$ and $\delta_{\rm CP}$, PDG 2026 m_μ and m_e, and the constituent m_u, typed `model_dependent_input` (a constituent value, not a measurement); S392 (working ledger, merged ahead of the Rev33.2 print): SL-37–SL-56 added — the JUNO 59.1 d rows and design-reach projections (SL-37–SL-47) and the inputs of the $\Phi_{\rm mass}$ scheme slice `s1336` (PDG 2026 $\alpha_s(M_Z)$, M_Z, M_W, M_H, $\alpha(0)$ and the Buttazzo *et al.* matching formulas, SL-48–SL-56), SL-50 a withdrawn duplicate of SL-06 kept as a tombstone so the ids stay contiguous; S395: SL-57 added — the measured-side $\overline{\rm MS}$ $y_t(M_Z)$, the SL-54 extraction run down to M_Z by the house kernel `s1337`, typed `derived_diagnostic` (a house-computed comparator, not a primary source), with SL-58 (the $\Delta m^2_{21}/\Delta m^2_{31}$ ratio from SL-01/SL-02, `s1330`/`s1359`) and SL-59 (the data-side $y_t(172.60\,{\rm GeV})$ from SL-54, `s1336`) of the same type — each row carries the printed rounding; S396: SL-60 added — $\Lambda_{G_2} = 260 \pm 0.5$ MeV, the rounding-rule band of R190, typed `declared_input` (a declared conversion scale: neither a measurement nor a derivation), with SL-61 ($\chi_c = 1.618 \pm 0.382$ under `s1182`'s $\kappa_{\rm odd}$, `s1218`) typed `derived_diagnostic`; Rev33.1 shipped 783e15283c443d7e…; Rev33.0 shipped ffe406dc8869e62a…; Rev32.11 shipped b0101556e1e2d20d…); gate `scripts/comparator_census.py`: 21 printed comparators resolved at the Rev32.6 cut, 20 at

the Rev32.9 cut, 21 at the Rev33.0 seal, 0 unmapped, 0 malformed fields. *The engine this suite ships (rolled at Rev33.1, unchanged in Rev33.2):* `Engine/SGTOE_Machine.xlsx` sha256 32875b1696a2… and `Engine/SGTOE_Outputs.xlsx` sha256 4e6e7c32b307… (`ENGINE_MANIFEST` 3/3; Rev33.1: the NuFIT 6.1 pair $\Delta m^2_{21}, \Delta m^2_{31}$ and $\sin^2\hat{\theta}_W(M_Z)$ moved onto the source ledger (SL-01, SL-02, SL-06) — the engine and its generator had carried 7.49×10^{-5}, 2.513×10^{-3} and the rounded 0.2312, so the generator printed a 9.27× mass-reading mismatch where the suite prints 9.21×; generator kernel v5.0.12; no ledger row was re-typed and no tolerance width moved; Rev33.0 shipped b47f7bc9db75… / a67bb0ebddb4…, where the comparators $|V_{cb}|$, $|V_{ub}|$, $\delta_{\rm CKM}$ and m_c moved from the superseded PDG 2025 values to the PDG 2026 rows of the source ledger, and the m_t comparator *value* followed the label and the κ reference that Rev32.11 had moved (R171); four own-output ledger rows were re-typed to the moved values and no tolerance width moved — two of them, $\kappa(t)$ and χ_c, recalculate to FAIL in the Rev32.11 engine behind a stale cached PASS, which this roll found; $|V_{cb}|$ now passes its $<1\sigma$ row at 0.997σ; Rev32.11 shipped 9ab03a2efac2… / 337d899aad01…, where ONE input moved — m_t from the superseded PDG 2025 172.56(31) to PDG 2026 172.60(27) GeV, completing on the top the PDG 2026 comparator roll Rev32.10 began on the bottom, so that the paper and the engine hold one comparator (S347.4); the two RCI tolerances were untouched and both still pass, at 0.67σ; Rev32.10 shipped 650634658022… / acd9a7f14da4…, where ONE input moved — m_b from the superseded PDG 2025 4183 ± 7 to PDG 2026 4186 ± 6 MeV, R140 — and three ledger rows were re-typed to PDG-σ comparators; Rev32.9 shipped 6d489626b664… / 26377c02e619…, the re-stamp that renamed the 27-row sheet a regression/comparator/identity ledger with no input moved; Rev32.6–32.8 shipped 7b09e1a2780b… / 81d3ba688401…); the public kernel `sgtoe_kernel_v5.0.12.py` sha256 d297f5b2… (v5.0.11, sha256 1b2f244d…, stays on disk as the Rev33.0 bundle's kernel; v5.0.10, sha256 33a1db19…, as the Rev32.10 and Rev32.11 bundles' kernel, and v5.0.9, sha256 7fcb12b3…, as the Rev32.9 bundle's; v5.0.8 stays sha-pinned as the dependency of `s1133`/`s1140`); the release MANIFEST count is printed in the bundle's `VERIFY_REPORT.md`. Every other engine hash in this suite (Paper 0's Rev19 box, Appendix O's S179–S192 pair) is dated to its era. *Scope of the resolver:* `scripts/resolve_proof_artifacts.py` certifies *presence and hash* of every print-cited kernel in a canonical home; it does not execute kernels or diff a certificate against current wording, so its pass line is a locator check, not a substantiation (A1572 F3) — a PDF-only reader can check the claim's shape here and its content from the bundle.

N-1 blind formula census (`s1206` v3; rulings R186–R188; the row-12a note is R187-b). Each of the 24 scorecard rows with an external comparator (rows 1, 2, 3, 14 and 15 are excluded by rule: they have no comparator) was priced against one fixed expression language — alphabet $\{\phi,\pi,2,3,5,7\}$, with $\{\phi,\sqrt{5},7,8,9,16,\pi\}$ as a second alphabet — of at most three binary operations and complexity ≤ 6, enumerated exhaustively (checked against brute force) and locked before any comparator was read. The suite's value lies inside its declared band on 13 of 24 rows. On two of them, $m_t/v_{\rm EW}=1/\sqrt{2}$ and $m_b/m_\tau=7/3$, it is the simplest in-band value in the first alphabet but not in the second (15 and 16 values there are as simple); both are cross-scheme rows scored at a ±1% tolerance, which both leave at ±0.8%, and their naive comparator distances are $d_{\rm cmp}=+5.57$ and -6.64 (experimental-error units only). Against comparators of one scheme and scale (`s1360`: the $\overline{\rm MS}$ $y_t(M_t)/\sqrt{2}$ for the first; $m_b(m_b)/m_\tau(m_b)$ at $\mu=m_b$, the scheme Table T5-A declares, for the second) neither value lies in the ±1% band (+7.2% and −1.5%), so the rank-one reading holds only against the cross-scheme comparators named here. The rank used here is an occupancy count of this language at the suite's own complexity, not a significance: the census prices the scorecard; it does not test it. *Row 12a:* the G_7-corrected $\delta_{\rm CKM}$ (68.13°, $+1.40\sigma$ from the global fit, outside 1σ) is reached in the census language only at complexity 6 — the ceiling — where every in-band value is at least as simple (rank = the whole band: 547,917 values in the first alphabet, 880,986 in the second); the bare-ϕ control (65.59°, -0.37σ, inside) is reached at complexity 3 with rank 396 in the first

alphabet (complexity 4, rank 10,257 in the second). The correction moves δ_{CKM} away from the fit and costs three units of complexity in the first alphabet (two in the second); if trigonometric functions are charged two units instead of one, it is not reached at all within the ceiling. The previous version of the census was reproduced independently by an executing reader who was never shown its answer hash; that reader's audit found the completeness defect and the unit and power defects repaired in this version. The reader had saved programme memory, so the read was not cold (R188). Record: `Kernels/current/s1206_n1_blind_formula_census.py` and its complexity-6 host record (two byte-identical runs).

Bottom line. The PMNS+CKM sector uses no fitted angle, phase, or mixing-strength parameter *inside* the loaded correspondence map — which, after the Rev29 re-tiering (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`), licenses no zero-parameter citation for any row: two of eight stand at DERIVED-CONDITIONAL or above ($\theta_{12}^{\mathrm{PMNS}}$, δ_{CP}; A1566 retype, Rev32.6), the physical θ_{23} row and $|V_{cb}|$ are LOADED-CORRESPONDENCE, the CKM first row is LOADED, and δ_{CKM} is COINCIDENCE-CLASS. The algebraic inputs are fixed by theorem-level Jordan/Freudenthal structure, while the measured anchors are standard physical constants and, for the charged-lepton mass subsection, the single tau-mass input plus the measured heavy-lepton dynamical scale where explicitly stated.

Time-layer inputs (Rev15, Sessions 106–108; see Appendix H). The post-Rev14 time layer adds no new admissible constant: the substrate clock frequency ω_0 is the electron Compton/zitterbewegung rate, i.e. the clock normalisation $q_0^{\mathrm{clk}} = m_e$ (a unit choice inside the one registered class — see the clarification below the input-count box; through Rev32.8 this read "the single dimensionful anchor") (the clock anchor, distinct from the KK tower quantum q_0^{KK} of Appendix G) read in the Compton frame ($m_e = \hbar\omega_0/c^2$ then *derives* $c, \hbar$ from ω_0), so the admissible-constant set is unchanged at $F_3 = \{q_0^{\mathrm{clk}}, \kappa, \Gamma\}$ with $\kappa = 1/(3\sqrt{2})$ exact. The time-reversal bridge is recorded as a structural fact on the certified substrate, not a new parameter: $T = \exp(\frac{\pi}{2}K)$ is a genuine element of $E_{7(7)}$ (symplectic) with $T^2 = -1$ (the Kramers/spinor half-turn). **Rev29 — the physical reading is stripped.** $T^2 = -1$ is an algebraic fact about the certified substrate and carries *no* readout licence. In particular it does **not** make the half-angle $\pi/8$ canonical for $\sin^2\theta_{13}$: it motivates a readout normalization that is not derived from the algebra, and a motivation is not a derivation. This reading was killed at S188, resurrected at S189, and re-killed at S261; none of the three reached print before Rev29 (the load-bearing site is the Appendix B readout lemma). No public observable moves; these are classical, finite statements (Appendix H, Definition-A posture).

Honest precision summary. The fermion mass ledger prints percentage offsets on twenty named rows, each $\lesssim 2\%$ from the PDG central value by percentage — offsets, not predictions with status: the σ-status of the heavy-quark rows is withheld pending a scheme-matched comparator (m_t, m_b, m_c are cross-scheme, no status), and the rows carry the tiers of their own tables. However, percentage agreement and σ-level tension are not equivalent when PDG measurements are sub-percent precise. Standard scientific measure (σ) gives:

- *Mixing sector* (PMNS+CKM, 8 observables): the current named comparator record is $\theta_{13} = -1.8\sigma$, $|V_{us}| = +1.11\sigma$, $|V_{cb}| = +1.00\sigma$, $|V_{ub}| = -0.18\sigma$, $\delta_{CP} = 0.36\sigma$, and $\delta_{\mathrm{CKM}} = +1.40\sigma$ against the global-fit phase (radians, R-29) or $+0.64\sigma$ against the direct UT angle. These are central-value comparator distances, not profile-likelihood confidence levels. The measured-angle $|V_{us}|$ readout and the 0.003855 $|V_{ub}|$ rounding are separately labelled controls. The pre-refresh $0.52/0.38/1.3\sigma$ stamps are historical audit entries, not current results.
- *Lepton masses*: m_e (-0.66%) and m_μ (phys, $+0.06\%$) are theoretically limited by matching-scale uncertainty; PDG experimental precision far exceeds current theory precision.

- *Top quark*: $m_t = 174{,}104\,\text{MeV}$, +0.87% above the PDG 2026 direct-measurement (MC) mass $172.60 \pm 0.27\,\text{GeV}$ (the cross-section pole mass is a distinct object); tree-level vs. pole-proxy, cross-scheme, declared. [LIB2-220, LIB2-312] Naive comparator distance $d_{\text{cmp}} = +5.57$ (experimental-error units only; not a tension, sets no status — Rev32 retype, A1531/T2). The mass row is STRUCTURAL (re-tiered Rev29); the algebraic statement $y_t = 1$ carries its own tier in the main ledger.

- *Bottom quark*: $m_b = 4146\,\text{MeV}$, −0.95% vs. PDG 2026 $m_b(m_b) = 4186 \pm 6\,\text{MeV}$; tau-pole-anchored bridge vs. $\overline{\text{MS}}$, cross-scheme, declared; $d_{\text{cmp}} = -6.67$ (not a tension, sets no status). STRUCTURAL ⋆⋆⋆⋆; formal S-matrix completion open [A640].

- *Charm quark*: the charm bridge value $m_c = 1291.9\,\text{MeV}$ lies +1.49% above the PDG 2026 $\overline{\text{MS}}$ comparator $m_c(m_c) = 1272.9 \pm 4.5\,\text{MeV}$. Dividing by the comparator's experimental error gives a naive cross-scheme comparator distance of +4.22, but the bridge is constituent-anchored and no constituent-to-$\overline{\text{MS}}$ conversion or scheme uncertainty is supplied; the number is therefore not a physics tension and does not carry a BLACK classification. A scheme-matched significance awaits an explicit conversion with uncertainty.

- *Atmospheric angle*: $\sin^2\theta_{23} = 7/16 = 0.4375$; NuFIT 6.1 NH (w/SK) global best fit $0.470^{+0.017}_{-0.014}$ (*lower octant*, 2.3σ); upper-octant local comparator ≈ 0.561 ($\approx 9.5\sigma$); 7/16 within current 3σ allowed range. **Not yet excluded; watch DUNE.**

The +0.87% and −0.95% offsets in m_t and m_b are the programme's most pressing quantitative challenges in the mass sector. **Rev32 retype (A1531/T2), superseding the Rev29 sigma framing.** The convention is now two-column: the *primary* statement is the signed percent offset with both schemes named inline; the *secondary audit* figure $d_{\text{cmp}} = (M_{\text{bridge}} - M_{\text{cmp}})/\sigma_{\text{exp}}$ is a naive cross-scheme comparator distance in experimental-error units only — not a tension, never status-setting. Only a scheme-matched $Z_{\text{matched}} = (M_{\text{bridge}} - M_{\text{cmp}})/\sqrt{\sigma^2_{\text{exp}} + \sigma^2_{\text{theory}} + \sigma^2_{\text{matching}} + \sigma^2_{\text{scheme}}}$ may set BLACK/RED, and no such conversion with uncertainty is currently supplied for these rows. [LIB2-312] This applies retroactively: the earlier bare $-5.3\sigma/+4.9\sigma$ (and the S205-era $+2.1\sigma$ "not BLACK" charm reading) were invalid on their own terms; the honesty-ledger BLACK rows are correspondingly retyped as comparator-distance records pending a scheme-matched Z. The percent offsets stand and are recorded here as *unresolved quantitative offsets of the mass sector*, not as incomplete standard matching (A713: standard pole/$\overline{\text{MS}}$ conversion moves the top the wrong way; m_b/m_c fit only on a tuned scale). The residual pattern carries a weak-isospin signature $\delta m_f/m_f = -\kappa T_3(f)$, $\kappa \simeq 1.824\%$ (equal-and-opposite t/b, charm needs a further factor χ_c, whose value depends on the κ normalisation, Paper 2 §3.6), which is STRUCTURAL, not derived: the carrier search (A715/A716) and a boundary-condition revision (A717) both closed negative, so κ and χ_c are phenomenological. Known electroweak mass-definition effects are percent-level and scheme-dependent in this sector (one-loop scheme band $[-0.8, +3.8]$ pp; A719b); the $-\kappa T_3$ reading is scheme-conditional, and the operative test remains the t/b equality at improved precision. m_τ is the external anchor and m_μ is STRUCTURAL (re-tiered Rev29; the resolvent short form is a loaded correspondence — see the provenance box below); m_e is KOIDE-CLOSED ⋆⋆⋆⋆ under external $K = 2/3$. Bottom and charm quarks STRUCTURAL ⋆⋆⋆⋆ pending formal S-matrix completion.

Scheme-label declaration (T5-A; S64, A724)

Every mass-sector comparison in this suite is now explicitly scheme-labelled. The declaration below states, for each framework object, which renormalized SM object it is, what it is compared against, and whether any scheme conversion is applied. No printed central value changes from these labels; they exist so that no comparison can be misread. Two rulings matter most. First, the 7/3 bridge value $(7/3)m_\tau = 4146\,\text{MeV}$ is a *tau-pole-anchored algebraic bridge value compared directly to* $m_b(m_b)_{\overline{\text{MS}}}$; it is *not* a bottom pole mass, and no pole$\to \overline{\text{MS}}$ conversion is applied (treating it as a pole mass and converting would shift it by -9% to -18% at one to four loops — a category error, not a property of the bridge). [LIB2-313] Second, $\kappa \simeq 1.824\%$ is a phenomenological dimensionless residual extracted in *mixed* comparison conventions (top tree-vs-direct/MC, transferred across a direct/MC$\to \overline{\text{MS}}$ boundary in the bottom bookkeeping); it is structural bookkeeping, not a scheme-clean prediction.

Rev33.2 declarations and their cells. The PI declared one scheme and one scale per structural number (S392, given blind — before any converted value existed), and `s1336` emits the cells. (i) $y_t = 1$ is $\overline{\text{MS}}$ at $\mu = M_{\text{Pl}}$; the Planck convention is not declared, so the non-reduced $1.2209 \times 10^{19}\,\text{GeV}$ and the reduced $2.435 \times 10^{18}\,\text{GeV}$ are both run and their spread is priced. Run in the SM with the gauge couplings and λ fixed by data at $\mu_0 = 172.60\,\text{GeV}$ (SL-55), it gives $y_t(\mu_0) = 1.211$ (conversion price 0.020) against the data value 0.9330 ± 0.0017 (SL-54): EXCLUDED AT THE DECLARED SCHEME ($Z = +14.0$). (ii) The 7/3 bridge relates $\overline{\text{MS}}$ running masses at $\mu = m_b$, $m_b(m_b) = (7/3)\,m_\tau(m_b)$, with m_τ taken pole$\to \overline{\text{MS}}$ (QED) and run to m_b: $4124.4\,\text{MeV}$ (conversion price $2.2\,\text{MeV}$) against $4186 \pm 6\,\text{MeV}$, -1.47%, EXCLUDED AT THE DECLARED SCHEME ($Z = -9.65$). (iii) ϕ^{-3} is the tree-level Lagrangian s_0^2 at the algebraic boundary (A698): UNSCALED, no SM comparator at a known scale; its $+2.10\%$ against $\hat{s}^2(M_Z)$ is an audit number. (iv) $m_c = (7/3)m_s^{\text{const}}$ is INCOMPARABLE BY SCHEME (constituent anchor, no conversion registered); its $+1.49\%$ is a cross-scheme audit number. The labels 'tree-level EW-boundary' (m_t) and 'tau-pole-anchored, direct' (m_b) in the table and in the first ruling above are the Rev33.1 bookkeeping; as declarations they are retired.

Table 2: Scheme-label declaration (T5-A). "Conv." = standard scheme conversion applied.

Object	Framework-side label	Comparator	Conv.?	Caveat
$v = (\sqrt{2}G_F)^{-1/2}$	G_F-derived, tree-level SM convention (246.21965 GeV)	— (input)	—	measured anchor
$m_t = v/\sqrt{2}$	*declaration retired (Rev33.2)*: tree-level Yukawa/EW-boundary value; no loop dressing; not pole, not $\overline{\text{MS}}$; declared $y_t(M_{\text{Pl}}) = 1$, $\overline{\text{MS}}$	PDG 2026 direct/MC $172.60 \pm 0.27\,\text{GeV}$ (pole proxy)	no	$+0.87\%$; $d_{\text{cmp}} = +5.57$ is a cross-scheme comparator distance only; declared cell excluded, $Z = +14.0$ (`s1336`)
m_μ/m_τ	charged-lepton pole-mass ratio	PDG pole masses	no	scheme-safe at quoted precision
m_e (QED–Koide)	physical pole mass after one-loop QED dressing, on-shell α, at Λ_{G_2}	CODATA/PDG pole	QED 1-loop	QED scheme/scale convention carried; T4-G open

Object	Framework-side label	Comparator	Conv.?	Caveat
$m_b = (7/3)m_\tau$	*declaration retired (Rev33.2)*: tau-pole-anchored algebraic bridge value; declared $\overline{\text{MS}}$ $m_b(m_b) = (7/3)m_\tau(m_b)$	$m_b(m_b)$, $\overline{\text{MS}}$ at $\mu = m_b$	m_τ: QED + running	**not a bottom pole mass**; declared cell 4124.4 MeV, −1.47%, excluded, $Z = -9.65$ (`s1336`)
$m_c = (7/3)m_s^{\text{const}}$	constituent-anchored algebraic bridge value	$m_c(m_c)$, $\overline{\text{MS}}$ at $\mu = m_c$	no	constituent→ $\overline{\text{MS}}$ not applied; INCOMPARABLE BY SCHEME (Rev33.2)
$m_{u,d,s}^{\text{const}}$	constituent masses (Λ_{G_2} tower)	—	—	declared incomparable to PDG current masses
$\Lambda_{G_2} = 260 \pm 0.5$ MeV	phenomenological G_2 confinement anchor	$\Lambda_{\overline{\text{MS}}}$ analogy only	no	scheme/n_f not derived; nearest n_f=4 $\Lambda_{\overline{\text{MS}}}$-like if forced; not a measured Λ_{QCD}
$\kappa \simeq 1.824\%$	dimensionless phenomenological residual in mixed conventions	top direct/MC; bottom $\overline{\text{MS}}$ bookkeeping	no	convention transfer declared; not scheme-clean; 0.0115 pp per 10 MeV of m_t
$\sin^2\theta_W = \phi^{-3}$	tree-level Lagrangian s_0^2 at the algebraic boundary (A698), UNSCALED; the $\overline{\text{MS}}$-curve reading $\hat{s}^2_W(\mu)$ is retired as a declaration (Rev33.2)	PDG 2026: $\hat{s}^2_W(M_Z) = 0.23122 \pm 0.00006$; on-shell 0.22348 ± 0.00010	no	$\mu_\star$ scheme-dependent, not derived; the effective and on-shell quantities are distinct scheme objects
PMNS/CKM	dimensionless fit observables	NuFIT 6.1 (NO, w/SK branch stated) / PDG	no	CKM global-fit phase vs tree-level γ kept separate

Provenance taxonomy: Proved / Structural / Input

The ledger above mixes three logically distinct kinds of ingredient, and a referee is entitled to see them separated. The active input ledger groups current quantities into exactly one of three operational classes (Rev32.1, A1538 F9: this is the ledger's operational grouping, not the suite-wide status taxonomy — the mathematical-status axis carries five tiers and Paper 0 §1 is authoritative). PROVED: forced by the Jordan/Freudenthal algebra, with no fit to particle data. STRUCTURAL: value correct and algebraically motivated, but *permitted* rather than *selected* — the algebra allows it and no other value competes numerically, yet no uniqueness/selection theorem closes it. INPUT: genuinely external — a measured dimensionful scale, an empirical selection rule, or a discrete datum the algebra does not fix. The dimensionless flavour observables are almost entirely PROVED or STRUCTURAL; the irreducible INPUT is concentrated in the dimensionful scales and a short list of discrete data.

Table 3: Provenance tiers across the suite.

Tier	Items	Source / status note
PROVED	Anomaly cancellation (SO(10) **16**); all SM hypercharges (under B+SU(5)); Yukawa tensor structure (cross-block $\varepsilon \cdot \varepsilon$); mixing observables (pre-Rev29: six of eight; **superseded** — two of eight at DERIVED-CONDITIONAL or above, namely θ_{12}, δ_{CP}; the internal 7/16 mismatch identity is theorem-grade with its physical θ_{23} attachment LOADED-CORRESPONDENCE; the CKM first row is LOADED and $\lvert V_{cb}\rvert$ LOADED-CORRESPONDENCE (physical) / Structural formula (A1566, Rev32.6), (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`)); quark gauge-protection	A696; A689; A686–A689; main ledger (Table 1); A630. Forced by algebra, no fit. *Rev29*: the mass exponent $p = 8/3$, the normalization $C = \sqrt{2}/10$ and the m_μ/m_τ formula are **removed from this tier** — "forced by algebra, no fit" does not hold of them (see the provenance box below).
STRUCTURAL	Mass exponent $p = 8/3$, normalization $C = \sqrt{2}/10$ and the m_μ/m_τ formula (*re-tiered Rev29* from PROVED; LOADED-CORRESPONDENCE / REPRODUCED — A616; A618); $\theta_{13} = \sin^4(\pi/8)$ (readout lemma B.5′); δ_{CKM} (COINCIDENCE-CLASS candidate construction; the Rev29 "conditional theorem under F-δ" wording retired Rev32.5); $\sin^2\theta_W = \phi^{-3}$ (2% target); 7/3 bridge (m_b, m_c); light-quark constituent masses; cross-sector colour/singlet mass envelope (octonion-index)	A678; A697/A698; A635; D702–D704. Right value, no selection theorem; readout-axiom claims name their axiom inline.
INPUT	Generation count = 3; Koide selector $K = 2/3$; dimensionful scales v_{EW} and Λ_{G_2}; measured constants α, M_{Pl}	A691–A694 (replication refuted; count external); empirical K; two external scales (calibration inventory — see the input-count convention box); PDG constants.

Global status of the B-frame (A707/A708). The hypercharge and Yukawa entries above are carried "under B+SU(5)": they use one explicit oriented SplitCD/$SU(5)$ frame B. That choice is no longer an independent axiom. The valid frames of the traced Yukawa skeleton form a 15-dimensional *manifold*, not a group (A1750): its tangent space at the identity is $\mathfrak{g}_{2(2)} \oplus \mathfrak{so}(1,1)_{34}$, but the extra $SO(1,1)_{34}$ boost is incompatible with the fixed compact SM gauge frame — it breaks the positive color norm and fails to commute with $U(1)_Y$ and $SU(2)_L$ (A707). An exhaustive finite audit of the e_0-fixing signed-permutation frames (A708, 645,120 enumerated; $\mathrm{Aut}(\mathbb{O}_s) = 192$ re-derived in house) gives

$$\mathrm{Valid}_{76} = \mathrm{Aut}(\mathbb{O}_s) \sqcup \mathrm{Aut}(\mathbb{O}_s)\circ\sigma \ (384 = 192{+}192), \qquad \sigma = \mathrm{diag}(1,1,1,-1,1,-1,-1,-1),$$

and, after the compact gauge frame is imposed, the valid finite frames reduce to the e_7-line-preserving subset ($96 = 48{+}48$) — a single component once one quotients by $G_{2(2)}$, compact SM gauge, and the

σ-rephasing; color conjugation $3 \leftrightarrow \bar{3}$ and the singlet swap $e_0 \leftrightarrow e_7$ both fail validity. Hence

$$A_B \text{ [DISCHARGED relative to the compact SM gauge embedding]},$$

so the PROVED hypercharge/Yukawa entries carry no separate frame axiom; the residual "B" qualifier names the compact SM embedding (a physical INPUT, listed below), not a frame ambiguity. Beyond the signed-permutation class the statement is now a theorem over all $O(4,4)$ frames (`s1213`; A1750): every frame keeping the 76 equations fixes e_0; the valid set is $S = \mathrm{Aut}(\mathbb{O}_s) \sqcup \mathrm{Aut}(\mathbb{O}_s) \circ \sigma$, of dimension 15 with exactly two components, and the compact gauge frame meets it in $K \cong SO(3) \times \mathbb{Z}_2$, entirely inside the identity component. The discharge therefore relies on one *additional* equivalence, named here: the σ-rephasing — an improper colour-frame reflection, $\psi = \det A$ of the real colour three-frame — which is a gauge-invariant label, not a gauge transformation.

Gauge-boundary status of the gauge-group input (S75–S77; Appendix F). The gauge-group entry in the INPUT tier is now carried with the upgraded label

$$A1: \text{ [INPUT + STRUCTURAL+empirical uniqueness-of-completion]},$$

bounded on both sides by theorem-grade results (statements, proofs, and computations in Appendix F). *Obstruction side (two walls):* the branching-wall theorem — no gauging of maximal N=8, D=4 supergravity, in any symplectic frame, contains a compactly embedded $\mathfrak{su}(3) \oplus \mathfrak{su}(2)$ (21/21 faithful embedding classes killed by exact integer branching multiplicities) — and the charge-wall theorem (S80) — no gauging contains a unitary-class $\mathfrak{su}(3)$ (matter colour, $\mathbf{8} = \mathbf{3} \oplus 5 \cdot \mathbf{1}$); only the vector and adjoint classes are ever gauging-realizable; both walls house-proved and independently blind-replicated. Emergence-as-gauging is closed negative, so the input is *unreachable*, not merely unselected: the right algebra, never the right representation. *Uniqueness side:* among compact low-energy completions gauging the full derived colour + weak action on the derived 16-fermion package with 10_H-only scalar closure, G_{SM} is unique up to $U(1)_{B-L}$ (extra Cartan computed unique = $3(B-L)$) and up to global structure Γ; the gauge-the-full-action and 10_H-closure clauses are load-bearing, the enumeration boundary is stated, and uniqueness-of-completion is NOT emergence. *Dynamical remnant:* a maximal derivable shadow-type $\mathfrak{su}(3) \oplus \mathfrak{u}(1) \oplus \mathfrak{u}(1)$ is selected by vacuum stability of the missing-16 scalar sector at a stable $SO(6,2)$ point (dyonic completion = published Minkowski vacuum, residual $SO(6) \times SO(2)$); family-relative. The SM charge-pattern identification was executed (S78): rep-level mismatch at the origin point (pre-registered demotion applied — the selected $\mathfrak{u}(1)$'s are not $Y_{\mathrm{SM}}/3(B-L)$ there); the labeling question was then closed *negative* in two strengthening steps: by counting for the entire 36-parameter vacuum family (S78b) and by the charge-wall theorem for every gauging (S80; Appendix F) — the shadow is structural, not the SM in disguise. No public number in the observable map moves on account of any of this; the entries of this ledger are unchanged except for the $A1$ label above.

Novelty labels (Rev33.0; R169)

Every theorem, lemma, proposition and corollary in the suite carries, immediately before it, one of three novelty labels, in the Track-2 vocabulary verbatim: *standard theorem used* (the result, or the step that carries it, is a named prior result applied to the suite's objects); *new specialization proved here* (a proof in the suite or a kernel certificate exists; where a general result is being specialized, it is named); *physical interpretation not established here* (the statement's content is a physical reading of the mathematics, not a mathematical fact). The label is a third axis: it does not replace the

provenance tiers above or the five-tier status axis of Paper 0, and it does not turn a standard theorem into a discovery. Each label has a written basis in `Trackers/NOVELTY_LABEL_WORKLIST_S361.tsv`; a result the house could not place stays *undetermined* and is never defaulted to the more flattering label. At Rev33.0 all 175 theorem-level environments are labelled: 131 new specialization, 22 standard theorem used, 22 physical interpretation not established here, none undetermined. At Rev33.2 the count, re-derived from the tex at the freeze, is 130 new specialization, 23 standard theorem used, 22 physical interpretation not established here, none undetermined, of the same 175: one theorem (App. F, `thm:framefree`) was relabelled *standard theorem used* after external review (A1739).

Dimensionless theory of ratios; the leap to units

Superseded note (Rev25). The paragraph below is the Rev19–Rev24 two-scale calibration statement, retained verbatim for the audit trail. It is **superseded** by the registration section that follows it: the "would reduce the genuinely new dimensionful input toward one" hope is now a *theorem chain* (two-parameter no-go + H^2 count + unconditional census; Appendix O, §"The scale endgame"), the framework's dimensionful input count is exactly *one*, and the second calibration scale is no longer available as an independent input: under the one-class statement, $\Lambda_{G_2} = \Lambda_{\rm QCD}$ is an identification of *role* (not an independent input, and not a numerical equality with a scheme-defined $\Lambda_{\rm QCD}$ — see the scheme table and the Λ_{G_2} note below the input-count box) and $v_{\rm EW}/\sqrt{\sigma}$ becomes theorem-mandated *derivable* (the one-input obligation below).

[Superseded, Rev19–Rev24 text:] The framework fixes every dimensionless flavour observable — mixing angles and mass ratios — from $J_3(\mathbb{O}_s)$ structure, with *no* dimensionful input. Absolute calibration, however, requires at least two external dimensionful scales: the electroweak vacuum $v_{\rm EW}$, which sets all Dirac masses through $m_f = y_f v_{\rm EW}/\sqrt{2}$, and a $\Lambda_{\rm QCD}$-like confinement scale Λ_{G_2} (dimensional transmutation), which sets the constituent masses and the hadronic sector. These two are logically independent — one is a potential VEV, the other a running-coupling scale — and their ratio $v_{\rm EW}/\Lambda_{G_2} \sim 10^3$ is itself a hierarchy that no scale-free algebra produces. This is not a gap in the derivation but a structural feature: by the Robertson constraint, fixing a single unit ($R \equiv 1$) sets *units* only and cannot, by itself, generate a dimensionless coupling or an exponentially large dimensionful ratio. The strong scale is plausibly identified with standard QCD dimensional transmutation ($\Lambda_{G_2} = \Lambda_{\rm QCD}$ under the $SU(3)_c \subset G_2$ confiner reading [D700/A700]), which would reduce the genuinely new dimensionful input toward one. Operationally the two scales function as *calibration*: once $v_{\rm EW}$ and Λ_{G_2} are set from two measurements, every other absolute mass is predicted from the algebraic ratios. The falsifiable core — all dimensionless observables — is therefore already exposed to test without the leap to units; only the absolute mass scale depends on the two calibration inputs.

The registered input (Rev25): the depth-cylinder Bargmann class

Input-count convention (Rev32 reconciliation; A1531/R1). This ledger prints three numerals about dimensionful inputs, and they are not competing counts — they count different nouns:

Count	Noun	What it inventories
one	independent dimensionful input *class*	the registered depth-cylinder Bargmann class ($L_{\text{depth}}/L_\sigma$, this section), after the post-Rev25 identifications and obligations
two	external dimensional *calibrations*	v_{EW} and Λ_{G_2}, required operationally to convert ratios into absolute masses
three	named operational *anchors*	m_τ, v_{EW}, Λ_{G_2}, as they appear in current formula chains (generation ledger)

Input-count convention. The suite has one independent dimensionful input class after registered identifications. Two external dimensional calibrations are required operationally, and three named dimensional anchors still appear in current formula chains; those are calibration and usage inventories, not competing counts of independent inputs. [LIB2-124, LIB2-299] Every other count printed in this suite cross-references this box.

Two clarifications of the box (Rev32.9; R84-h, PRM-029; A1642 O4). (i) $\Lambda_{G_2} = 260 \pm 0.5\,MeV$ *is one object with one type:* an external dimensional *calibration* (row two of the box). Its value is taken from hadronic phenomenology; where a row calls it "measured" or "fitted" that names the source of the value, not a measured Λ_{QCD} in any scheme. Under the one-class statement it is not an independent input, and its Rev25 identification with the QCD confinement scale is an identification of role, not of number. It is used in two sector roles — the heavy-lepton anchor (Appendix A) and the light-quark constituent anchor (Paper 2) — which are two uses of one calibration, not two inputs. Its registered uncertainty is the rounding rule (R190; ledger SL-60). At this precision the τ-'5' bridge test (SL-27) is uninformative: the rounding band alone moves the fitted coefficient by 3.3, against its own error 0.087 (`s1211`). (ii) *The clock normalisation* $q_0^{\text{clk}} = m_e$ (Appendix H; the time-layer paragraph above) is a unit choice made inside the one registered class. It is not a fourth anchor, and it is not the Kaluza–Klein tower quantum q_0^{KK} of Appendix G. Scorecard row 18 outputs m_e from m_τ; the clock reads that output as its unit.

Name. $L_{\text{depth}}/L_\sigma$ — the Bargmann/central-charge class of the winding category as banked in the corpus (A1347, the object on which `s748` computed it); the framework's *one* dimensionful input. On the evaluation category $\text{Wind}_{\text{ev}} := T_\delta$ registered at Rev33.2 the same count gives $\dim H^2 = 0$: the class lives in A1347's two-cylinder decorations and transfers only when those windings are shipped into Wind_{ev} (open, C1; `s1341`).

Canonical statement (s748 card). The winding category admits exactly one central positive scale class ($\dim H^2 = C(2,2) = 1$, unconditional on the banked symmetry census); no homogeneous machinery can fix it (two-parameter no-go); its value is fixed by measurement as $m_e/\sqrt{\sigma}$ and is not

ϕ-menu-expressible (sealed null). Registering it is registering the Bargmann central charge: one number, one contact point with experiment, theorem-counted.

Operational value (inputs pinned: $m_e = 0.51099895$ MeV, $\sqrt{\sigma} = 445(3)(6)$ MeV):

$$\frac{\sqrt{\sigma}}{m_e} = 870.843277 \quad (\pm 1.51\%,\ \sqrt{\sigma}\text{-dominated}), \qquad \frac{m_e}{\sqrt{\sigma}} = 1.1483122 \times 10^{-3}.$$

ϕ-ladder position: $\log_\phi(\sqrt{\sigma}/m_e) = 14.0675$ ($+0.0675$ off-integer rungs). The `s721` null model is **SEALED**: no ϕ-menu form within 1% at frozen base rates — the value is *registrable, not derivable*, exactly as the theorem demands; the seal is kept *after* registration as the standing guard against menu-numerology relapse.

Theorem chain (each element banked and verified): `s696` Möbius forced → `s697` exactly two cylinders → `s713` substrate blindness → A1353 two-parameter no-go (an input is *required*) → A1354 two doors (scale separated from the charge kernel) → A1355 $H^2(\mathrm{Sym}; \mathbb{R}_+) = \mathbb{R}$ (area cocycle) → `s741` unconditional census, $\dim H^2 = C(n_{\mathrm{cyl}}, 2) = 1$, single-flip symmetries forbidden → `s740` loaded fiber admits no dynamical stabilization (the radion met honestly).

The one-input obligation. $\dim H^2 = 1$ forbids a second independent dimensionful input. Every other dimensionful ratio in the suite is therefore theorem-mandated *derivable*. Targets: (i) $v_{\mathrm{EW}}/\sqrt{\sigma} = 553.3026$ ($\log_\phi = 13.1250$; not ϕ-menu-clean, consistent with a dynamical origin) — **mandated-derivable**; the σ-bridge program is the designated post-freeze flagship. (Pencil curio, not a claim: the off-integer part $+0.1250 \approx 1/8$ rung carries ± 0.03-rung uncertainty.) (ii) Λ_{G_2}: if $\Lambda_{G_2} = \Lambda_{\mathrm{QCD}}$ (D700/A700 reading), it is *not* an independent input — this closes the superseded two-scale paragraph above into the one-class statement. **Falsification clause:** exhibiting a second underivable dimensionful ratio falsifies the one-input theorem chain.

[LIB2-299]

Ledger consequence. In the provenance taxonomy above, the INPUT tier's "dimensionful scales v_{EW} and Λ_{G_2}" entry is read, from Rev25 on, as: *one* registered dimensionful input ($L_{\mathrm{depth}}/L_\sigma$, operationally $m_e/\sqrt{\sigma}$), with $v_{\mathrm{EW}}/\sqrt{\sigma}$ reclassified from input to obligation (mandated-derivable) and Λ_{G_2} carried under the identification reading. Rev32.1 (A1538 F8): these are two readouts that must not be collapsed — ONE abstract central scale class is registered by the theorem, while THREE empirical dimensional anchors (m_τ, v_{EW}, Λ_{G_2}) are currently consumed by the operative formula chains; the physical "one-input" reading becomes available only when derived relations with uncertainties tie v_{EW} and Λ_{G_2} to the registered class. [LIB2-124, LIB2-299] The registration is an administrative act on a theorem chain; no public observable moves.

Color/singlet localization envelope (cross-sector mass bands)

The B-map and octonionic colour decomposition give a genuine structural distinction between colour-singlet and colour-triplet directions,

$$\mathbb{O}_s \to 1(e_0) \oplus 1(e_7) \oplus 3 \oplus \bar{3} \quad \text{under } SU(3) \subset G_2,$$

with H_u, H_d, L, e^c, ν^c on the singlet directions and Q, u^c, d^c on the colour $3 \oplus \bar{3}$ directions [A677, A688/A689]. This band *label* is rigorous.

Split-frame reading of "$\bar{3}$" (wording correction, A1426-K2, Rev28). The real carrier here is $1 \oplus 3 \oplus 3^*$ of *split* $SL(3,\mathbb{R})$ with the contragredient action. "$\bar{3}$" in this and neighbouring prose must therefore be read either as complexified notation, or as requiring a *separately loaded* compact-frame transport — and **compact-Casimir or physical-colour language may not be inferred directly from the split fibre**. The band label survives this reading; what does not survive is any argument that walks from the split decomposition to a compact $SU(3)$ Casimir without paying for the transport. Verified house-side (`s905`, 15/15 $\times 2$); the $b = a$ result is basis- and intertwiner-invariant and is **unaffected**.

A localization law $c_f = c_0 + \alpha\, C_2^{SU(3)}(f)$, with $C_2(1) = 0,\ C_2(3) = C_2(\bar{3}) = 4/3$, would reduce the lepton/quark mass envelope to a single coefficient α. An explicit $SU(3)$-structure Dirac-operator analysis [D702–D704] establishes two facts. First, the Casimir-weighted *form* $\delta c_R \propto C_2(R)$ is geometrically natural only when the internal $SU(3)$-curvature is adjoint-isotropic; this is realized by homogeneous orbit-averaging on a nearly-Kähler coset, but *not* on a generic background, where the connection splits weights inside the multiplet (a partial/Cartan holonomy does not isotropize) and the band law fails. Second, even where the form holds, the amplitude $\alpha k R = \eta\, s\, \gamma\, \hat{\tau}^2(kR)$ and its sign remain free moduli; the phenomenological value $\alpha k R \simeq 9.55$ (from m_t/m_e) is *not* derived, and a single α cannot in any case fit the raw cross-sector ratios (the required value spans a $15\times$ range), so the envelope can only be a band multiplier on top of the algebraic within-sector factors. The within-sector ratios (ϕ-ladder, Koide, 7/3) are untouched, since the Casimir acts by representation only.

Envelope status. $\mathbb{O}_s \to 1 \oplus 1 \oplus 3 \oplus \bar{3}$ band label: PROVED (under B), read in the split frame per the correction above (A1426-K2) — the compact-frame transport is a separately loaded step, not a corollary. Casimir form $\delta c_R \propto C_2(R)$: PROVED under a homogeneous adjoint-isotropy condition; PROVED-NO for generic $SU(3)$-structure curvature. Coefficient $\alpha k R$ and sign: OPEN (free moduli). Cross-sector envelope: REDUCED TO ONE GEOMETRIC MODULUS under the isotropy assumption, NOT DERIVED; absent that assumption, a relabelled fit.

Three-generation ratio ledger: what varies across the generations

The generation *count* ($N_g = 3$) is an INPUT: algebraic replication is refuted [A691–A694], so the algebra does not derive *why* the one-generation Yukawa structure repeats. Given the count, however, the full spectrum is populated by anchored algebraic ratios, and the economy is sharp: *exactly one object varies across the three generations.*

The single generation-varying object — the resolvent index d. Every inter-generation ratio derives from the one-parameter resolvent family

$$R_d = \frac{1}{\phi^{2d} - 1}, \qquad R_1 = \phi^{-1} = 0.6180, \quad R_2 = 0.1708, \quad R_3 = 0.0590, \quad R_4 = 0.0217,$$

with the index d fixed *structurally* by the Peirce eigenvalue spectrum $\{\phi, 1, \phi^{-1}\}$ of J_{vac} — not fitted:

$$d = 1:\ \text{CKM transfer}; \qquad d = 2:\ m_{\nu_2}/m_{\nu_3}; \qquad d = 3:\ m_\mu/m_\tau; \qquad d = 4:\ m_s/m_b.$$

(Rev33.0 update to the Rev32.2 note on the $d = 2$ rung: R_2^2 is now typed as a $\Delta m^2_{21}/\Delta m^2_{31}$ ratio prediction asserting $m_1 = 0$ — an assertion of the framework, not a derivation, superseding the former edge-target typing. $d(\nu) = 2$ remains SELECTED by the post-hoc feature rule $d = 2 + [Y_R \neq 0] + [\text{coloured}]$, the χ^2 minimum of an injective assignment, not derived — Paper 5; `s1104`/A1546 for the one-family result.) The charged-lepton ordering is reversed by the $U_{T_{1/2}}$ Jordan-automorphism theorem ($d = 1 \to \tau,\ 2 \to \mu,\ 3 \to e$). The index d — equivalently the ϕ-ladder rung, or the topological winding level of the Peirce idempotent — is the *only* quantity that carries a generation label.

What does NOT vary (generation-invariant, proved). The dynamics is permutation-invariant across the three idempotents $\{f_1, f_2, f_3\}$ [A637/Q1B]: the winding exponent $p = 8/3 = \dim(\mathbb{O}_s)/\text{rank}(J_3)$ [A616]; the normalization $C = \sqrt{2}/10 = \sqrt{2}/\dim(J_2(\mathbb{O}_s))$ [A618]; the lepton–quark bridge $7/3 = n_{23}/\text{rank}(J_3)$ [A635/A637]; the cross-block Yukawa tensor $\varepsilon\cdot\varepsilon$ [A686–A689]; the SM hypercharges [A689]; and the Koide selector $K = 2/3$. None carries a generation index. *Rev29 qualifier*: what is proved here is generation-*invariance* [A637/Q1B]. The mathematical tier of p and C (exact dimension ratio, exact dimension count) is not in dispute; their physical attachment to the mass formula is LOADED-CORRESPONDENCE, not PROVED — see "Provenance of the charged-lepton ratio".

Table 4: Spectrum from three anchors + generation-indexed algebraic ratios. **Rev32.1 reading rule: tiers name mathematical provenance; physical attachments are stated separately; no row may be cited as zero-parameter; displayed σ values are named comparator distances, not profile likelihoods.**

Fermion	Index	Formula (anchor × ratio)	Predicted	PDG (miss)	Status [source]
τ	anchor	$m_\tau \equiv 1776.93\,\text{MeV}$	—	—	INPUT (lepton dial)
μ	$d = 3$ (a label; the monomial is not R_3 — Paper 2 §3.1, Rev32.9)	$m_\tau(\phi/\sqrt{5})^{8/3}(\sqrt{2}/10)$	106.05	105.66 (+0.37%)	STRUCTURAL / LOADED-CORR. (*Rev29*) [A616,A618]
e	Koide	$m_\mu^{\text{tree}} \to \delta_{\text{QED}} \to K{=}2/3$	0.508	0.511 (−0.66%)	KOIDE-CONSISTENT [A636,A637]
t	anchor	$v_{\text{EW}}/\sqrt{2}$ $(y_t{=}1)$	174,104	172,600 (+0.87%; d_{cmp}=+5.57, cross-scheme, no status)	STRUCTURAL (*re-tiered Rev29*) [A632]

Fermion	Index	Formula (anchor × ratio)	Predicted	PDG (miss)	Status [source]
c	7/3	$(7/3)\, m_s^{\text{const}}$	1291.9	1272.9 (+1.49%; cross-scheme, declared)	STRUCTURAL [A635,A637]
u	const	$\Lambda_{G_2}\, \phi^{1/2}$	331	$\sim$ 336 const (−1.57% at full precision 330.73)	STRUCTURAL [A618]
b	7/3	$(7/3)\, m_\tau$	4146	4186 (−0.95%; cross-scheme, declared)	STRUCTURAL [A635,A637]
s	const	$\Lambda_{G_2}\, \phi\, (K_{G_2}/K_{SU(3)})^{1/4}$	553.7	$\sim$ 540 const	STRUCTURAL [A631]
d	const	$\Lambda_{G_2}\, \phi^{1/2}$	331	$\sim$ 340 const (−2.73% at full precision 330.73)	STRUCTURAL [A618]

Anchors (dimensionful dials — the "three named anchors" usage inventory of the input-count convention box; see scales subsection above): $m_\tau = 1776.93\,\text{MeV}$ (lepton tower + m_b via 7/3); $v_{\text{EW}} = 246.22\,\text{GeV}$ (top, pinned by G_F); $\Lambda_{G_2} \approx 260\,\text{MeV}$ (light-quark constituents + m_c via 7/3). [LIB2-124] Light-quark entries are *constituent* masses, not directly comparable to the current-quark PDG values ($m_u \approx 2$, $m_d \approx 5$, $m_s \approx 93\,\text{MeV}$). [LIB2-128, LIB2-129]

Provenance of the charged-lepton ratio m_μ/m_τ (*Rev29* re-tiering)

The ratio is not proved, and its own sources never claimed it was. Through Rev28 the formula $m_\mu/m_\tau = (\phi/\sqrt{5})^{8/3} \cdot \sqrt{2}/10$ printed at PROVED / zero-parameter / "forced by algebra, no fit". That typing is **withdrawn**. The provenance is target-first, and the chain is short enough to print in full:

- `D616:99-102` takes the PDG numbers, computes $0.0595/0.4220 = 0.141$, and asks whether 0.141 is algebraic in φ. The target came first.
- `A616:50-63` offers $\sqrt{2}/10$ as the answer, agreeing to 0.37%, and states in the same breath that *the algebraic origin is not yet proved.*
- `A618:47-51` supplies $\dim J_2(\mathbb{O}_s) = 10$ *afterward*: the dimension is fetched to explain a number already in hand.
- `A618:134-140` concedes that the divide-by-dimension step "is argued by analogy".
- `A618:147` lists the formal derivation as BLOCKING. It is closed nowhere in the corpus.

The +0.37% residual IS the fit gap. C enters the ratio *multiplicatively*, so the residual measures C and nothing else: reproducing the measured ratio requires $C = 0.140894$, whereas the offered constant is $\sqrt{2}/10 = 0.141421$. The constant is +0.37% high and the predicted muon mass is +0.37% high — the same number twice, because it is the same number. There is no independent determination of C against which 0.37% could be a residual discrepancy.

Re-tiered typing (*Rev29*). Mathematical tier STRUCTURAL; physical attachment LOADED-CORRESPONDENCE; reproduction status REPRODUCED — the arithmetic is exact and reproduces in house, and what is missing is a derivation of C, not a computation. This brings Appendix X into line with the two-axis row already printed in Paper 0 ($8/3 = \dim \mathbb{O}_s/\text{rank } J_3$ is an exact dimension ratio and LOADED-CORRESPONDENCE: "not, alone, a mass exponent"), and supersedes every "zero free parameters" citation of this ratio elsewhere in the suite.

Honest boundary. Populating generations 2 and 3 by anchored ratios is phenomenological *accommodation* — legitimate, and exactly parallel to the dimensionful calibration above — *not* derived replication: $N_g = 3$ is an input and algebraic replication is refuted [A691–A694]. The inter-generation ledger has mixed provenance: theorem-grade internal maps, DERIVED-CONDITIONAL physical rows (**two** of eight angles after the Rev29 re-tiering and the A1566 retype (Rev29 re-tiering, Paper 3 § "Row-by-row re-tiering"; kernel `s959`; Rev32.6) — the pre-Rev29 "six of eight at theorem level" is superseded; Table 1), Structural rows, and LOADED-CORRESPONDENCE rows. The full matrix is not a derived physical object, and what survives is not tuned. In one line: the *vertical* ladders (masses within a sector) are anchored ratios on top of an input count, while the *horizontal* structure (mixing between generations) is of mixed provenance — theorem-grade internal maps attached to physical rows by correspondences of stated tier.

Update (s496/s498, S206) — a conditional forcing skeleton. A forcing argument now exists alongside the input label. Requiring an ergodically-mixing four-dimensional Lorentzian continuum to emerge forces the boundary monodromy to be hyperbolic, and the minimal hyperbolic seed trace is 3 (the trace of the $\mathrm{SL}(2,\mathbb{Z})$ seed M_1; its Sym^3 lift in $\mathrm{Sp}(4,\mathbb{Z})$ has trace $t^3 - 2t = 21$ — two objects, Rev32.9) (s496 lower-bound theorem; s498 least-dilatation/least-entropy minimality). [LIB2-344] Hence $N_g = 3$ is FORCED-CONDITIONAL: the lower bound $|\text{trace}| \geq 3$ is *proven*, while $= 3$ minimality is conditional on least-action selection and the smooth-continuum limit. The engine still *takes* 3 as an

input; this is a separate, conditional derivation, not an unconditional theorem.

Within-generation Yukawa coefficients: a scoped honest negative (O-YUKAWA-COEFFICIENTS, conditionally closed)

The exceptional-Jordan cubic derives the *four* Standard-Model Yukawa tensors $\{Qu^cH,\ L\nu^cH,\ Qd^cH^c,\ Le^cH^c\}$ together with a single *democratic* $(1,1,1,1)$ common normalization, and nothing finer (Appendices J, K). The four distinct physical coefficients $\{y_u, y_d, y_e, y_\nu\}$ — i.e. the within-generation hierarchy — are *irreducibly external to the banked parent action.* Five independent lines converge on this: the $G_2/U(1)_Y$ 7–3 fence (s238); the exact $\frac{1}{2}$-norm $Y = \pm 1$ mirror leakage removed by the $U(1)_Y$ Reynolds projector (A1048); the Reynolds-forced democracy that *requires* a non-invariant compensator (s239/s240, Appendix J); the failed candidate sweep $F_4/E_6/B{-}L/J_{\text{vac}}$ (A1049/A1050); and the rank-4 residue map with order-of-magnitude RG running failure (A1051). This is recorded as the INPUT/open item `O-YUKAWA-COEFFICIENTS`, now *conditionally closed* as a scoped honest negative. *Scope fence:* the no-go is over the *banked* exceptional-Jordan framework only; non-perturbative or composite dynamics are not banked and so this is *not* a universal no-go. The honest published claim is the four SM Yukawa tensors + democratic normalization, full stop; no public observable moves.

Rest-mass selector and selector fences (Rev21 fold; cross-reference)

Two further inputs are now named explicitly elsewhere in the suite and recorded here for the ledger.

The rest-mass occupation $O = \{0, 3, 11, 17\}$ (Appendix M). The integer rungs at which the charged leptons and up quark sit on the golden ladder were carried as an INPUT through Rev24. Appendix M establishes the surrounding *structure* target-independently — the exact parity $\mathbb{Z}_2$ grading, the clock modulus $M = 60 = 2h(E_8)$ (two independent derivations), and the exact Galois localization $\text{Stab}_{(\mathbb{Z}/60)^\times}(O) = \{1\}$. **Rev25 update:** the selector is now derived in all three layers (mod-2 Möbius parity, mod-5 torsion walls, mod-7 Fano/Artin adjacency; A1349/A1350-verified), and the occupation is promoted to DERIVED-CONDITIONAL (Appendix M, Rev25 section), with the hash-stamped continuation $\{31, 39\}$ as the live public falsifier. The engine's occupation column keeps its Input label until the kill condition has been exposed to test; the ledger records the derivation chain.

The charged-lepton mass gaps (Appendix O). Beyond the integer windings, the charged-lepton mass *ratios* require two further numbers — the democratic-singlet and doublet "gaps" μ_d, μ_p of the outer mirror. Appendix O derives the surrounding *shape* (Koide $Q = 2/3 \Leftrightarrow r = \sqrt{2}$) and the forced two-gap operator structure, but on all present evidence the two gap *values* are INPUT of Yukawa type: the loaded action gives a uniform gap that does not reproduce the spectrum, and the conditional 3/5 closure is inexact. They route to one open object — a valid gauge-preserving S_{125} carrier, the same gate as the colour charges. Nothing is promoted.

Selector fences (Appendix N). The fine-structure constant α is an INPUT modulus, fenced on **five** independent legs (Wyler route falsified; golden numerology density; noble/KAM robustness-not-selection; the selector is topological, not an offset; and the dynamical-scale-via-decay route is closed at $S_{\text{bounce}} = 0$ — Appendix N, Legs 1–5, `s256`/`s258`–`s262`/`s175`). The compact colour real form is an INPUT: compact $\mathfrak{su}(3)$ (dimension 8) cannot embed in the available maximal compact part (dimension 6).

Engine-side presentation tabs. The Outputs workbook carries three presentation-only tabs (Electron

Shell Levels, Winding→Mass, Exact Windings↔Energies, sessions S164/S166/S167) that tabulate the descriptive golden-ladder read-offs; they move no public observable and are derived from the banked kernels only.

S198–S207 clean negatives (banked, no observable moves).

- The σ mass-gap is *not* an $F_4 = \mathrm{Der}(J_3)$ Cartan object: it is the $L_{J_{\mathrm{vac}}}$ multiplication in the $\mathfrak{e}_6/\mathfrak{f}_4$ (Str/Der) complement ($\|\mathrm{proj}_{\mathrm{Der}}\| = 10^{-16}$).
- An action-level *charged* Dirac mass is forbidden (no-Q theorem, triple-confirmed); a neutral 2×2 Majorana mass is allowed.
- The **912** $\Theta(A)$ gauging is blind to the hypercharge residue A: the compact $\mathfrak{g}_{\mathrm{SM}}$ is excluded for all A (including the SM value $A = 2$).
- The S125 extraction is retired with proof: the Majorana/Ward/reciprocity/bridge defect ranks are congruence (frame-orbit) invariants, so no legal frame change repairs the registered obstruction (re-confirmed A1208).

S209–S226 clean negatives (Rev25 fold; banked, no observable moves).

- α **is not derived — confirmed three independent ways** (S214): boundary-volume route ($\alpha^{-1} \approx 14.5$, `s593`); bulk-residue route (1/163, `s595`); KK-ratio route (`s597`). All three fail cleanly. S226 adds the *typing*: α belongs to the Door-2 / dimensionless-coherence class (Appendix O, §"The scale endgame"), not the scale-datum class — which is *why* no ruler-type route can produce it. The α fence (Appendix N) stands unchanged.
- $N_\Lambda = 33$ **lepton-cutoff form is a one-parameter fit**, not a prediction (`s600`, honest fail: the form breaks at the electron).
- δ **sign = generation chirality** (`s598`): banked *with* the preceding caveat — the identification inherits the fitted status of the N_Λ form it rides on.
- **Radion first harmonic killed** (`s743`): charge-face defects are not tick-organized. **Scale-menu null sealed** (`s721`): no ϕ-menu form for 870.84 within 1% (kept sealed post-registration; see the registration section).

Rev26 fold: the dynamics frontier and the paid scale obligation (S228–S240)

Ledger status (Rev26). The one-input theorem chain of Rev25 is unchanged; what Rev26 adds is the *payment* of its obligation at conditional/loaded-depth tier: $v_{\mathrm{EW}}/\sqrt{\sigma} = \phi^{105/8} = 553.30$ (framed-depth operator $D = 13 + \frac{1}{8}$; Appendix O) (registered at $\beta = 1$ from the two-element menu $\beta \in \{1, 7\}$ of Appendix N's electroweak-permanence proposition — no intrinsic selector fixes $\beta = 1$), with $m_\tau/\sqrt{\sigma} = 4 = \phi^2 + 1 + \phi^{-2}$ the exact dimensionless companion. This is a registration *relative* to the marked compact-positive carrier (Paper 4): the frame-free route is closed *negative* (the $E_{7(7)}$ permanence theorem, Appendices F/N). The interacting-vertex frontier (Appendix R) banks the first derived parameter-free (discrete, derived) relative vertex sign (-1, amputated, F4-fenced) and re-counts the head wall as one merged loaded import + three continuous data $\{g_3, C_{\mathrm{sub}}, r_C\}$ + the

loaded placement kernel $\Pi_{5\leftarrow 6}$. Theorem-tier upgrades this fold: the frame-free Higgs-line no-go is now *derived* (Appendix F); the Marked Carrier Theorem (Paper 4); the 7/3 finite-carrier theorem (Appendix K). No dimensionless observable moves; α, colour, $V_{\mathrm{CKM}} = I$ intact.

S228–S240 clean negatives (Rev26 fold; banked, no observable moves).

- **Static baryon geometry is exhausted** (S235): the curved KV three-body shape functional makes the equilateral configuration a curved-length *maximum*, curvature-robust ($K = -\frac{1}{4}$ softens by $\approx 8\%$, cannot flip the sign) — the remaining baryon wall is the quantum source/propagator problem, not geometry.
- **The free contact measure does not select the threshold readout** (S238): in the uncompressed $R \to \infty$ limit the edge atom carries zero weight and the continuum saturates the full resolvent; atomization and the KV-flow route are dead at shipped tier.
- **The** 10=10 **chamber/torsor coincidence is not a bulk transfer** (S240): $W_0^{A_4} = 0$ and all $S_4 \to D_5$ maps factor through $\mathbb{Z}_2$, so the equivariant intertwiner space vanishes; only the $\mathbb{Z}_2$ orientation fibre is shared.
- **Two house over-scopes, corrected** (S240): full-**27** trilinear uniqueness (the **27** is reducible, rank-3 invariant matrix; repaired on the traceless doorway) and the hope that the kinetic UNINORM fixes the cubic coupling g_3 (it fixes only the tensor norm; g_3 is structurally free).

Rev27 fold: the priced-selector arc — the wall as a closed ledger (S242–S245)

Ledger status (Rev27). No dimensionless observable moves; the fold characterizes the discrete half of the parent-action wall (Appendix S) and re-prices, honestly downward, one hoped-for completion (Appendix O). The engine is unchanged; α, colour, $V_{\mathrm{CKM}} = I$ intact; KILL-NODE 2 remains fired. The D1116–D1121 hexad closes the selector question at a *finite priced ledger*: the missing datum is $[j_A] \in \mathbb{R}\mathrm{P}^2$ (or a single-summand sparsity axiom + a branch bit), one relative datum (the common-lift axiom **or** the unpinned lift Δ_θ), two magnitudes, and the standing orientation $\mathbb{Z}_2$; the one datum *derived* beyond counting is the V-side quadrature $z_- = \pm i\, z_+$, conditional on the candidate shape action. Structural theorems banked this fold (Appendix S): the quadratic-action no-go (invariant census 0/5/3); $J(A)$ typed to $\mathbb{C}^2$ with rank $= 2 \iff \mathrm{Im}(\bar{z}_+ z_-) \neq 0$; the placement–charge table ($\iota = 16p$, $V_{\pm 1}$ excluded, attachment forced-free at 0 new bits/dims); the independence ladder at three/five/six tiers; the paired-source CASE-D root-line locus; the evaluation relation $d = -5s + 18o$ (kernel $(5, -18, 1)$); the copy-blindness split (target-Gram actions copy-blind; the missing selector a non-scalar tensor $A \in \mathrm{Sym}_3$); and minimal height *refuted-as-canonical*.

S242–S245 clean negatives and re-pricings (Rev27 fold; banked, no observable moves).

- **The exact additive-χ mass completion is refuted at the ratio tier** (S245): on the occupancy depth support $\{0, 3, 11, 17\}$ no single-χ closure $r = \chi^2/(\phi^{2N} + \chi^2)$ admits an exact bijection (0/24); the ordered ϕ-depth skeleton and the few-percent shape survive (Appendix O). The support $\{3, 11, 17\}$ is independently corroborated as the adjacent-integer minimax (floor 0.0795; Appendix M).

- **No banked principle selects the arrow** (S245): equivariance, Kan extension, the paired-source theorem, the CRT tick classes, the weld, and the χ-verdict each leave j_A free — six independence theorems, the axiom stack minimal at six tiers.
- **Minimal height is not canonical** (S245): the house ordering survives only in the ray-wise-primitive convention; the common A_2-lattice selector is shorter, and 13 coefficient rays tie at output height 1, so minimal height cannot even replace the sparsity axiom.
- **Four house over-claims, lane-caught and corrected** (S244–S245, casualties #4–#7): the independent-channel doorway (retired for the paired-source theorem); "the action votes for the root line" (common-lift-conditional only); the blanket ≤ 2-parameter completion no-go (*withdrawn* — a fitted period-70 two-parameter harmonic passes the 0.1% bar at 3.04×10^{-4}; a watch item is banked and reopens only if its two coefficients are ever derived); and the depth-exclusion cut (retired — $[M, Q_{\mathrm{em}}] = 0$ forbids identifying the two $\mathbb{Z}_4$ labels but not depth sourcing).
- **The 7/3 metric-weighted-residue route is a structural no-go** (S246, `s902`): a unitary residue over $\mathrm{Im}\,\mathbb{O}_s$ (signature $(3,4)$) lies in $[-4,+3]$, so the integer 7 cannot be a signed physical residue ($|\text{residue}| \leq 4 < 7$); it is realized only as the unweighted multiplicity. This *upgrades* the former "well-posed open" status of the 7/3 S-matrix bridge (Appendix I) to closed-negative *for the residue route*, with the finite-carrier *index* reading (Appendix K) the unique survivor. No observable moves: 7/3 was never a promoted amplitude.

1 The Rev28 fold ledger

Rev28 folds twenty verified rounds (D1126–D1146; assessments A1430–A1449; house verifiers `s909`–`s929`, each $\times 2$ byte-identical, four of them adjudicated against gates pre-registered before the returns landed). The physics sits in Appendix S (the boundary arc) and Appendix T (the Cartan carrier, the observation functor, the record). This section carries the *ledger*: what the architecture now is, what freedom is left and how it is typed, what died, and what the house got wrong.

1.1 The architecture (Rev28 box top)

The even/odd = shape/selection reading is retired. The suite's structure is stated as six layers:

L0	substrate	split octonions and the Albert algebra; the $(4,4)$ model choice is an *adopted premise*
L1	representations, support	Peirce decomposition; $27 \downarrow \mathfrak{so}(5)$; the annular C_5 carrier and its categorical support ($q = \pm 1$ absent); the affine cover $\ell^2(\mathbb{Z})$
L2	kinematic invariants	trace-derived metric and orientation; triality as the imaginary unit; signature as chirality; the $\mathfrak{so}(5)$ closure; one shared radius
L3	boundary and export	$P_{\text{ann}}/P_{\text{aff}}$, caps and interfaces, AX-MG, ρ, AX-AFF-ONSET
L4	dynamics and state	L_{eff} (exact conditional operator law, dynamics-tier *candidate*); the reflection-positive Cartan state at finite regulator; the positive completion cone; the record process
L5	observable attachment	$\mathcal{O}_{\text{MOS}}$: modular score, unital-CP normalization, POVM, minimal dilation, Pin/OS evenization; AX-MASS-LABEL; the faithfulness wall

One sentence. One $E_{6(6)}$ KK annulus carrying two opposite signature-chart trivializations and two grading orientations, with triality cycling the internal Peirce presentations and leaving the KK generator fixed; the boundary exports through a pointed affine interface priced by AX-MG and one registration coordinate; and every number a reader can see arrives through one explicitly constructed observation functor whose *selection* by the microscopic theory is the open question.

1.2 The observation-functor faithfulness wall

Which loaded choices can any admitted observable distinguish? Two independent theorems answer instances of this and none answers it in general: lift degeneracy (Appendix S) and the even-sector invisibility of the signature transition (Appendix T). Distinguishing the latter costs one of four named new structures. Until the wall's kernel is computed, **no scalar bit-or-dimension total is available for the selector sector, and none is printed.** [LIB2-259]

1.3 The typed freedom ledger (replaces bits+dims)

The Rev27-era headline (a bit total plus a dimension count, with a surplus) is **withdrawn pending typed rebuild**: its 17-dimensional decomposition is internally inconsistent as typed — confirmed at primary source, lane-caught, and logged as a house casualty below. It is replaced by typed rows. [LIB2-259]

Object	Moduli	Assumption / attachment	Falsifier
boundary state $[g_\rho]$	$\rho \in \mathbb{R}/20\mathbb{Z}$ (one continuous)	AX-MG, two clauses; cross-layer interface	a derived edge normalization (unitality, trace-preservation or idempotence) collapses $c = 1$ without the axiom
onset marking	none continuous	AX-AFF-ONSET; $\mathbb{Z}_2$ defect at level 7	a level-structure principle that selects 7 (parity currently allows 7, 8, 9)
source arrow $[j_A]$	$\mathbb{RP}^2$ off shell; collapsed to the order line on shell	AX-COT; log-Hodge naturality, absorbed into AX-MOS	an off-shell arrow differing by $\ker J$ that moves an exported quantity
chamber lift	zero; residual $O(2)$ is unphysical gauge	minimality (minimal-dilation theorem)	an exported quantity that moves under the complement frame
branch / orientation	none continuous (S_3 orbit absent the branch)	branch Real clause; standing loading; boundary-condition selected	a continuum SSB limit that selects a branch (impossible at regulator tier)
mass label	none	AX-MASS-LABEL, one-particle	an interacting pole differing from the quasi-free gap
realized record	none — it is data	sample	none possible: a deterministic natural section is a category error

No total. What can be said without the wall's kernel is this: *exactly one continuous modulus survives anywhere in the end-state ledger (ρ), and every other priced row is zero-dimensional.* Any reader who wants a scalar "freedom number" should read that sentence instead.

1.4 The Rev28 kill list

Structural. kill-node 2 (unconditional; fired off shell *and* for the full microscopic loading) · the D1125 selector derivation (loaded-effective survives) · AX-SEQ · the unpointed interface · $\mathfrak{T}_{AM}$ naturality · one-global-bridge · triality-as-class-fix · triality-as-third-cascade · the natural A_4 completion · multi-bulk independent radii · sheet-gluing at the F_4 tier · automatic principal-F_4 nontriviality · the relative radion (three independent times) · direct spin–charge locking · the deterministic natural section of the actualization morphism · the tail/Poisson/Martin actuality rescue.

Routes. the 7/3 signed-residue route (index reading survives) · the spectral selector (general r) · positivity-uniqueness · AX-MG derivation at the Markov/annular/affine tier, both clauses · every AX-COT derivation route · minimal-height-as-canonical · depth-cut · monomial selectors · quadratic invariant actions · AXIOM-K1-as-derivable · the kernel axis (twice) · constant pumps · $V_{\pm 1}$ fill · sedenion/16-dimensional substrate extensions · the phase-only bridge · the linear projector for simplicity · the smooth-elliptic route around the gravitational sign obstruction · pure MM/EH as a continuously reachable target · ghost-safety alone as a metric selector · the equivariant selector for the extremal branch · the χ-completion route through L_{eff}.

Numerical. 0.0331562% (off-support, twice killed, final) · the exact single-χ ratio on $\{0, 3, 11, 17\}$ ·

χ additive-completion-as-exact. The 0.2744490308% affine-safe row is a **legal miss**, not a kill: it stands in the miss column at 2.74449 times the frozen bar.

Retirements (zero-cost). the orientation bit · AX-GOLD-PROFILE · TBD-7 and TBD-12 · the $\mathbb{CP}^1$ continuous price · the "one W bit" charge · the wrong-carrier finite-sign residue (erratum) · the "positive chiral determinant" target · g_χ · AX-KUBO-MASS · AX-RIESZ-CLIFFORD-MASS · log-Hodge naturality (absorbed into AX-MOS).

Watches (not kills). the period-70 two-parameter harmonic (adoption refused) · NuFIT $\theta_{23} = 7/16$ · the T-U1U1-1 enhancement · the two-cylinder $\leftrightarrow$ two-chart identification, still the sharpest single open test.

1.5 House casualties and the standing rule

Four corrections were caught against house-signed text in this arc and are recorded rather than absorbed. (i) A dispatch summary asserted that "the continuum already died spectrally"; the banked result had *constructed* the spectral continuum and refuted only selection. A pre-send grep rule against banked results was adopted in consequence. (ii) The 17-dimensional decomposition of the freedom headline is internally inconsistent as typed; the headline is withdrawn (above). (iii) The single label "W" denoted two operators; it is split suite-wide. (iv) A banked row priced the chamber lift by a commutant computed on the wrong carrier; the finite-sign residue is retired and replaced by the minimal-dilation theorem (Appendix T, erratum box). In all four cases the review lane caught the house.

Standing rule (process, binding). No further χ-completion or χ-expression rounds run against the `s900` ratio vector until a physical generator and response map exist; the route is generator $\to$ semigroup/two-point $\to$ normalized response $\to$ χ, or nothing. This is the lesson of the off-support kill and the legal miss, stated once as a rule.

1.6 Provenance: the laboratory notebook

Appendix Y lists every adversarial round from S228 to S253 (A1358–A1449) with its dispatch, its headline result and the component carrying it, together with a reverse index from printed object to certifying round and the full casualty register. It also discloses the provenance gap it exists to close: before Rev28 no assessment above A1357 was cited anywhere in this suite, so two folds' worth of certification was present in substance and absent in citation. Four rounds (A1414, A1416–A1418) had no ledger row at all and are named there.

1.7 What is open, named

$\Pi^{\mathrm{full}}_{5\leftarrow 6}$ (the microscopic loading) · Einstein universality (the AdS linearized spectrum of the sourced heat-kernel action) · $\mathfrak{N}_{\mathrm{mass}}/\mathfrak{R}_\chi$ (a normalized physical response map into the mass quadrature) · the interacting mass attachment · the conserved-$\mathbb{Z}_2$ sector identification · one realized record · AX-MOS realization — equivalently, *is the physical observation functor the logarithmic Cartan spectral functor?* · $\mathfrak{L}_{\mathrm{phys}}$ · H2, still unbridged.

2 Real-form declaration: which signature invariants survive $\mathbb{O}_s \to \mathbb{O}$

Declaration. Every Jordan/FTS *signature* invariant quoted in this suite is computed over the **split** octonions $\mathbb{O}_s$. Some of those signatures are properties of the construction; others are properties of the *real form* and change if the identical construction is run over the division octonions $\mathbb{O}$. This section declares which is which, so that no signature in this suite is read as a real-form-independent invariant unless it is one.

The discipline is not new here — Paper 0's Step 2 already argues the fibre case explicitly, selecting $\mathbb{O}_s$ because its norm form has neutral signature $(4, 4)$ and excluding the compact $\mathbb{O}$ because its norm is positive definite. What follows extends that same declaration to the FTS-level signatures, which had been carried as bare invariants.

House kernels `s973` (independent FTS build, 25/27 gates) and `s976` (constrained-action real-form control) run the *identical* construction over both real forms. Results:

Object	**over $\mathbb{O}_s$ (split)**	**over $\mathbb{O}$ (division)**	**Status**
Full FTS Hessian, rank 56	$(29, 27, 0)$	$(29, 27, 0)$	**real-form independent**
Normal 28-block	$(14, 14)$	$(2, 26)$	real-form *dependent*
Boundary 28-block	$(15, 13)$	$(27, 1)$	real-form *dependent*
Constrained-action full Hessian	$(15, 15, 26)$	$(3, 27, 26)$	real-form *dependent*
Constraint tangent	$(14, 14, 26)$	$(2, 26, 26)$	real-form *dependent*

Two structural facts hold in *both* real forms and are therefore not real-form artifacts: the full Hessian has rank 56 with signature $(29, 27, 0)$, and the two 28-blocks' signatures sum to the full signature. The stationarity of the golden point is also common to both ($\|\nabla S_{\min}(z_0)\| \sim 3.5 \times 10^{-14}$ in each).

Analytic cause. The three Peirce blocks contribute $-|a|^2/\varphi$, $-|b|^2$ and $-\varphi|c|^2$; over the division octonions this is negative definite on all 24 octonionic directions, which is exactly what drives $(14, 14) \to (2, 26)$ and $(15, 13) \to (27, 1)$.

Scope, binding. This declaration does *not* weaken any result whose statement is real-form independent, and in particular the rank-56/$(29, 27, 0)$ statement is **unaffected** — it survives the change of real form. What it forbids is quoting $(14, 14)$, $(15, 13)$, $(15, 15, 26)$ or $(14, 14, 26)$ as if they were intrinsic to the construction rather than to $\mathbb{O}_s$. Where a downstream argument uses one of those four, the split real form is a *declared premise* of that argument and must be named as one.

3 The Rev29 fold ledger

Rev28 closed on an architecture. Rev29 opens on its price list.

The twenty-three-round derivation dive of S255–S257 (Appendix Y, Era 5) moved no observable and fitted no quantity. What it did was convert a set of hopes into a set of *named, discharge-conditioned clauses*. This section prints them. A reader who wants to know what this programme is still assuming,

and exactly what would settle each assumption, should be able to find the whole of it here rather than reconstructing it from twenty-four assessments.

3.1 The tiered symmetry-breaking ledger (replaces the D1100 price row)

The Rev26-era ledger carried a single blunt row: a twenty-five-parameter price on the microscopic loading. That row is **withdrawn**. The D1147 dilation retyping, house-verified exact at s930, replaces it with a tiered statement.

The full microscopic loading is an isometry together with a fixed-point / conditional-expectation map, carrying three *forced* dark character sectors; a faithful classical 5×6 kernel is rank-obstructed. The symmetry-breaking cost is therefore not one number but a graded one:

$$\dim \mathrm{Hom} \;=\; 0 \;/\; (3,3,3) \;/\; 9,$$

zero at the tier where the loading is forced, $(3,3,3)$ across the three dark character sectors, and nine in total. The three sectors are exactly the three nontrivial V_4 characters, and their forcing is a theorem, not a choice.

> Why this matters more than an arithmetic improvement: a single twenty-five-parameter row invited the reading that the programme had twenty-five knobs. It never did — it had a tiered obstruction whose top tier is *empty*. The honest cost is that nine dimensions of hom-space remain, in a place where the structure says which nine.

3.2 The priced-clause register

Twenty-six clauses, plus one added when the ω round closed — twenty-seven. Every one carries where it first appeared, what it is now, and — the column that matters — what would discharge it. Nothing in this table is banked physics; the table is the programme's outstanding debt, stated so that it can be paid or defaulted on in public.

Clause	**Status**	**What would discharge it**
Open head walls		
Module-naturality premise (**the promotion gate**)	OPEN HEAD WALL	Derive from the microscopic parent action that the physical D_F is recovery-fixed and commutes with the exact Fibonacci parent module action.
Six microscopic boundary gauge coefficients (**the boundary law**)	OPEN: six coefficients, three zero-mode sums	Compute the complete localized divergences and specify the UV boundary renormalization condition, or derive an endpoint-exchange / unified boundary symmetry fixing the finite parts.
Open, one object or one datum each		

Clause	Status	What would discharge it
X-semion / $Spin(10)$-center microscopic weld	Local representation EXACT; microscopic attachment OPEN	Construct the tube/clock-to-field recipient functor whose central action on $16 \oplus 10$ is $\exp(i\pi X/2)$.
X radial magnitude s	OPEN; one positive modulus	Derive a radial parent equation, a quantized holonomy, or a normalization law fixing s *without* a fitted coefficient ratio.
Recipient clause: physical transfer is the simple τ image	OPEN	Construct the full tube-module recipient and show the physical transfer represents the simple object τ, not a composite τ^m.
Polar-holonomy weld	OPEN / conditional	Prove that one primitive localization step is the positive polar image of the physical Fibonacci transfer.
Peirce-middle marking P_m	OPEN / loaded	Derive from the parent/module boundary why the middle Peirce record is the unique pointed defect and hidden-state recovery target.
Recovery rate / time registration γ	OPEN; one positive rate/time datum	Derive γ from the parent clock/transfer generator, or prove it is pure time reparameterization after an independently fixed physical clock.
Zero-dimensional weld: NS sign *and* mouth exchange in one transport	OPEN; one C_2 choice	Derive $T_m = -\tau_1$ rather than $+\tau_1$ from one microscopic 5D transport, or identify the choice with a previously derived orientation element.
$m_n - m_p = 1.2933$ MeV (Paper 9)	UNPRICED; sign wrong without a flip energy (Rev32.2)	The framework carries the QED half (Coulomb, α external) and no QCD half ($m_u = m_d$ at constituent level; the e_7 flip $H_u \leftrightarrow H_d$ carries no energy). Derive a flip energy for the $\mathbb{O}_s$ conjugation, or register an $SU(2)$ isospin object (only the $\mathbb{Z}_2$ parity is registered; `s1122`–`s1124`, `s1129`).
AX-LEP-PRIM	OPEN zero-dimensional clause	Derive from the B-map-to-clock observation functor that the two nonzero charged-lepton rungs are primitive generators of $\mathbb{Z}_{60}$.
Primitive polar degree $m = 1$	EXACT on the simple-object module tier; physical recipient OPEN	Discharged automatically once the recipient clause is proved; only $m = 1$ satisfies the primitive Fibonacci relation.
Conditional — true given a named premise		
AX-MOS minimal modular-OS observation	CONDITIONAL	Derive the ordered golden/inverse experiment and its minimal UCP/OS readout from the microscopic parent dynamics.

Clause	Status	What would discharge it
LOG-HODGE NATURALITY	CONDITIONAL; folded into AX-MOS	Derive from the microscopic state/readout functor that the ordered golden pair is exported through its relative modular logarithm.
Single-background hypothesis (Q_Y reduction)	CONDITIONAL	Derive from the 5D parent action that *one* charge-valued odd background, rather than six unrelated singlet masses, generates the profiles.
Branch-Real WH/BH mouth pairing	CONDITIONAL	Derive the two mouths as the anti-linear Real-conjugate parent pair in the microscopic action.
Literal MRSS 5D/orbifold attachment	CONDITIONAL; Appendix-P fence	Derive the physical interval/orbifold realization from the exact internal fifth momentum and NS carrier, rather than interpreting it.
Physical family attachment of the three polar characters	CONDITIONAL	Prove module naturality from the parent action, and show the interacting finite Dirac / two-point operator preserves and physically reads the three record projectors as families.
Loaded — carried as a choice, with the alternative named		
DIRAC-LOADED neutrino treatment	LOADED	Derive Dirac versus Majorana from the parent action. A Majorana branch requires a $\Delta(B-L) = 2$ source/seesaw block *and* a complete re-audit of every ν-dependent row.
Loaded low-energy scalar spectrum (one weak doublet, no coloured triplet)	LOADED / conditional	Derive the low-energy scalar content and the Higgs parity from the parent boundary dynamics, rather than selecting the parity by the desired spectrum.
Active c_3/c_2 Peirce ordering	LOADED	Derive the active Peirce branch and the ordered golden/inverse boundary state as the unique extremal physical state.
Retired, superseded, or refuted — do not cite as live		
GEN-SEPARABILITY	REFUTED; retired price	Not applicable. Replaced by the weaker generation-algebra commutation theorem plus two operator-valued generation contrasts.
AX-RIESZ-CLIFFORD-MASS	Retired as a standalone price; survives as a stage of graded H_{tot}	No separate discharge; what remains is sector-specific pole/physical mass attachment, not the coefficient of the normalized Clifford map.

Clause	Status	What would discharge it
AX-MASS-LABEL	Retired as a standalone clause	Replaced by the quasi-free pole theorem and the sector-specific interacting spectral-atom question.
AX-MAP-HISTORY	Retired as the actuality law; optional zero-noise extremal candidate only	Not applicable. The stochastic history law is derived; one realized sample remains record data unless a new deterministic extremal law is explicitly adopted.
Golden-unit holonomy law $h \in \mathbb{Z}[\varphi]^\times$	SUPERSEDED as the sharpest form	Replaced by the polar-holonomy plus simple-τ recipient statement; it remains an optional arithmetic compression, not an independent requirement.
Strict no-brane / complete-multiplet boundary branch	Retired as a generic assumption; remains a special branch	Derive the full Higgs-10 boundary field content and the boundary counterterm law. The split Higgs branch and B-RANK3 are selected under the loaded spectrum.

3.3 The two head walls, stated once

Everything above except two entries is a clause. Two are walls — they are what the next revision is actually for.

The promotion gate — not yet instantiated (*Rev29*). Derive from the parent action that $D_F \in \mathrm{Fix}(\Phi^*) \cap \mathrm{Comm}(\hat{M})$, where $\hat{M}$ is the exact Fibonacci parent module operator on the record space — the one canonical meaning of the symbol in this suite from Rev32.2 (five historical meanings existed across kernels and boards; every other use of $\hat{M}$ is retired, and a kernel that spells it $\hat{H}$ names this operator). Without it, the three golden records remain *conditional* family sectors rather than families. *Correction to the Rev28 wording*, which said the canonical test functional S_{promo} "is constructed" and its zero set "house-verified". [LIB2-264] An AST audit of the 1037 banked kernels finds **zero bindings of `S_promo`**: the functional is *specified*, not built, and no run of it exists. [LIB2-264] What *is* built is narrower, and is stated exactly here because the record is better than a flat negative would suggest: the recovery map Φ_η is constructed and executable (`s953:91-95`), and Fix ∩ Comm is solved, 9-dimensional down to 3-dimensional (`s954:84-95`). **That zero-set result stands** — at **scalar tier**, with $\hat{H} = H_4$, the qualifier carried at `s954:9` and dropped in transmission to the paper; it is restored here. So: the zero-set computation is real and qualified, and the gate itself is NOT YET INSTANTIATED. The stronger reviewer formulation — that the gate "is not an executable object" — is *too strong*, is not adopted, and should not be repeated.

The boundary law. Six finite counterterms, three visible combinations. What is missing is a UV condition, an endpoint-exchange theorem, or a unified boundary symmetry that fixes the finite parts. The price corrected to 6/3 at A1471; the A1463-era endpoint shorthand is retired.

3.4 Closing mathematics of the era, exact

Two small results close the dive and are recorded because they are exact and because neither moves an observable.

The least-cost mixing defect is the $(1 \leftrightarrow \varphi^{-1})$ link, with penalty

$$\Delta_{\min} \;=\; \tfrac{1}{2}\,\varphi^{-4}\varepsilon^{2}\|K\|^{2},$$

house-verified symbolically via the banked Hilbert–Schmidt identity. And generation localization splits as $6 = 3 + 3$.

> **Read the first of these carefully.** It is a statement about the *cost of the cheapest defect* in a frame where strict naturality forbids mixing altogether. It is not a mixing prediction, it is not a CKM matrix element, and φ appearing in it is a consequence of the module structure the theorem is stated in — not evidence for anything. The programme's standing rule against numerological support applies here with full force.

3.5 P0-1: the ω question, answered

One item of the Rev29 queue was deliberately not settled when this fold opened. It is settled now, and the way it was settled is worth as much as the answer.

At S67 two independent lanes converged on the Freudenthal Triple System's native symplectic form ω as "the single new ingredient that turns the static algebra dynamical". The claim was never tested. It sat for nineteen sessions, survived every fold by not being examined, and became the last Sargasso item.

The programme declined to rule on it from the chair. A dedicated round ran instead under freeze-first: the house computed its own answer in exact $\mathbb{Q}(\sqrt{5})$ arithmetic and *sealed* it before the dispatch was written; the dispatch posed six questions and disclosed none of the answers; the return was adjudicated blind against the sealed table.

The result

ω is a genuine symplectic form on the full 56 — non-degenerate, antisymmetric, $\det \Omega = 1$. That much is true by definition of an FTS and was declared a control in advance, because it proves nothing about dynamics.

The decisive computation is where the mass proxies live. Writing $L_J = \mathbb{R}e_{-} \oplus J_{+2}$ for the slice carrying the golden ladder:

$$\dim L_J = 28 = \tfrac{1}{2}\dim M, \qquad \omega|_{L_J} \equiv 0, \qquad L_J^{\omega} = L_J.$$

L_J is a **Lagrangian subspace** — maximal isotropic as an identity, not by a dimension count. [LIB2-011, LIB2-331] Two independent computations, one of them sealed in advance, agree on this exactly.

Three consequences follow, each verified separately rather than inferred:

- ω **cannot resolve** the golden ladder. The full pairing matrix among the directions carrying $(\varphi, 1, \varphi^{-1})$ is exactly $0_{3\times 3}$.

- The induced Poisson bracket on functions of the mass coordinates is **identically zero**. No Hamiltonian flow is generated within the mass sector by ω alone.
- The FTS's own native quartic does not rescue it: $I_4(a, 0, X, 0) = -4aN(X)$, but every derivative in the conjugate directions vanishes on the mass boundary, so mass-boundary initial data give zero initial velocity. This is an *independent* route to the same no-go — it does not rest on the degeneracy of ω.

And ω is blind to the exact grading $e_- \to r^{-6}$, $X \to r^2$, $e_+ \to r^6$, $Y \to r^{-2}$, which preserves both ω and I_4 while sending $(\varphi, 1, \varphi^{-1}) \mapsto r^2(\varphi, 1, \varphi^{-1})$. Ratios survive; absolute scale does not. Scale remains an external boundary condition, exactly as recorded nineteen sessions earlier.

All of it is real-form independent: the same statements hold for $J_3(\mathbb{O})$ with $E_{7(-25)}$ as for $J_3(\mathbb{O}_s)$ with $E_{7(7)}$, because only the symmetry and non-degeneracy of the Jordan trace pairing enter.

The disposition. The broad claim is REFUTED AT THE MASS SECTOR: ω is not the ingredient that turns the static algebra dynamical. **P0-1 retires as an independent dynamics-selector item.** What survives is narrower, true, and worth printing:

The FTS is the symplectic bulk vectorization of the cubic Jordan Lagrangian boundary.

ω contributes **kinematics**, not dynamics. The algebra where the mass proxies live is not merely a subspace of the 56; it is a Lagrangian *boundary* of it, which is precisely why the symplectic form is blind there.

Clause 27, and a correction the house owes

The sealed house table predicted that this outcome would leave nothing behind — that if ω selects no dynamics, the promotion gate already owns whatever remains. **That reasoning was wrong, and the lane refuted it under blind conditions.** "ω contributes no dynamics selector" does not entail "the promotion gate owns the residual question". S_{promo}, *as specified*, constrains D_F; it says nothing about a polarization, a Legendre map, or a Hamiltonian on the FTS. The house verdict is withdrawn and logged as a casualty in Appendix Y.

The residue is therefore *named and printed* rather than quietly absorbed, and the priced-clause register above goes from twenty-six entries to twenty-seven:

Clause	Status	What would discharge it
FTS symplectic-lift / Hamiltonian attachment	OPEN; parent-action / physical-attachment tier	Select the conjugate section or Legendre map, and a Hamiltonian whose vector field carries the required physical mass-sector attachment. Sibling of the promotion-gate wall; *not* owned by S_{promo}.

Why this episode is printed at length. The cheap path was to retire ω quietly on the grounds that the parent-action work had probably absorbed it. That would have been a ruling dressed as bookkeeping, on the one item the programme had avoided testing for nineteen sessions — and it would have been wrong in a way nobody would ever have noticed, because the residue it silently absorbed is real. What actually happened instead: the house sealed a table it believed, the lane reproduced five of its six results independently and refuted the sixth, and the programme ended the round down one unexamined claim and up one honestly named open clause. That is what the sealed-table protocol is for.

Carrier fence and dynamics-seed reconciliation (Rev32; beside clause 27)

Three tens — every ten names its ten (Rev32 fence). Three ten-dimensional carriers circulate near clause 27, and equal dimension identifies nothing. The *physical/vector ten* (the $\mathfrak{so}(5,5)$ vector inside V_{56}, K_{dyn}-Casimir $1/6$) is isomorphic to the *selector ten* (two-host certified; kernel `s1067`); the *boundary ten* $\mathrm{ad}\, K_{\mathrm{dyn}}$ (Casimir $1/4$, self-intertwiner $\mathbb{R}\cdot I$, Hom $= 0$ to the physical ten, obstruction numeral $1/\sqrt{6}$) is a different module entirely. [LIB2-016, LIB2-017] Every ten-claim in this suite names its ten; a ten-claim that does not is ill-typed. [A1522 B1; A1523 B1; kernels `s1067`/`s1070`; the adjoint-wall census is printed in Appendix S.]

The no-energy-order fence and the dynamics seed are compatible (Rev32). The A1489 order theorems stand: the $\otimes$-layers carry no derived energy order. [LIB2-167] *Interpretive corollary, printed beside the fence only (Rev32.3, PI wording S300): the $\otimes$-layers are order-free by theorem; apparent motion is re-presentation along exchange isomorphisms; dynamics is priced extra structure.* The clock generator H_{clk} (L-DynamicsSeed; A1268–A1270, kernels s483–s485, MOTIVATED tier) is *licensed extra structure* on the symplectic carrier — a Cartan grading, not a $\otimes$-layer order — so citing the seed does not violate the fence and the fence does not retire the seed (first suite citation of the seed since S210). [LIB2-167] Independent corroboration, printed both ways: the A1512 golden bridge H_{φ} re-derives a face of the seed's golden Cartan flow (the $\exp(2\ln\varphi\, H)$ ladder) seventy-four sessions later with no shared derivation path; each corroborates, neither derives, the other. [LIB2-167] [INTERCONNECT_SWEEP_S281, Finds 1a/1b.]

3.6 What Rev29 opens with, stated plainly

The engine is unchanged. Rev28 remains sealed. No observable moved in twenty-three rounds and none moves in this fold. The bits-plus-dims headline stays withdrawn, and the honest replacement sentence is unchanged: exactly one continuous modulus (ρ) survives anywhere in the end-state ledger. χ has no value, and none will be computed until a physical generator and response map exist. $V_{\mathrm{CKM}} = I$; α and colour remain fenced; KILL-NODE 2 remains fired.

What is new is that the programme can now say, in one table, exactly what it is assuming — and for each assumption, what would end the argument.

4 The Rev30 fold ledger: the freeze-first arc's typed inputs

The freeze-first arc (A1475–A1500) moved no observable and changed no engine value. What it produced that belongs in an *input* ledger is a set of exact type counts for the data the selector construction consumes — and one correction to a type that had been printed wrongly and propagated.

4.1 A corrected type: the sign datum is connected

Correction (Rev30). The source-layer type $\mathcal{D}^{\text{sign}}_{\text{phys}} = \mathbb{RP}^{15} \times \mathbb{R}_{\geq 0} \times \mathbb{Z}_2$ — "direction, magnitude, orientation" — is **wrong as a global space**, and it was carried through two house documents before it was caught. The oriented-line double cover of $\mathbb{RP}^{15}$ is S^{15}, which is **connected**, and a connected space is not a product with $\mathbb{Z}_2$.

The type of record. The functional consumes simply $j \in \mathcal{F}_B^{16} \cong \mathbb{R}^{16}$: 16 **continuous,** 0 **independent discrete**. An orientation bit exists *only* over an already-fixed ray — it is ray-relative, not an independent factor, and must never be counted as one. House-verified. Any count that adds a separate $\mathbb{Z}_2$ to this datum is double-counting a bit that is not there.

4.2 The two irreducibly-Input data, with exact counts

A homogeneity argument settles both of the inhabited data at once: the relevant action is transitive on each fibre, so every invariant scalar is constant on it, and *no selection law exists in the frozen frame*. This upgrades an earlier half-statement about frame insufficiency into a two-datum theorem with exact types:

Datum	Continuous	Discrete
Boundary datum L_∂	10	0
Non-equivariant reduction σ_{sel}	43 (53 once the source is fixed)	0
Source j (sign datum, §4.1)	16	0 independent

The 43 and 53 are *alternative stages of the same count*, not additive. The residual stabilizer of the loaded data has dimension exactly 10.

Scope, binding — and it is narrow. These are counts of what the construction *consumes*, not of what it derives, and the accompanying reduced action lives in an explicitly declared homogeneous (0+1) reduction. **The four-dimensional continuum lift is open and is not relabelled here.** A (0+1) result quoted as a spacetime result would be exactly the interface-type erasure this appendix exists to prevent: an object built in one layer, carried across an unproved interface, and renamed. Nothing in this section promotes an observable, moves a tier, or touches the engine.

4.3 A declared-class exclusion: the compact two-scale Gram ansatz collapses to the scalar line

One further typed result from the same arc (the D1180 finish-the-string round, adjudicated A1487) is printed here for the first time; it has appeared in no prior revision of this suite. Intersecting the

compact two-scale Gram ansatz

$$g_Q = g_u = g_d = c, \qquad g_L = g_\nu = g_e = l$$

with the plane-preserving, coset-compatible, SM-invariant Gram family leaves **only the scalar line** $l = c$. Hence $r = l/c = 1$ exactly, and **the previously priced point $r = 1/2$ is excluded in this declared natural class**. [LIB2-138] The exclusion closes the particular compact two-scale route to a 7:3 pole-strength ratio *within the declared class* — it does not prove every possible physical Gram democratic, and it is not a universal statement. [LIB2-138]

Three fences, printed with the result. (i) *Disambiguation:* the "two-scale" here is the compact two-scale *Gram ansatz* of the D1176–D1180 string. It is not the retired Rev19–24 two-scale *calibration* recorded earlier in this appendix, and the two must not be conflated. Likewise the 7/3 printed elsewhere in this suite is the unification ratio $n_{23}/\mathrm{rank}(J_3)$ — a different object; this exclusion attaches to no printed formula, and no suite claim is corrected by it, because $r = 1/2$ was never asserted in print. (ii) *Provenance:* the collapse-to-scalar-line theorem is the **lane's**, from the D1180 return; the house independently verified the supporting first-order arithmetic (commutant dimension 43, Gram response rank 26, all nonzero singular values $2/\sqrt{3}$) but did *not* independently delimit the plane-preserving/coset-compatible/SM-invariant family, and the lane kernel s989 has run on one host. The result is class-conditional. [LIB2-138] (iii) *Scoring:* the earlier house ruling that the B-map Gram would not land at $l/c = 1/2$ (D1176 R4) was scored VOID because its subject was ill-typed when scored; it stays VOID and no bar moves. The physics went the house's way; that is recorded here, separately from the score.

4.4 The tautology charge reduces to one number: the price of a continuous anchor

The D1102 loading-budget round is recorded in Appendix Y (row 1402) as *verdict line not proven.* That typing is correct and is not revisited here. What was never printed is the *reason* it is not proven, which is sharper than the refutation and is the useful object: the entire disagreement collapses onto a single methodological number.

Write I_{out} for the information credited to the outputs and L_{in} for the loading charged to the inputs, and let b be the average price, in bits, of one continuous anchor dimension. Then $I_{\mathrm{out}} > L_{\mathrm{in}}$ holds exactly when $b < b_\star$, and $b_\star$ is fixed by the output accounting alone:

Output accounting	I_{out} **(bits)**	$b_\star$
As claimed by the round	36.27	3.85
As claimed, m_e Koide line corrected	39.61	4.46
Strict floor + m_e Koide restored ($= 27.27 + 3.34$)	30.61	2.82
Strict floor (argued lines only)	27.27	2.22

Two of these rows were corrected in revision, and the corrections run in opposite directions. The third row was previously labelled "Koide credit restored to the house value", which reads as the first row adjusted; it is not. It is the *strict floor* plus the 3.34-bit Koide restoration, $27.27 + 3.34 = 30.61$ — the asserted lines are struck in that row too. And the second row, "as claimed with the m_e Koide line

corrected" ($36.27 + 3.34$), had never been printed at all. It is the reading the old label most naturally suggested, and it is **the most favourable of the four to the position argued here**; its omission understated this appendix's own case and is repaired rather than left standing.

The round's own conservative conversion prices a continuous anchor at ≥ 6 bits/dim, giving $L_{\text{in}} \in [48.1, 59.1]$ against $I_{\text{out}} = 36.27$ — i.e. **its own numbers give $I_{\text{out}} < L_{\text{in}}$ while its verdict line asserts the reverse**, the flip being attributed to unexplained discounts. Hence *not proven by the shown table*: the arithmetic reproduces, the conclusion does not follow from it.

Two accounting defects drive the spread between the rows. First, 9.0 of the claimed 36.27 bits (25%) sit on asserted CKM and mass-proxy lines carrying no line argument; striking them is what produces the strict floor. Second, the m_e Koide line is credited 3.9 bits against a house value of 7.24 at a 0.66% match — under-credited by ≈ 3.3 bits with no stated discount rule. (By contrast m_μ/m_τ, credited 7.7 against house 8.04 at 0.38%, is consistent, and the three mixing angles agree to the digit.)

The rule is forced by the output side, not chosen

The whole budget is one affine relation. Reading the round's own two conversions ($L_{\text{in}} = 48.08$ at 6 bits/dim, 59.08 at 8) back gives

$$L_{\text{in}} \;=\; 15.077 \;+\; 5.5\,b, \qquad \text{hence} \qquad b_\star \;=\; \frac{I_{\text{out}} - 15.077}{5.5},$$

with 15.077 the discrete subtotal and 5.5 the number of continuous anchor dimensions. This reproduces all three published crossovers to the digit (3.853, 2.824, 2.217), so nothing in the dispute is arithmetical. *(Rev33.0, R174.)* The count 5.5 is the round's D1102 convention, and it charges the loaded depth $13 + \frac{1}{8}$ twice: once as a continuous anchor inside the 5.5, and once inside the 15.077, whose ten itemised choices already include β, the depth registration (`s1197`). The three crossovers are kept as that convention's arithmetic; the budget that governs is the re-scoped one below, which charges the depth neither way (its measured continuous requirement is zero, and β selects none of the credited lines, `s1199`). *(Rev33.0.)* This affine relation charges every continuous anchor the *same* b and the whole discrete subtotal whatever the outputs. It is kept because it is how the round and the house argued; the verdict no longer rests on it. The sensitivity computation it deferred has since been run anchor by anchor and re-scoped to the governing output lines — see *Run and ruled (Rev33.0, R166)* below.

What fixes b is that the output credits are *already* in a stated currency. Decoding them: θ_{12} at 4% is credited 4.64; θ_{13} at 3.2%, 4.97; θ_{23} at 12%, 3.06; δ_{CP} at 12.5%, 3.00. Each is exactly $-\log_2$(relative precision). So I_{out} is a **description length**: bits of the observable actually pinned down. Consistency then forces the input side to be the same quantity, and b is not free:

$$b \;=\; -\log_2(\text{relative precision to which one continuous anchor must be specified}).$$

Two consequences, and they are not the ones the round expected (binding).

(1) The model-selection penalties are the wrong currency, and are excluded — not adopted. AIC gives $1/\ln 2 = 1.44$ bits per parameter; BIC and two-part MDL give $\frac{1}{2}\log_2 N$, i.e. 1.50 at $N = 8$ observables and 2.16 at $N = 20$. All of these fall *below every* $b_\star$ in the table, so on those rules $I_{\text{out}} > L_{\text{in}}$ in all three accountings and the tautology charge would simply fail. But they measure the price of *having* a free parameter in a model comparison, not the description length of its value, and I_{out} here is the latter. Reaching $b = 6$ under BIC would require $N = 4096$ observables. The earlier

pointer to the complexity-penalty/information-criterion gap was therefore a *mis-identification*, and is withdrawn here.

(2) The accounting dispute does not decide the charge — ***at the two extreme scopings, and only there****.* At the charged count $n = 5.5$, $b_\star$ moves only over [2.22, 4.46] across every accounting on offer, while the round's own convention is $b \geq 6$: every accounting lands on the same side, so the itemization quarrel is not what the verdict turns on there. The same holds, in the opposite sense, at $n = 2.5$.

> **But this claim was printed unqualified, and unqualified it is false.** At the intermediate scoping $n = 3.5$ of §4.4 the four accountings give $b_\star = 7.01, 6.06, 4.44, 3.48$ — a range that *straddles* $b = 6$. There the choice of accounting decides the verdict outright, and the m_e Koide line, questioned two paragraphs above, is exactly what moves it. **The general claim was an over-reach licensed by testing only the two scopings this appendix happened to have printed**, and it is corrected here rather than carried. What survives is narrower and still useful: the itemization quarrel is not decisive *at the endpoints*. In the middle it is the whole question. **The omission of the third scoping was the house's own** — and when the third column was shipped to both external lanes in the scope round (§4.4), it was disclosed as the house's omission, not presented as new ground.

What is genuinely open, now sharply. $b_\star = 3.85$ is a relative precision of 6.9%; $b = 6$ is 1.6%. So: **if each continuous anchor must be known to better than $\approx 7\%$ for the outputs to hold, the charge stands; if 7% sloppiness suffices, it fails.** That is a sensitivity computation, not a matter of taste, and it was the single outstanding item (run at Rev33.0 — see *Run and ruled* below). Nothing here credits or debits any observable, promotes anything, or touches the engine.

The house charging convention, stated once and applied uniformly (Rev32.3, PI ruling S300; called a "rule" at Rev32.3, renamed at Rev32.5 — the two-sided discount for definitional inputs that the box on the next page calls owed is still owed). The charged count n was never given a rule in print; here it is. A continuous anchor is charged at weight 1 when it is *internal* to the readout chain, at weight $\frac{1}{2}$ when it is an *external* empirical anchor consumed as a datum, and at weight 0 when it is a *transfer-sector* quantity on which no scored output depends. Applied to D1102's own six entries: the m_τ anchor (1); the Koide $K = 2/3$ line ($\frac{1}{2}$ — the only external anchor, and exactly the half in $n = 5.5$, previously unstated); the loaded depth $13 + \frac{1}{8}$ (1); and g_3, C_{sub}, r_C (0 each — the named-open data of the interacting-vertex programme, banked in Appendix R as structurally free and invisible from the boundary; a description-length rule cannot charge bits to specify a quantity no scored output depends on). Under the rule the house's own count is $n = 2.5$ — the charging party's withdrawal count in the scope table below, reached on the same principle (chargeable only if the map to some scored output is non-constant). The blind lane's reservation stands beside it: it declined to exclude r_C without a proof of analytic orthogonality, giving $n = 3.0$, and that proof has not been supplied. This paragraph fixes what the house charges and why; it does *not* lift the fence below. No verdict on the tautology charge is asserted here in either direction, the intermediate scoping is still decided by the sensitivity computation, and the acquittal the charging party offered is still not taken.

Corollary — the m_e Koide anomaly is explained. Under the decoded rule the house values are exact: $-\log_2(0.0066) = 7.24$ and $-\log_2(0.0038) = 8.04$, matching the house column on both lines. The

lane's 3.9 for m_e Koide is therefore not a missing discount rule but an **unexplained departure from the rule its own table applies everywhere else**.

Receipts: house verifier `s864` (arithmetic PASS, verdict-follows-from-table FALSE, sha `5a5ba03dfaaba55d`); register at the head of `Trackers/DISPATCH_INDEX.md`.

Ruled (Rev32.10, R140 D-4, S343). The one methodological choice that the `s864` contradiction reduces to — the claimed table $b_\star = 3.85$ against the strict floor $b_\star = 2.22$ — is ruled: **the strict floor governs.** The return's verdict line "$I_{\text{out}} > L_{\text{in}}$ materially (+8–12 bits)" is struck as not following from its own table (its conservative conversion gives $L_{\text{in}} \in [48.1, 59.1] > I_{\text{out}} = 36.27$). This ruling fixes the standard; it does not assert a verdict on the tautology charge, which the paragraph above leaves open in either direction.

Run and ruled (Rev33.0, R166, S360). The sensitivity computation has been run, and like for like against the governing row. **Output side:** the strict floor, $I_{\text{out}} = 27.27$ (θ_{12} 4.64 + θ_{13} 4.97 + θ_{23} 3.06 + δ_{CP} 3.00 + m_μ/m_τ 7.70 + m_e Koide 3.90). **Continuous side, measured per anchor** against those six lines (house kernels `s1187`, `s1198`): 13.97 bits if the Koide line is read as a ratio, 18.70 if read on the absolute mass. This charges v, Λ_{G_2} and α wherever the six lines depend on them — anchors the six-entry ledger above does not name — and charges 0 to the loaded depth and the transfer trio, whose measured requirement is zero. **Discrete side, re-scoped:** the 15.077 is itemised as ten $\log_2 N$ choices (`s864`, `s1197`); a choice that selects none of the six credited lines is not charged, by the same principle that zeroes the transfer trio (`s1199`). Five are removed — the AX6 orbit selector (the weak-angle branch), Route B (the CKM first row), β (the depth registration, reaching only $\sqrt{\sigma}$), and two transfer-sector choices absent from the scorecard generator — and a def-use trace of the generator confirms that none of them, nor the Dirac/Majorana choice, feeds any credited line (`s1200`). Left: 7.17 bits. **Result:** $L_{\text{in}} = 21.14$ (ratio reading) and 25.87 (absolute reading) against $I_{\text{out}} = 27.27$ — **both clear, by** 6.13 **and** 1.40 **bits.** The ruling (R166) answers the description-length charge at this accounting, and it is stated with its weaknesses, which travel with it: (i) the absolute margin is thin — one uncharged anchor bit found anywhere nearly erases it; (ii) one line sets every continuous requirement: each anchor's bits are driven by the m_e Koide match (0.66%); (iii) the computation is the house's own and has had no external hostile review; (iv) which Koide reading governs is not ruled — both clear, so it does not decide this. Dirac/Majorana stays charged (1.000 bit): no generator value depends on it, but a choice that acts before the generator is outside a def-use test; if it is later shown to reach no credited line both margins rise by 1.00. **Reopener:** any finding that adds ≥ 1.40 bits to the absolute reading, or removes a line from the strict floor. Nothing here credits a new observable, promotes anything, or touches the engine.

What the 5.5 **actually is — and the scope question that outranks the precision question**

The dimension count is not a free parameter either. The round's own K1 ledger names six continuous/anchor entries: the m_τ anchor, the Koide rule $K = 2/3$, the three transfer constants $\{g_3, C_{\text{sub}}, r_C\}$, and the loaded depth 13+1/8. Five at full weight plus Koide at the half-dimension external-anchor discount gives exactly 5.5. The half-dimension is that discount and nothing else.

What K costs, and how well it is known. The Koide line is the framework's most sensitive anchor: $\partial \ln m_e / \partial \ln K = -76.13$, so K must be known to 8.8×10^{-5} relative for m_e to hold at its claimed 0.67 %. The rule is supplied externally and is known better than that: across the whole span of the two competing τ-mass determinations this appendix already prints side by side in Table 1 —

1776.93(9) and 1776.86(12) — and both their error bars, the measured Q stays within 2.0×10^{-5} of 2/3 (at the anchored 1776.93, $Q = 0.6666645$, a shortfall of 3.3×10^{-6}). The framework's use of exactly 2/3 therefore sits inside its own tolerance by a factor of ≈4.5 (`s1365`, which parses both determinations from Table 1 rather than re-entering them). **It remains an external empirical input, charged:** nothing here derives K, and no print in this suite calls it derived, algebraic or zero-parameter.

But three of those six — g_3, $C_{\rm sub}$, r_C — are the named-open data of the *transfer / interacting-vertex* sector (Appendix R), not anchors of the committed zero-fit readout. That appendix banks g_3 structurally free as a theorem, proves $\mathcal{N}_{\rm abs} = g_3\, Z_{\rm sub}(C_{\rm sub})$ invisible from the boundary, and closes with the statement that nothing in it is a world-scored observable. Under a description-length rule one cannot charge bits for specifying a quantity on which no scored output depends, so the charged count and the effective count can differ — and the gap is decisive:

	$b_\star$ (claimed) $I_{\rm out} = 36.27$	$b_\star$ (m_e corr.) 39.61	$b_\star$ (floor + Koide) 30.61	$b_\star$ (floor) 27.27
As charged, $n = 5.5$	3.85	4.46	2.82	2.22
g_3, $C_{\rm sub}$ excluded; r_C retained, $n = 3.5$	6.06	7.01	4.44	3.48
Transfer sector excluded, $n = 2.5$	8.48	9.81	6.21	4.88

The middle row is not a compromise position but the one the sector's *own* theorems most directly license: g_3 and $C_{\rm sub}$ are *proven* to cancel and to be boundary-invisible, whereas Appendix R types r_C only as "continuous, OPEN" — not proven to cancel, merely not fed into any scored formula. Whether that difference is a difference *in kind* is the question on which the whole table turns, and it is the one this appendix is least entitled to answer for itself.

At $n = 5.5$ the round's own $b \in [6, 8]$ exceeds every $b_\star$ and the charge stands on every accounting. At $n = 2.5$ it falls below $b_\star$ on the claimed accounting and the charge fails — **on the round's own convention**. *The scope question therefore outranks the precision question: which parameters are in the budget moves the verdict further than what each one costs.*

A self-directed problem, recorded because it cuts against this appendix's own case (Rev31.2): the Koide rule appears on both sides of the budget. The input side charges $K = 2/3$ at the half-dimension external-anchor discount — nearly free, as befits an exact rational — while the output side collects $-\log_2(0.0066) = 7.24$ and $-\log_2(0.0038) = 8.04$ bits for the two mass lines that same rational generates. If a definitional input is priced at ≈ 0 while its consequences are credited in full, **the ledger manufactures information out of a definition** — the tautology charge in miniature, relocated to the Koide entry. Neither lane raised it: the referee that audited the arithmetic held the output ledger but not the charge framing, so it could not see the double-appearance. What is owed is a discount rule for definitional inputs, stated once and applied uniformly to both sides. Until one exists, this entry is a named open problem against the house's own accounting; nothing is credited, debited, or promoted on its account.

This is fenced, and the fence is about who may answer it. The house does **not** rule here, and the reason is not modesty. The scoping that shrinks n to 2.5 is the one that *refutes a charge against this programme*, so a house ruling in that direction is exactly the conflict of interest the coalition's blind-lane discipline exists to prevent. Nothing above is banked: no observable is credited or debited, no tier moves, the engine is untouched, and **no verdict on the tautology charge is asserted in either direction**. What is established is narrower and, we think, more useful: the charge is decided by two declarations — the currency (fixed: description length, forced by the output column) and the scope (open) — and *not* by the line-itemization quarrel the round was fought over. The scope declaration is owed to an external lane, with every column shipped so it cannot be settled by whichever number is quoted first.

The scope declaration, as it came back

The declaration was put to two external lanes simultaneously, under a house pre-commit sealed before either was sent (`s1036`, sha `cd3cb1ab7847...`) whose first entry is not a prediction but a *disclosure*: that the house's interest lay with $n = 2.5$, and that the house believed the exclusion argument correct *and* believed itself disqualified from saying so. One lane received the question with the framing redacted — no identities, no $b \geq 6$ convention, no indication which answer favoured whom. The other was the party that brought the charge, and received everything, including the statement that exclusion would defeat its own case.

	redacted lane	**charging party**
g_3, C_{sub}	exclude	exclude
r_C	**not** excluded	exclude
Koide half-dimension	strike (exact rational is discrete)	retain
resulting n	3.0	2.5

The charging party withdrew the charge, declaring $n = 2.5$ on a named principle — chargeable only if the map to some scored output is non-constant — and excluding r_C *further than the blind lane was willing to go*. The blind lane refused r_C without a proof that the outputs are analytically orthogonal to it across its domain, a bar this appendix has not met and does not claim to.

And the house did not take the acquittal. Two things forbid it. First, the withdrawal was argued from a single cell: at the conceded $n = 2.5$, on the round's own $b \in [6, 8]$ and its own accountings, $I_{\text{out}} > L_{\text{in}}$ fails in **five of nine** cells — the declaration quoted the most favourable one. **A concession is not a proof**, and the verdict line remains *not proven*, exactly as Appendix Y row 1402 has always had it. Second, and worse for us: **the number the house wanted came from the party with a stake in giving it, and was refused by the party with none**. That is the precise configuration the fence above was erected to distrust, and it would be a poor use of a fence to dismantle it on arrival. The scope therefore stands **contested**, not settled — and the reader should note that the two rulings disagree in part because a row of *our* ledger, "Koide rule $K = 2/3$, external anchor, discounted", let one lane price an exact rational and the other an empirical relation. That ambiguity was ours; it is resolved by ruling (Rev32.10, R140 D-5, S343): **the** 0.5 **charged the empirical fact that the Koide relation holds of the observed charged-lepton masses — the external anchor — not the exact rational** $K = 2/3$**.** Grok's reading (retain the 0.5) is the one the row meant; Gemini's pricing of the rational answered a different question, and A1501's conditional resolves on this referent. The lesson stands: name the object, not the label.

5 The Rev31 register clause: the boundary object, priced once (S274–S275)

The parent-action closure (Appendix E, §"The parent-action closure") changes nothing in the tables above and adds exactly one register entry, stated here so that its price is written in the ledger that exists to carry prices.

Register clause (Rev31). The selector parent action is complete-conditional on **one ten-dimensional boundary object with three exact faces**,

$$[j] \iff L_\partial \iff E^* = \text{span}\{i_B j,\ \Omega i_B j\},$$

priced **once**: 10 continuous for the object, plus 1 continuous for the positive dark coupling κ_d, with *zero* discrete data consumed. The three faces are one loaded object, not three inputs — any count that prices a face twice is double-counting. Status of the object itself: INPUT (loaded); the $O(16)/SO(16)$ obstruction theorem (A1509/A1510) proves the price *irreducible within the grammar*, so this row cannot be discharged from inside; only a genuinely external category could move it. Plan-v2 Row 1 stands at CANDIDATE-INHABITED (NON-BLIND), on the adopted register sentence: "The declared braid grammar contains a primitive carrier object whose loaded orbit and stabilizer profile exactly match the required boundary datum; that object is another realization of the existing pure-spinor boundary choice." (A1508, certified non-blind after a scope hold and PI-ruled rerun.) Physical boundary selection remains OPEN/LOADED. The dark projective ray $[W_0]$, by contrast, enters at *no* register cost: it is grammar-derived (unique top eigenspace of the prefrozen detector spectrum, A1509 Thm. B), and the selector coefficients $(1, -\frac{2}{9})$ are forced (A1511) — a parameter-free selector. Fences: selector tier only; continuum/pole/LSZ untouched; ρ, κ_d, U positivity-only, no numeric values claimed.

Leibniz Quantum Beats Newton

Appendix Y — The Laboratory Notebook: Round-by-Round Provenance, A1300–A1572

Rev33.2. From the S224 head band (A1300) to the close of the freeze-first era (A1500, S270, 2026-08-05); the core span is one hundred and fourteen consecutive adversarial rounds from the first Rev26 assessment (S228, 2026-07-05) to the post-Rev27 derivation dive (S257, 2026-07-27), with the A1300–A1357 head band and the A1475–A1500 freeze-first era added at Rev30. This appendix exists because the suite's printed claims and the rounds that certified them had drifted apart: Rev26 and Rev27 folded their physics into the tex without carrying their citations, so a reader could not walk from a printed statement back to the round that verified it. Nothing printed elsewhere in this suite is unsupported — but until this appendix, much of it was unfindable. Here every round is listed with its dispatch, its session, its headline result and the component that carries its content. This is a locator and an audit trail, not a second statement of the physics.

Tom O'Sieg

August 2026

1 What this appendix is, and why it exists

The provenance finding (disclosed, S254). Before Rev28, the highest assessment number cited anywhere in this suite was **A1357** — the Rev25 nuclear arc. The sealed Rev27 suite (246 pp) cites nothing above it. That means the entire **Rev26 dynamics-frontier fold** (S228–S240) and the entire **Rev27 priced-selector fold** (S242–S246) reached the tex *by content only, with no citation back to the assessments that certified them* — roughly seventy rounds in the gap. This is not a content loss: the physics is in Appendices R, S and X, and every printed row traces to a banked source when checked by hand. It is a *provenance* loss — the audit trail that the programme's own honesty machinery depends on. Rev28 is the first fold in that stretch to carry its citations into the tex, and this appendix retrofits the locator layer for everything behind it.

The programme runs an adversarial protocol: a dispatch is drafted and its sha frozen, an external lane (ChatGPT, Gemini, Grok) returns an executed or analytic answer, the house independently re-verifies every load-bearing claim with its own kernel, and the result is banked at the tier the *verification* supports — not the tier the return claimed. The assessment file is the primary record of that adjudication. The A1358–A1473 core span is listed below — 114 rounds over 116 numbers (two reserved) — together with the A1300–A1357 head band and the A1475–A1500 freeze-first era, both added at Rev30.

Three properties of this record are worth stating before the table, because they are what make it usable as evidence rather than as narrative.

1. **House verification is independent, not confirmatory.** In this era every executed (MODE-X)

round carries a house verifier kernel, recorded in that round's own assessment header, run twice and required to be byte-identical. Lane modules are not imported; the house reimplements. The kernel identifiers for the S249–S253 rounds are printed in the table; for earlier rounds they live in the assessment headers and are not reproduced here, because this appendix will not print a hash it has not re-checked.

2. **The house is wrong on the record.** Rounds in which the lane caught a house error are marked CAS in the table and collected in §16. There are eight in this era. They are printed because a protocol that only records its wins is not evidence of anything.

3. **Tier labels travel.** Where the table says REFUTED, OBSTRUCTED or NO-GO, the round's product was a negative result, and the negative is banked as firmly as any positive. Roughly a third of this notebook is negative results.

2 Reading the ledger

Round: the dispatch answered. "content" marks a return accepted against a dispatch that was never formally filed (an informal-thread round); those carry a receipt variance, logged in §16. **S**: session number. **Where it lives**: the component carrying that round's content — a locator at section granularity, *not* a per-claim citation. A dash means the round's product was editorial, protocol-level, or a negative that closed a route without adding printed text. **Marks**: CAS = house casualty logged this round; PRE = the round was adjudicated against a house prebuild registered *before* the return landed; COLD = external hostile review; ORPH = the round had no dispatch-ledger row at all before S258, when the four were retrofitted.

Convention on "where it lives". Appendix R = the interacting vertex and dynamics frontier. Appendix S = the priced selector and the boundary arc. Appendix T = the Cartan carrier, the observation functor and the record. Appendix O = the rest-mass programme and the scale registration. Appendix M = the order-five clock and occupation. Appendix L = quantum foundations. Appendix I/H = dynamics, throat and knot geometry. Appendix X = the input ledger, kill list, priced-clause register and casualty register. Paper 9 = the nuclear cluster tier. Era 5 adds five destinations: Appendix P (and its sidebar) = the fifth direction, the chiral lift and the X derivation. Appendix Q = the golden vacuum spectrum and the pointed Fibonacci operator. Appendix F = the gauge boundary and its counterterm law. Appendix K = the Yukawa coefficient. And the papers themselves take Era-5 content for the first time: Paper 3 (the controlled-defect frame), Paper 4 (doublet–triplet), Paper 5 (the Dirac-loaded posture), Paper 6 (thresholds).

3 The head band: A1300–A1357 (recorded S270)

The ledger below opened at A1358. Everything before it — the whole S222–S227 head of the band — had no row, while the rounds themselves are filed and readable. Fifty-five rows are added here; they were drafted from the assessment files, and each carries the round's *outcome*, not its title.

Three numbers are absent by fact, not omission: A1325, A1326 and A1327 have no assessment on disk. They are listed as absent rather than skipped, so the gap is a record and not a silence. A separate check: A1415, sometimes read as a gap, does not exist either — the sequence runs from A1414 straight to A1416.

A	Round	S	Outcome
1300	D1008	224	$V_{\omega=\pi/4}$ + complex 56-bein + C_{iab} preflight — ACCEPT: an *honest no-close*, and a real advance
1301	D1009	224	matched-unit bosonic C_{iab} completion — ACCEPT (AMBER) + ⋆ a decisive in-house finding
1302	D1010	224	plan confirmation before the `s546` heavy build — ACCEPT, with two corrections
1303	D1011	224	the Appendix-H T-bridge construction — ACCEPT: built and validated in house to 10^{-13}
1304	D1012	224	golden-adapted grading / σ-eight — ACCEPT: diagnosis confirmed, path set
1305	D1013	224	golden VEV / flat-valley flow — ACCEPT: flow picture confirmed
1306	D1014	224	the C_{iab} endgame recipe — ACCEPT: last mile specified, in-house probe agrees
1307	D1015	224	fork decision (A vs B) + the rank-4 / ten-physical revised endgame — ACCEPT
1308	D1016	224	two-neutral / rank-4 Cartan module + the H^{CW} decider — ACCEPT
1309	D1017	224	*Grok:* is the rank-4 golden vacuum right? — ACCEPT (literature/SUGRA cross-check)
1310	D1018	224	*Gemini:* what the rank-4 golden vacuum *is* — ACCEPT
1311	D1019	225	the fermionic C_{iab} endgame route — ACCEPT
1312	D1020	225	*Grok:* the $2 \leftrightarrow 2$ bridge — ACCEPT; PLAUSIBLE→STRUCTURAL
1313	D1021	225	*Gemini:* bulk + defect — ACCEPT + ⋆ the pivotal physical reading
1314	D1022	225	the κ_{phys} caveat-lift review — ACCEPT
1315	D1023	225	baryon-pole / trefoil-sector verdict + a deterministic kernel bundle
1316	D1024	225	*Gemini:* the $SU(2)_L = \kappa_{\mathrm{phys}}$ unification lead — one wall or two?
1317	D1025	225	*Grok:* the named proof — soliton throat action from gauged-SUGRA reduction
1318	—	225	*unsolicited, no dispatch:* minimal one-current boundary action $\Rightarrow \chi = 1/2$ (conditional)
1319	D1026	225	exact DWS vector-mass operator + locality projector + Goldstone
1320	D1027	225	*Grok:* the three load-bearing algebra steps of the vector-mass close
1321	D1028	225	*Gemini:* alternative spurious-mode mechanisms + supertrace bypass routes
1322	D1026	226	bundle-review return: the full S215 vector-mass gate verdict
1323	—	226	narrows the supertrace wall to the physical sector
1324	—	226	rebuild of the A2/L2 → spin-$\frac{1}{2}$ machinery the bundle lacked — ACCEPT
1325–27	—	—	*absent on disk — no assessment filed at these numbers*
1328	D1032	226	⋆ exact closed-form A-tensor projector DELIVERED
1329	D1033	226	⋆ pre-registered Gate-H prediction — **blind, on record before the run**
1330	D1034	226	⋆ Gate-D puzzle ANSWERED: (A.6) is exact for dyonic ω
1331	D1035	226	⋆ the compensator DERIVED: $K = \mathrm{ad}_{H^+}(R_6)$ with centralizer
1332	D1036	226	⋆⋆⋆ **the vector fix**: $B(c) \cdot \mathcal{M}^{-1}(c)$ replaces the full-P Gram
1333	D1037	226	quasi-flats identified as FD/section *artifacts*
1334	D1038	226	⋆⋆⋆ blind tower test = **direct hit** (4 levels, all doublets)
1335	D1039	226	⋆⋆ **two blind wins, one blind miss** — all scored against the seal
1336	D1040	226	×× **seven-for-seven blind MISS**; the corrected $\kappa(w)$ fails
1337	D1041	226	⋆ blind digit-window **win**: all three e_i to $\leq 4 \times 10^{-14}$
1338	D1042	226	× cross-curvature blind **4/4 MISS**
1339	D1043	226	dictionary delivered as an executable recipe — but the **sealed window is UNSCORED**
1340	D1044	226	×× classification **exactly INVERTED** — 0/6
1341	D1045	226	⋆ the strongest return of the arc: C_ω delivered (zip not received; receipt correctly declared)
1342	D1046	222	⋆ B-anatomy **structural win** · window B d_2-zeros **blind hit** · window A **honest refusal**
1343	D1047	223	E7 proof **cross-check** · window C **3/5 partial**
1344	D1048	223	⋆ fingerprint form = **structure win**, identical to the in-house construction
1345	D1049	223	⋆ **window E VOID** — in-house error, owned

A	Round	S	Outcome
1346	D1050	223	⋆ window F: the **first certified-blind scoring**
1347	D1051	223	⋆ window F′ **sharp 4/9** (was 0/9)
1348	D1052	223	⋆⋆ selector **exact** on 0..19 — $\{0, 3, 11, 17\}$, no partial credit needed
1349	D1053	223	⋆⋆⋆ second nucleus **3/3 sharp** at the frozen 1% bar
1350	D1054	223	⋆⋆⋆ board sweep — ^{6}Li **sharp** (0.112% vs frozen 1%)
1351	D1055	225	⋆⋆⋆ ^{8}Be **survived the killer test** — unbound *by typing*
1352	D1056	225	⋆⋆⋆ ^{7}Li 0.642% + ^{7}Be 0.028% **both sharp**
1353	D1057	225	⋆⋆⋆ ^{9}Be 0.177% + ^{12}C 0.103% **sharp**; ^{9}B mirror sign **pass**
1354	D1058	226	⋆⋆⋆ ^{16}O 0.167% **sharp**, the f_4 paradox **resolved**
1355	D1059	226	⋆⋆⋆ the answer-sheet round: ^{20}Ne 0.00031% **sharp**
1356	D1060	227	⋆⋆⋆ the ladder rule in **closed form**: $\mathcal{T}_n = (n-2) + \varepsilon_n$
1357	D1061	227	⋆⋆⋆ the breathing kernel landed, $\Delta_{\mathrm{br}} = \varphi^{-15} - \varphi^{-18}$; ^{28}Si **full credit via the obstruction branch**; ^{40}Ca −0.032%

What the head band shows. It is the blind-scoring era, and read down the outcome column the misses are as loud as the hits — a seven-for-seven blind miss (A1336), a 4/4 cross-curvature miss (A1338), a classification returned *exactly inverted* (A1340), a window voided by an in-house error the house owned (A1345), and a sealed window left UNSCORED (A1339). They sit beside the nuclear sweep that lands ten consecutive sharp passes at a frozen 1% bar. Both halves are the same era; printing only one of them would have been the more comfortable and the less true record.

4 The round ledger

A	Round	S	Headline result (tier as banked)	Where it lives	Marks
Era 1 — the Rev26 dynamics frontier (S227–S240): the two-circulation, continuum and 7/3 rotations					
1358	D1062	227	Track-2 series update v0.8.0→v0.9.0 — editorial, no scored physics	—	
1359	D1063	228	$A = 8$ naturality square + second-circulation operator; same-object conjecture	App. R, Paper 9	
1360	D1064	228	annular spectral transfer functor; π-coefficient; channel ladder; unit interface	App. R	
1361	D1065	228	the 7/3 retype: naturality + consistency sweep after hostile review	App. K	
1362	D1066	228	orbit certificate; K–B lemma; Möbius delay	App. R	
1363	D1067	228	transfer certificate; arrow ledger; the σ-bridge doorstep	App. R, O	
1364	D1068	228	the export-functor species (rotation cycle 2 complete)	App. R	
1365	D1069	229	the open carrier round — capture + independent verification, ungraded by design	—	
1366	D1070	229	does the Zorn product force $b = a$? finite-carrier round	App. R	
1367	D1071	229	κ_{load}: constructible or a genuine input?	App. R, O	
1368	D1072	229	load-HQ parent action — closes or *relocates* κ_{attach}	App. R	
1369	D1073	230	the σ-bridge flagship: $v_{\mathrm{EW}}/\sqrt{\sigma} = \varphi^{105/8}$ as framed registration	App. O §σ-bridge	
1370	D1074	230	depth-selector round — forcing 105/8 REFUTED; no-go returned	App. O, N	
1371	D1075	230	LOAD-EW-DEPTH open construction	App. O	

continued

A	Round	S	Headline result (tier as banked)	Where it lives	Marks
1372	D1076	231	the framed-Higgs-depth-selector kill shot	App. N	
1373	D1077	231	the reversal spec: permanence fixed-point OBSTRUCTION	App. N	
1374	—	231	lane self-initiated "shape of the missing piece" — no matching dispatch	—	
1375	D1078	231	the loaded-carrier intersection, frame-relative	App. R	
1376	D1079	231	finishing the loadings: HQ + YM/colour	App. R	
1378	D1081	233	arc review of the WH–BH neck geometry (s804–s816)	App. H, I	
1379	D1082	233	the three-tube build: baryon = 3 point masses + 3 flux tubes	App. I	
1380	D1083	233	audit of the completed baryon programme (6 kernels)	App. I	
1381	D1084	233	promotion gate: scale triangulation (s822+s823)	App. O, X	
1382	D1085	233	baryon completion v2 double-check (s824–s828)	App. I	
1383	D1086	235	the curved-KV three-body quantum Hamiltonian	App. I	
1384	D1087	235	the quantum three-body object: twist sectors + quaternionic alternative	App. I	
1385	D1088	235	wallbreaker: twist-evenness / golden magnitude / selection-vs-doubling	App. I, R	
1386	D1089	237	the measure theorem under attack: universality / order-selection / stratification	App. R	
1387	D1090	237	Horn D discriminator: parity-resolved couplings	App. R	
1388	D1091	237	the order-selection vertex (MODE-A, analytic)	App. R	
1389	D1092	237	derive Σ: analytic pass — **superseded by A1390 wherever execution overturns it**	App. R	
1390	D1092	237	derive Σ, executed. **Threshold-edge routing theorem BANKED** — conditional on three named imports (deck holonomy, circle dispersion, edge readout). Σ_{full} **REFUTED** as a pure-BC consequence (the half-integer tower propagates); **P3 and P4 DEAD** (conjugate zero refuted); **radial INVERTED** — retained by the periodic BC, not eliminated. New named wall: $\mathcal{P}_{\text{th}}$ (threshold-residue projection). Receipts: bundle sha exact, three kernels bit-portable, two standing RECORDED-UNVERIFIED items CLOSE.	App. R	
1391	D1093	238	$\mathcal{P}_{\text{th}}$: the threshold-residue projection (capstone)	App. R	
1392	D1094	238	the interacting measure: dressing typed, odd protection adjudicated, pole classified	App. R	
1393	D1095	239	the linear zero + exit gap; edge-alignment theorem; magnitude NO-GO	App. R	
1394	D1096	239	the chamber functor: the L3→L4 shortcut KILLED; functor decomposed	App. R	
1395	D1097	240	the parent vertex: η_+ weld lands conditional; **first derived relative vertex sign** -1; two house overclaims refuted	App. R	CAS
1396	D1098	240	the ten-dimensional deficit adjudicated: 10=10 dead; vertex readout built at rank 7; g_3 structurally free	App. R	
Era 2 — the Rev26 freeze (S241): three independent cold reviews					
1397	D1099	241	ChatGPT COLD referee, 36/100 — one real fix (CW index tier), zero new physics	Rev26 freeze fixes	COLD
1398	D1099	241	Gemini COLD referee, 35/100 — converged on the suite's own named head wall	Rev26 freeze fixes	COLD

continued

A	Round	S	Headline result (tier as banked)	Where it lives	Marks
1399	D1099	241	Grok COLD referee, 11/100 — its one concrete error catch REFUTED in-house	Rev26 freeze fixes	COLD
Era 3 — the selector arc (S242–S248): the wall priced to its last coefficient					
1400	D1100	242	the five-state closure REFUTED: the unique fully equivariant unital kernel is uniform rank-1	App. S	
1401	D1101	242	the readout trichotomy; house "primitive quartic" REFUTED	App. S	CAS
1402	D1102	242	the price of the map: loading budget itemized; its verdict line *not* proven (`s864`; the charge reduces to one number, $b_\star \in [2.22, 3.85]$ — App. X, *The tautology charge reduces to one number*)	App. S, X	
1403	D1103	242	the golden annulus decides against the pretty exit: Tube(Fib) builds, does not derive H_{Mkv}; Fourier support $\{0, \pm 2\}$	App. S	
1404	D1104	242	the chain lands, the weld dies by its own twist list; "proved (welded)" REFUTED in-house (`s867`: $Z(\mathrm{Fib})$ has **three** distinct object twists $\{1, e^{\pm 4\pi i/5}\}$ over four objects, not five; $e^{\pm 2\pi i/5}$ do not occur, so they cannot parameterize the $\mathbb{Z}/5$ winding sectors — corroborated by `s865`-P4 contrast rank 2 of 4)	App. S	
1405	D1105	242	the pre-registered RDL protocol lands; lane reverses itself; tautology charge reinstated	App. S, X	
1406	D1106	242	the geometric clock returns four pieces, not one; underdetermination certificate ($w_1 = 0$ or 1)	App. S, M	
1407	D1107	243	**the quantization theorem**: finite graded odd sector, winding $z \in \{\pm i\}$, no continuous fugacity — and the missing-tube NO-GO	App. S, M	
1408	D1108	243	the three odd lemmas — **W1 and W2 CONCUR; W1b phase DINGED** (`S873_W1_W2_CONCUR_W1B_PHASE_DINGED`)	App. S	
1409	D1109	243	the two-sided audit — **TABLE FAILS RECOUNT**; ledger corrected; dimensions dominate (`S874_TABLE_FAILS_RECOUNT_LEDGER_CORRECTED_DIMS_DOMINATE`)	App. S	
1410	D1110	244	the chamber intertwiner	App. S	
1411	D1111	243	the χ row	App. S, O	
1412	D1112	243	the even→SM assignment audit; price ASSERTED, not proved	App. S, X	
1413	D1113	244	the occupancy round: census honest-null; tautology guard adopted	App. M, X	
1414	D1114	244	the S244 double-check: 13.907 untied / 11.907 tied ledger headline; T1/T2 *not* tube-forced; s882 SO(6,2) correction	App. X (headline now **withdrawn**)	ORPH
1416	D1116	244	**the parent action**: first two exact equations on C; S_{shape} conditional candidate	App. S §source	ORPH
1417	D1117	244	**the mixed tensor**: placement-charge table $\iota = 16p$; $V_{\pm 1}$ exclusion	App. S §charge	ORPH
1418	D1118	244	**the orientation-odd phase**: V-side quadrature $z_- = \pm i z_+$, derived-conditional (blind-adjudicated)	App. S §quadrature	ORPH
1419	D1119	245	the anti-alignment round	App. S	
1420	D1120	245	the j_A round: $[j_A] \in \mathbb{RP}^2$ typed; null axis $(5, -18, 1)$	App. S	

continued

A	Round	S	Headline result (tier as banked)	Where it lives	Marks
1421	D1121	245	minimal height REFUTED-AS-CANONICAL; the support floor $\{3, 11, 17\}$	App. S, M	
Era 4 — the Rev27 freeze and follow-up (S246–S248)					
1422	—	246	ChatGPT COLD review of the Rev27-WIP suite — the only round with fixable defects	Rev27 freeze fixes	COLD
1423	—	246	Gemini COLD review — zero undisclosed overclaims found	Rev27 freeze fixes	COLD
1424	—	246	Grok COLD review (Paper 0 focus) — zero undisclosed overclaims found	Rev27 freeze fixes	COLD
1425	—	248	in-house adjudication of the three follow-up reviews; **the ^{28}Si archaeology** that Gate 1 of Rev28 executes	Paper 9 §typed	
1426	D1122	248	the cascade/equivariance round: six gates banked; one house prediction corrected	App. S	CAS
1427	D1123	248	the two-rescues round: six gates banked; *two* house casualties	App. S	CAS
1428	D1124	248	the one-action / kernel-axis round: axiom ledger compresses to one row; kernel axis dies	App. S	
1429	D1125	248	**the parent-action moonshot**: the dihedral–Markov superconnection candidate, all ten C exact *inside* the candidate	App. S §D1125	
Era 5 — the boundary arc (S249–S251): six rounds, closed at its tier — folded at Rev28					
1430	D1126	249	the $\mathfrak{T}_{AM}$ round: **transgression obstruction theorem**; D1125 KILLED as selector-deriving; AX-AM is a choice *s909 8/8*	App. S §4→S	
1431	D1127	249	support-export: P_{ann} derived, $q = \pm 1$ categorically absent, ray collapse, non-factorization; AX-SEQ KILLED *s910 8/8*	App. S	
1432	D1128	249	the ladder interface OBSTRUCTED + pointed rescue (ρ); affine support theorem; **final kill** of 0.0331562%; AX-AFF-ONSET *s911 8/8*	App. S	
1433	D1129	250	**non-selection theorem**: every $r \in (0, 1)$ a bound-state ray; positivity-uniqueness closed; AX-MG named *s912 33/33 + prebuild 27/27*	App. S	PRE
1434	D1130	250	AX-MG typing: split quantization + **degree independence**; the "continuum died spectrally" wording reversed *s913 32/32*	App. S	CAS
1435	D1131	251	the edge-functor round: **coefficient independence** + reduction theorem + lift degeneracy *s914 46/46*	App. S	
Era 6 — architecture and geometry (S251–S252): folded at Rev28					
1436	D1132	251	the wide-lens round: **the 17-dim map defect confirmed**; the observation-functor faithfulness wall named; the χ moratorium adopted	App. X §Rev28	CAS
1437	D1133c	251	**the two-W separation theorem**; no third cascade; A_4 REFUTED; one radius *s915 27/27 + prebuild 25/25*	App. T §2	PRE
1438	D1134c	251	**the transition cocycle derived at zero continuous parameters**; non-gluing obstruction; $\mathfrak{B}_W$ bibundle; the wall derived a second way *s916 24/24 + prebuild 14/14*	App. T §3	PRE
1439	D1135c	251	the carrier half of $\mathcal{O}_{\text{sig}}$: Π_{sig}, the metric, the connection family, **the $\mathfrak{so}(5)$ Cartan closure** *s917 28/28*	App. T §4	

continued

A	Round	S	Headline result (tier as banked)	Where it lives	Marks
1440	D1136c	252	$27 \downarrow \mathfrak{so}(5)$; the internal fifth direction; triality as the imaginary unit; MacDowell–Mansouri candidate; the solder NO-GO *s918 36/36 + prebuild 38/38*	App. T §5	PRE
1441	D1137c	252	faithfulness retypes the solder problem; branch Real clause; **the spin–charge no-go**; the conformal-mode obstruction machine-proved *s919 22/22*	App. T §6	
1442	D1138c	252	**a reflection-positive state exists at finite regulator**; the gravitational sign problem; the positive cone *s920 27/27*	App. T §7	
1443	D1139c	252	AX-COT named; the conditional arrow; nonlinear OS positivity; associative envelope; Born/Tsirelson at regulator tier *s921 30/30*	App. T §8, L	
1444	D1140/41c	252	**AX-COT independence theorem**; nonlinear simplicity measure; $\mathbb{Z}_6$ under local faithfulness; the χ source–response wall *s922 45/45*	App. T §8	CAS
Era 7 — the sponsored string (S253): five rounds in one session, two blind prebuilds — folded at Rev28					
1445	D1142	253	the §8 attack: the AX-COT stabilizer obstruction becomes a **theorem**; $B-L$/parity/$\mathbb{Z}_4$ closed; two doc-gates *s923 71/71*	App. T §8	
1446	D1143	253	the three-price closure: $o = -D_\varphi$ **exact**; the minimal dilation theorem; g_χ derived away; **the A1444-G2 erratum** *s924 47/47*	App. T §9	CAS
1447	D1144	253	**the observation functor** $\mathcal{O}_{\mathrm{MOS}}$: $K_{\mathrm{rel}} = D_\varphi$ a theorem; unital-CP forces the scale; POVM exact; OS evenization *s926 30/30 vs pre-registered s925 15/15*	App. T §10	PRE
1448	D1145	253	the four residuals: AX-MOS *compressed*; $\Pi^{\mathrm{MOS}}_{5\leftarrow 6}$ **rank-3 quotient, provably $\neq$ microscopic**; the one-particle gap; the record process *s927 46/46*	App. T §11	
1449	D1146	253	actuality: history $\Longleftrightarrow$ character $\Longleftrightarrow$ record are **one object**; tail/Poisson REFUTED; X_{actual} **splits** *s929 27/27 vs pre-registered s928 12/12*	App. T §12, L	PRE

Era 5 — the post-Rev27 derivation dive (S255–S257, A1450–A1473)

Twenty-three lane rounds and two house outbounds ran across three sessions of one continuous day. The arc took the rest-mass problem from a typed pipeline to a derived architecture to a closed, priced ledger. **No observable moved in any of the twenty-four rounds, and no quantity was fitted.** Every row below is house-verified: each kernel ran twice, byte-identical, with empty `stderr`, and every lane bundle was re-run in house against its banked hash.

Rounds ran under a temporary regime, the P0-17 DISPENSATION, in which the lane chose its own next object. That regime ended at A1473 and freeze-first resumed. Where a round executed without the gate freeze its item required, the row says so — those are priced variances, not silent ones.

A	D	s	Headline result	Carried by
A1450	D1147	s930	Canonical minimal microscopic dilation $U_{\text{micro}} \in SO(6)$ confirmed exact; a forced three-dimensional dark sector carrying exactly the three nontrivial V_4 characters. Prose-only dispatch; house built the verifier.	App. O
A1451	D1148	s931	The 1+3+3 rank-seven anatomy is REAL and exact. Instrument identity REFUTED-AS-GAUGE. The faithful Klein weld REFUTED; survivor is one global orientation C_2. $\star$ Correction: *two* $Spin(5)$ form factors, not one — supersedes A1450 claim 9.	App. O
A1452	D1149	s932	$\{S, H\} = 0$ DERIVED from the branch-Real pairing of the two mouths: the scalar form factor dies and the τ_2/τ_3 phase is fixed in one stroke, at the price of one zero-dimensional clause.	App. O, App. P sidebar
A1453	D1150	s933	Typing round: *one* lattice with a forced dark/null complement and a separately attached mass operator — not two alternative lattices. Three inequivalent meanings of "null" held apart.	App. O
A1454	D1151	s934	$\ker \Pi = \ker H^2_{\text{tot}}$ refuted as ill-typed; the correct pullback $\ker M^2_{\text{full}} = \ker \Pi \otimes F$ is derived-conditional and exact. The MASS–EXPORT ORDERING FUNCTOR. One named falsifier: $P_1 M^2_{\text{micro}} P_1 \stackrel{?}{=} 0$.	App. O
A1455	D1152	s935	Mass loading precedes annular export — mathematics confirmed, *verdict withheld*: the round introduced its selection criteria in the same bundle that used them. Sent to the PI as a ratification question.	App. O
A1456	D1153	—	The rest-mass attack map: the operator-to-pole pipeline with gates R1–R9. Synthesis round; every formula re-checked against s930–s935.	App. O
A1457	D1154	s936	The graded quadrature survives arbitrary positive noncommuting kinetic dressing after canonical Riesz normalization — house-exact and *generic*, stronger than the lane's witness. Tangent/normal protection of the $\frac{1}{4}$ REFUTED by explicit countermodel. Two residual ratios (r_q, r_ℓ) remain.	App. O, App. K
A1458	D1155	s937	Spurionic $Spin(5)$ covariance does *not* protect the $\frac{1}{2}$ — REFUTED. Replacement: the BULK-LOCAL NORMAL-BLIND COUNTERTERM THEOREM. The Yukawa problem retyped as one democratic scalar.	App. F
A1459	D1156	s938	The smooth-circle normal-blindness theorem (analytic, named tier); hierarchy characters close at the invariant-ring level; the three-way assignment residue compresses to one zero-dimensional clause, AX-LEP-PRIM; the radius modulus conditionally quantizes to odd integers.	App. F, App. P

A	D	s	Headline result	Carried by
A1460	D1157	s940	House-run against the FROZEN s939 gate table. Outcome class O-e: the canonical map is *not* derived, but the two cheapest canonical families are PROVABLY DEAD and the missing structure is exactly named. AX-LEP-PRIM stays NAMED, NOT ADOPTED.	App. S
A1461	D1158	s941	⋆ **The L_{eff} semigroup/OS kill-test fires.** $e^{tL_{\text{eff}}}$ is conservative and self-adjoint but **not positivity-preserving**: the nearest-neighbour tap of $L_{\text{eff}} = L+\varphi^{-2}L^2$ is $1-4\varphi^{-2} = 4\varphi-7 < 0$ in closed golden form, and a bounded generator with a negative off-diagonal cannot generate an entrywise-positive semigroup. The Markov/stochastic dynamics-tier candidacy of L_{eff} DIES. The exact operator law (s912-X7) is untouched.	App. I, App. X kill list
A1462	D1159	s942	Free compactification is hierarchy-blind (it moves only the common scale). The universal-profile gauge threshold has rank two with the testable branch relation $a_X = -2a_T$; the leg-factorized profile split restores full rank three with $a_M := 2a_T + a_X$.	P6
A1463	D1160	s944	First return adjudicated against *pre-frozen* house gates (s943) — outcome O-I. $\mathfrak{L}_{\text{MRSS}}$ is the unique minimal admissible chiral lift, zero continuous parity moduli, house-proved; one branch-Real chiral zero channel plus an NS partner tower at $\lvert p_5\rvert R = \frac{1}{2}$.	App. P
A1464	D1161	s945	$\mathbb{R}^6 \to Q_Y \cong \mathbb{R}^3$ reduction; the BPS kink wall derived; ⋆ the kink *direction* is INDEPENDENT, backed by an explicit countermodel pair. Executed without its gate freeze — variance priced, verdict class least gate-sensitive.	App. P
A1465	D1162	s947	On the pre-frozen s946 gates; outcome O-X. The free mouth-Real determinant is *globally* hierarchy-dead (Jensen); the 11-ray quartic census shows full-hierarchy rays are maxima; ⋆ the conditional primitive-X theorem (norm-80, $\perp$ Y); falsifier $y_d = y_e \iff a_C = a_X$.	App. P, App. X
A1466	D1163	s948	⋆⋆ **The centralizer theorem**: X primitive and Y-orthogonal collapse into *one* theorem — $X = 2(1,1,1,1,1)$ is the unique primitive D_5-lattice vector on the A_4-centralizer line. The $16\oplus 10$ branching is the B-map; $\Omega^2 = (-1)^F$; z_{10} forced, so **the weld is derived at representation tier**. ⋆ The radial-shadow theorem. Gate freeze knowingly waived by the PI, recorded.	App. P, App. S

A	D	s	Headline result	Carried by
A1467	D1164	s950	On the full frozen s949 table — and **the fit trap came back clean**: no measured input anywhere, three inequivalent normalization candidates listed, none selected. The central C_4^F functor is forced and equals the frozen reference; $\det T_X = -\frac{27}{16}A(A-B)^2$ exact; the minimal $X \otimes D_\varphi$ generation lift REFUTED.	App. T
A1468	D1165	s951	A category correction — it closes questions by proving they were the wrong shape. No Fibonacci fiber functor into $\mathrm{Hilb}_{\mathrm{fd}}$ ($n^2 = 1+n$ has no nonnegative integer root); no nonzero $M_2(\mathbb{C}) \to \mathbb{C}$ against the $\mathbb{C}^7$ gauge commutant; the even-rank theorem; the commuting generation rescue DEAD. The pointed Fibonacci operator appears, with golden singular spectrum $\{\varphi, 1, \varphi^{-1}\}$.	App. Q, App. T
A1469	D1166	s952	$\star\star\star$ All three $\mathfrak{F}_{\mathrm{gen}}$ gates pass. $M^2 = M+I-P_m$ is a **theorem** — the unique minimal pointed Fibonacci completion. $L^\top L = P_m$. And $O_\varphi^\top \lvert M \rvert O_\varphi = J_{\mathrm{vac}}$ **exactly**, so the tube and generation walls **merge** at structural tier.	App. Q, App. O
A1470	D1167	s953	Scorecard round, six items: the marked erasure acquires a *parent dynamics* (a canonical recovery semigroup, Kraus-complete for every η) derived-conditionally; GEN-SEPARABILITY REFUTED and retired as a price; $m = 1$ at module tier; the boundary classification is two-branch; the environment weld REFUTED-AS-TYPED; the generation question retyped.	App. O, App. S
A1471	D1169	s954	$\star\star\star$ $[D_F, \mathcal{A}_{\mathrm{gen}}] = 0$ is **forced** under recovery-fixed module naturality: the census cuts $9 \to 3$, spanned by $\hat{D}(A, B, C)$. Strict naturality *forbids* mixing, so CKM/PMNS is a **controlled defect** — the structural home for $V_{\mathrm{CKM}} = I$. $\star\star$ **Doublet–triplet splitting derived** from the unique MRSS Higgs parity. Boundary price corrected to 6/3.	P3, P4, App. F
A1472	D1168	s955	The first house outbound complied with in full. $\star$ **The ν rider is answered: dirac-loaded**, explicit, with charge controls $X(L) + X(H_u) + X(\nu^c) = 0$ versus $X(\nu^c\nu^c) = 10$; the SD16 "Majorana pair" is disambiguated as a parent-layer Real condition ($\mathcal{C} = \tau_1 K$), *not* an SM mass term. C-FORCED tier-qualified; B-RANK3 with $\Delta_2 = 0,\ 4/5,\ 2/5$. $\star$ Parity correction: uniqueness is *conditional* on the loaded scalar spectrum.	P5, App. X

A	D	s	Headline result	Carried by
A1473	D1170	s956	The closeout. ⋆ **Receipt proof**: the lane's recorded dispatch hash equals sha256 of the house's dispatch exactly — cryptographic verbatim receipt, a coalition first, now standard. Four deliverables land: the 23-row dive ledger (banked-hash crosscheck 10/10), the 26-clause priced register, a 26-row supersession self-audit, a 12-row live falsifier list. ⋆ **Head walls banked**: the promotion gate and the boundary law. $\Delta_{\min}$ is the $(1 \leftrightarrow \varphi^{-1})$ link with penalty $\frac{1}{2}\varphi^{-4}\varepsilon^2\|K\|^2$; $6 = 3 + 3$.	App. X, App. O
A1474	D1171	s958	⋆ **The ω round — the last Sargasso item, answered.** First round of the restored freeze-first regime, adjudicated BLIND against a house table sealed before the dispatch was written. Outcome class **C · free overlay**: $L_J = \mathbb{R}e_- \oplus J_{+2}$ is a 28-dimensional **Lagrangian** subspace of the 56, so ω vanishes identically where the mass proxies live — it cannot resolve the golden ladder, generates no Poisson flow there, and does not fix scale. The native quartic gives the same no-go independently. **P0-1 retires**; what survives is kinematic: *the FTS is the symplectic bulk vectorization of the cubic Jordan Lagrangian boundary.* ⋆ House casualty at the sealed table's own Q6 (see §16); ⋆ the lane declined a golden-ratio reading it could have led with. Second consecutive receipt-proof; house reproduced the lane's results hash byte-for-byte.	App. X

What the era cost, honestly. Twenty-three lane rounds produced no new observable and no fitted quantity. What it produced is architecture: a centralizer theorem where there had been a candidate direction, an exact center computation where there had been a weld hypothesis, a uniqueness theorem where there had been a generation guess, and a forced preservation result resting on one named premise. Every amplitude in that architecture remains an open price, and the register in Appendix X lists all twenty-six of them by name. The arc also fired one kill-test against its own leading dynamics candidate (A1461) and refuted three of its own earlier claims (A1451 on A1450, A1472 on A1471, and the A1463-era endpoint shorthand). That is the denominator this appendix exists to make visible.

5 The freeze-first era: A1475–A1500

The P0-17 dispensation ended at A1473. Everything below ran under restored freeze-first discipline: the house sealed its expectations before the dispatch was written, and each row's outcome is scored against that seal rather than against hindsight. Twenty-six rounds are recorded here.

This section was written at S270 and is a catch-up. The notebook had stood at A1474 while the corpus reached A1500 — twenty-six rounds with no entry, including the four that built Plan v2

and the round that adjudicated it. The rows below were drafted from the assessment files themselves, not from recollection; where an assessment carries a retraction or correction header, the row says so rather than repeating the retracted claim.

The *Carried by* column began reading "unpapered" throughout, and that was the honest entry. Every one of these adjudications records *nothing promoted, engine unchanged, no observable moved, 0 tex* — so what the arc produced is architecture, not physics, and the fold records it as such. **The fold landed at S271 in three tranches**: eighteen rows now name a home (A1495 appears twice in the list below because it is homed in two components; it is one row). A1483, A1488, A1492, A1493, A1495 → Appendix E, "Six declared-class negatives on selector laws"; A1484, A1485, A1490, A1491 → Appendix M, the two-lane blind nonexistence and the what-does-carry-an-order remarks; A1496 → Appendix F, "The invariant-connection kill: a holonomy no-go"; A1494, A1495, A1497 → Appendix X, the Rev30 fold ledger (the corrected sign-datum type and the two irreducibly-Input counts); A1499 → Appendix T, the Clifford isotropy law and the fenced braid representation; A1500 → Appendix N, Part N.D, the dual ruling with all four of its statuses printed together; A1479, A1480 → Appendix H, the $\mathcal{D}_J$ clock kill (with the $F^2 = I$ fence in Paper 9); A1481 → Appendix Q, the two golden-boundary negatives; A1482 → Appendix L, the polarization/unitarity obstruction. **Five still read "unpapered", and for those five that is not a backlog but a finding** (the twenty-sixth row, A1475, is a VOID tombstone and carries "n/a", so twenty-five rows ever carried the tag). Four of the five — A1476, A1477, A1478 and A1498 — carry corrections whose targets are session notes and dispatches, *not* printed text in this suite, so folding them here would assert a paper correction that does not exist. A1486 rests on an object whose equivariance was checked arithmetically rather than structurally. **Each of the five is unfolded for a stated reason, and the reasons are recorded rather than the rows being quietly closed.** *Two rows left this list, and the correction is recorded here rather than made silently.* Through Rev32.11 this paragraph read "seven", and gave reasons for A1487 ("its correction likewise targets a dispatch") and A1489 ("no blind corroboration… a single-lane result"). Both had since been folded: A1487 is printed in Appendix X, in the half-R section, which states it appears there "for the first time", and the A1489 order theorems are carried in Appendix X's Rev32 block and in the Appendix M energy-order fence (S280, closed S297). The notebook of record had therefore understated its own fold by two and printed two reasons that were no longer true — found by the S264–S352 master sweep at S353, not by any instrument (`Trackers/LOST_SHEEP_MASTER_SWEEP_S353.md`, row S353-5). **This is the fourth time this notebook has been found behind its own frontier, and the first time the finding was that it was behind in the *closing* direction** — claiming open what was already shut. A reader should take the unhomed rows as a record of *rounds run*, not of results carried into the papers.

One caution the S271 triage produced, recorded because it changes how this table should be read. The claim that this era is *uniformly* unpapered was checked round by round against the tree, and it does not hold in one direction: the largest correction the arc identified — the retirement of the "zero-parameter, forced by algebra, no fit" framing of the charged-lepton mass ratio — had *already* been folded at Rev29 and is printed today in Paper 7, Paper 2, Appendix X, Appendix O and Appendix A, each carrying its re-tier and its provenance. An "unpapered" entry in this column means the round's own architecture has no home yet; it does not mean the round's findings are absent from the suite.

A	D	s	Headline result	Carried by
A1475	—	—	⋆ **VOID — number reserved, never spent.** A tombstone, not a lost assessment. Recorded because a number with no artifact is *voided with a record*, never silently omitted.	*n/a*

A	D	s	Headline result	Carried by
A1476	D1173	—	**Retrospectively filed S266, filing-cliff recovery.** Written at the time, never landed at its canonical path; recovered byte-for-byte from session notes. Nothing re-adjudicated, no tier moved.	*unpapered*
A1477	D1174	—	**Retrospectively filed S266, filing-cliff recovery.** As A1476 — recovery only.	*unpapered*
A1478	D1175	—	**Retrospectively filed S266, filing-cliff recovery.** As A1476 — recovery only.	*unpapered*
A1479	—	s970	PI-initiated unprompted string (the "missing $E = mc^2$" opener). **Not a return against any dispatch**: no D-number, no pre-commit, no receipt protocol — and therefore **not scored as a round**. Logged so the inbound is findable.	App. H
A1480	—	—	HRHP five-shot inbound, no D-number. Receipt verified: bundle sha256 matched the covering text exactly.	App. H, P9
A1481	—	—	⋆ N1–N5 campaign inbound, no D-number. **The programme's first clean sealed prospective loss** — a pre-registered prediction that failed against its own seal, recorded as such.	App. Q
A1482	—	s975, s976	Post-N1–N5 continuation. Adversarial kernel plus a constrained-action real-form control.	App. L
A1483	D1176	s978, s981	⋆ **The compression round; the lane went 5/5 on the frozen bars** — "the strongest single dispatch of the arc: every demand met, two no-go theorems delivered where one was asked".	App. E
A1484	D1177	s983	Madelung-obstruction, Gemini lane. **R5 refuted**: the lane named a framework object carrying an intrinsic partial order on a state set, free of atomic physics — the weight-lattice dominance order. ⋆ Carries a **retraction header**: its s979 character claim ($\chi = (10, 2, 1)$ against appendix S) is **false**; the plan and the papers were right, the kernel compared two different modules.	App. M
A1485	D1178	s984	Madelung-obstruction, Grok lane. **R4 and R5 both refuted — the house put two risky bets on the table and lost both.** ⋆ Same retraction header as A1484.	App. M

A	D	s	Headline result	Carried by
A1486	D1179	s986, s985	Domain-and-injection. **The house won two risky bets, R3 and R4** — its first wins in the arc after six placed and none won across three previous rounds. ⋆ Same retraction header as A1484; ⋆ lane serial collision recorded.	*unpapered*
A1487	D1180	s989, s990	Finish-the-string. **The dispatch asked the lane to find a third house error, and it found a real one**: a commutant is a *subspace*; the house had counted basis vectors.	*unpapered*
A1488	D1181	s993, s994	Second-order / selection. **Three risky bets won in one round (R2, R3, R4)** — the best round of the arc on that measure.	App. E
A1489	D1182	s996	Roadmap round. **The sealed design worked**: two parties ranked five targets independently, neither seeing the other, both hash-committed first — ranks 1 and 2 identical. Lane correctly consumed no serial (descriptive audit).	*unpapered*
A1490	D1183	s999	Blind two-lane round, lane 1 of 2 (Gemini). **Nonexistence proved** within the declared class of canonical categorical invariants, on a two-legged argument.	App. M
A1491	D1184	s1000	Blind two-lane round, lane 2 of 2 (Grok). Nonexistence inside an explicitly declared class — independent of, and convergent with, A1490.	App. M
A1492	D1185	s1002, s1003	**The full-carrier selection law.** ⋆ Fourth subspace-vs-set instance — and the first *committed inside the dispatch that priced it.*	App. E
A1493	D1186	s1005, s1006	**The physical tensor — and the error streak ends: the arc's first clean house dispatch, 5/5 bars.**	App. E
A1494	D1187	s1008, s1009	Grade-moving current. **Second consecutive clean house dispatch.** ⋆ Carries a **same-session correction**: the source-layer typing $D^{\mathrm{sign}}_{\mathrm{phys}} = \mathbb{RP}^{15} \times \mathbb{R}_{\geq 0} \times \mathbb{Z}_2$ is **corrected** one round later at A1495 — the oriented-line double cover of $\mathbb{RP}^{15}$ is S^{15}, which is *connected*, not a $\mathbb{Z}_2$ product.	App. X
A1495	D1188	s1011, s1012	**The transported parent action.** 5/5 bars. The session's title question answered: **the remaining wall is exactly three open rows**, each with a decidable success and kill condition.	App. E, X

A	D	s	Headline result	Carried by
A1496	D1189	s1014, s1015	Rows 1 ‖ 2 of the decidable plan. **Row 2 KILL — the invariant connection class is closed by a holonomy theorem**; the dependency column survives. 5/5 bars. ⋆⋆ Two-host: lane disclosed single-host honestly, house re-ran on a second host, all six outputs byte-identical.	App. F
A1497	D1190	s1017, s1018	Row 3 + the provenance sub-bar. **First-class constraints, positive semigroup, interacting reflection-positive two-point** — and **both provenances close by invariance no-go**, exactly as the seal anticipated. ⋆ House serial collision recorded. ⋆⋆ Two-host, 36/36.	App. X
A1498	D1191	s1020, s1024	The two-sheet audit. Five of six house sentences corrected (R3 won as hoped); the three decidables all land (R4, R5 won); the sealed s1022 answer key unsealed against them. ⋆⋆ Two-host, 32/32.	*unpapered*
A1499	D1192	s1026, s1032	**The synthesis — Plan v2 — the CT/knot lenses.** 5/5 bars; house risky 4W/0L, the second clean sweep. ⋆ The unsealing produced a **genuine braid representation**. ⋆⋆⋆ Two-host, now exact.	App. T
A1500	D1193	s1034	⋆⋆ **Plan-v2 rows 1 ‖ 2 + orphan typing — the dual ruling. Row 1**: the declared frozen-V_{56} orphan class is **killed as a sole parent datum** (orbit 43, required rank 53, residual boundary fibre **10**; Stab = K^0_{sel} dim 20 against the required K^0_{dyn} dim 10) — the orphan is the carrier-level shadow of σ_{sel}, not the missing boundary datum. **Row 2**: conditional finite/regulator subgate **PASS**, **original Plan-v2 row OPEN** — no stable renormalized 4D continuum trajectory. ⋆ The numerically perfect "orphan + L_∂" object is exactly the *forbidden circular control*, and was typed as such rather than banked. 20/20 gates, 20/20 should-fire controls; ⋆⋆ two-host across two Pythons and two NUMPY majors.	App. N

What this era has produced, honestly. Twenty-six rounds, no new observable, no fitted quantity, no tier moved. What they produced is a decidable plan with named kill conditions, and then two of its rows actually decided — one killed (A1496), one killed-as-a-class with the residual fibre counted exactly (A1500) — plus a Row-2 finite subgate that passed while *the row it belongs to was held open.* The house lost risky bets in three consecutive rounds before winning any (A1484–A1486), found and corrected its own errors twice within a session (A1487, A1494), and on the last round declined a numerically perfect result because it was the forbidden circular control. That denominator is the point of this appendix.

6 Era 6: A1501–A1572 (S273–S302; added Rev32.5; A1563–A1572 added Rev32.6)

Sixty-two rounds ran between the close of the freeze-first era (A1500, S270) and the Rev32.4 seal, and none had a row here until Rev32.5: the notebook of record had fallen sixty-two rounds behind its own frontier, a gap the house found in its own hostile pass at the Rev32.4 cut (Rev32.5 changelog, H1). The rows below are generated from the filed adjudications (`Coalition/assessments/A1501-A1572`); each carries the round's dispatch number, the house kernels it cites, the adjudication headline with its outcome word, and the component that carries the content — or "—" where nothing carries it yet. Four arcs are recorded: the Rev30 cold review and the parent-action closure that became Rev31 (A1501–A1516); the Rev31.2 and Rev32 cold rounds and the Rev32 hardening arc (A1517–A1538); the Track-2 synchronization and the nuclear/mixing rulings arc that became Rev32.2–32.3 (A1539–A1552); and the carrier arc that Appendix T now prints as a fold (A1553–A1562). Every "⋆⋆" is the house grade at intake, not a promotion of anything printed.

A	D	s	Headline result	Carried by
A1501	D1194	s1036, s1037, s1038	ADJUDICATION of the Gemini return on D1194	App. X §x-bstar (scope round)
A1502	D1195	s1036, s1037, s1039	ADJUDICATION of the Grok return on D1195	App. X §x-bstar (scope round)
A1503	—	—	GEMINI · Rev30 COLD REVIEW RETURN · house verification	Rev30 changelog (ledgers)
A1504	—	—	GROK · Rev30 COLD REVIEW RETURN · house verification	Rev30 changelog (ledgers)
A1505	—	—	CHATGPT · Rev30 COLD REVIEW RETURN (PDF-ONLY PASS) · house verification	Rev30 changelog; Engine (S274 re-stamp)
A1506	D1196	s1014, s1041, s1042	CHATGPT · THE PARENT-ACTION ASSEMBLY RETURN · house verification	App. S / App. R (parent-action closure, Rev31)
A1507	D1197	s1014, s1017, s1043	HOUSE ASSESSMENT OF THE D1197 RETURN (ChatGPT lane): THE PURE-SPINOR	App. S (pure-spinor leg)
A1508	D1196	s1041, s1046	HOUSE ASSESSMENT OF THE D1196 NON-BLIND EXHAUSTIVE RETURN (ChatGPT lane).	App. S / App. R
A1509	D1198	s1044, s1045	HOUSE ASSESSMENT OF THE D1198 CLOSURE RETURN (ChatGPT lane). S275	App. S (10+1 closure)
A1510	D1199	s1042, s1045	HOUSE ASSESSMENT: GEMINI ANALYTIC REFEREE RETURN on the parent-action	App. S (analytic referee)

A	D	s	Headline result	Carried by
A1511	D1199	s1042, s1045	HOUSE ASSESSMENT: GROK ANALYTIC REFEREE RETURN on the parent-action	App. S (analytic referee)
A1512	D1200	s1017, s1042, s1047	HOUSE ASSESSMENT OF THE D1200 RETURN (ChatGPT lane): clock identity /	App. H ($\sqrt{6}$ clock identity, golden bridge)
A1513	D1201	s1048, s1049	HOUSE ASSESSMENT OF THE D1201 RETURN (ChatGPT lane): the golden-carrier	App. H / App. X (golden carrier)
A1514	D1202	s1051	HOUSE ASSESSMENT OF THE D1202 RETURN (ChatGPT lane): the 26⊕10	App. S (26⊕10 grammar-forcing)
A1515	D1203	s1052	HOUSE ASSESSMENT OF THE D1203 RETURN (ChatGPT lane): the character-	App. S (rank-character variational theorem)
A1516	D1204	s1054	HOUSE ASSESSMENT OF THE D1204 RETURN (ChatGPT lane): the selection	App. S (selection; F-SEL menu)
A1517	—	—	GEMINI · Rev31.2 COLD REVIEW RETURN · house verification	Rev31.2 changelog
A1518	—	—	GROK · Rev31.2 COLD REVIEW RETURN · house verification	Rev31.2 changelog
A1519	—	—	CHATGPT · Rev31.2 COLD REVIEW RETURN · house verification	Rev31.2 changelog
A1520	D1206	s1042	ChatGPT D1206 RETURN, COMPLETED VIA THE D1208 COLLECTION ROUND —	Track 2 (not this suite)
A1521	D1207	s1042, s1056	ChatGPT return to D1207 (HAMILTONIAN LOOSE-END SYNTHESIS) — INTAKE ASSESSMENT	App. S (Hamiltonian loose ends)
A1522	D1209	s1005, s1052, s1053	House assessment of the D1209 RETURN (problem-board adjudication ×	boards (Trackers), not print
A1523	D1210	s1063, s1064, s1066	House assessment of the D1210 RETURN (hostile carrier pass ×	App. S (carrier grammar, hostile pass)
A1524	D1211	s1068, s1069, s1070	House assessment of the D1211 RETURN (freeze-prep hostile pass ×	Rev32 freeze prep (ledgers)
A1525	D1212	s1071, s1072	INTAKE OF THE D1212 RETURN (ChatGPT lane, "THE EXPORT ROUND")	App. S (export schema; Kernels)
A1526	D1213	s1073, s1074	INTAKE OF THE D1213 RETURN (ChatGPT lane, "THE REPAIR ROUND")	App. S (repair round)

A	D	s	Headline result	Carried by
A1527	D1214	s1073	INTAKE OF THE D1214 RETURN ("THE DECLARATION-PREP ROUND")	App. S (declaration prep)
A1528	D1215	s1072, s1073, s1074	INTAKE OF THE D1215 RETURN ("THE HAMILTONIAN LEDGER ROUND")	App. S (Hamiltonian ledger)
A1529	D1216	s1059, s1074, s1076	INTAKE OF THE D1216 RETURN ("THE SIX WALLS ROUND")	App. S (six walls)
A1530	D1217	s1075, s1077	INTAKE OF THE D1217 RETURN ("THE PRE-DECLARATION HARDENING +	App. S (hardening)
A1531	D1218	s1058, s1079	INTAKE OF THE D1218 RETURN ("THE CATCH-UP + NEXT-RUNG ROUND")	App. S / P3 (catch-up, next rung)
A1532	D1219	s1058	INTAKE OF THE D1219 RETURN ("SM-1 TYPING + FRAMED-WINDING")	App. S (SM-1 typing; framed winding)
A1533	D1220	s1052, s1053, s1054	intake of the D1220 return (ChatGPT lane): F-SEL(c), artifact recovery, refined-family census	App. S / P3 (F-SEL(c) closed negative, Rev32)
A1534	—	—	GROK REV32 COLD RETURN — INTAKE VERIFICATION + ADJUDICATION	Rev32.1 changelog
A1535	—	—	GEMINI REV32 COLD RETURN — INTAKE VERIFICATION + ADJUDICATION	Rev32.1 changelog
A1536	D1221	s1085	GROK D1221 BRAINSTORM RETURN — INTAKE VET + ROUTING RECOMMENDATIONS	App. E (s1085/s1086 builds)
A1537	D1221	—	GEMINI D1221 BRAINSTORM RETURN — INTAKE VET + ROUTING RECOMMENDATIONS	App. E (s1085/s1086 builds)
A1538	—	—	CHATGPT REV32 COLD RETURN — INTAKE VERIFICATION + ADJUDICATION + ROUND CLOSE	Rev32.1 changelog
A1539	D1222	s1042, s1085, s1086	ChatGPT D1222 RETURN — TRACK-2 v1.2 / REV32.1 SYNCHRONIZATION —	Track 2 v1.2 (not this suite)
A1540	D1186	s1085, s1086	Independent hostile review — SGTOE Track-2 v1.2	Track 2 (not this suite)
A1541	—	s1086	ChatGPT TRACK-2 v1.2.1 FIX-CAPTURE POINT RELEASE — ⋆⋆ PASS	Track 2 v1.2.1 (not this suite)
A1542	D1223	s1091, s1092, s1093	ChatGPT D1223 PERIODIC-TABLE EXPANSION RETURN — ⋆⋆ PASS	P9 §typed (^{28}Si retype, Rev32.3)

A	D	s	Headline result	Carried by
A1543	D1224	s1091, s1096, s1098	ChatGPT D1224 P3 BUILD ROUND RETURN — ⋆⋆ PASS	P9 (annular order defined)
A1544	D1225	s1090, s1091, s1096	ChatGPT D1225 PT PUSH RETURN — ⋆⋆ PASS (THREE SCOPED NEGATIVES + ONE ERRATUM CAUGHT)	P9 §typed (Rev32.3)
A1545	D1226	s1101, s1102, s1103	ChatGPT D1226 THREE MAPS RETURN — ⋆⋆ PASS (FOUR SCOPED NO-GOS, EACH WITH ITS OBSTRUCTION NAMED;	P9 §typed (annular closure)
A1546	D1227	s1102, s1104, s1110	ChatGPT D1227 HOUSE PROGRESS REVIEW RETURN — ⋆⋆ PASS (ALL HOUSE ARITHMETIC REPRODUCED 7/7 + 5/5;	P3 / P5 (S293 rulings, Rev32.2)
A1547	D1228	s1088, s1090, s1100	ChatGPT D1228 CLOCK-LENGTH MAP RETURN — ⋆⋆ PASS (SCOPED NO-GO: THE COUNT MAP IS NOT A MAP ON THE	— (count map: control, nothing prints)
A1548	D1229	s1079, s1087, s1089	ChatGPT D1229 OBSERVATION FUNCTOR RETURN — ⋆⋆ PASS (SCOPED NO-GO, DECIDED ON THE REGISTERED TABLE:	App. H remarks (Rev32.3)
A1549	D1230	s1079, s1133, s1134	House adjudication of the ChatGPT D1230 return (TWO WALLS: the replication map · the weak angle) — ⋆⋆ PASS — four frozen exits, all honest, all reproduced; the lane found the same AX6 label collision the house found in parallel (s1135) and ADDED an independe	P4 AX6 / P0 moiré (Rev32.2–.3)
A1550	D1231	s1079, s1133, s1140	House adjudication of the ChatGPT D1231 return (THE NONCENTRAL DEFECT under the two-lattice lens) — ⋆⋆ PASS — the round's headline is a HOUSE CORRECTION, delivered with three exact counterexamples the house has now reproduced to the digit: the "Peirce catego	P4 $AX6_{pol}$ (Rev32.2)
A1551	D1232	s1079, s1142, s1145	House adjudication of the ChatGPT D1232 return (THE ORIENTED ENVELOPE: the noncentral defect posed in the 56) — ⋆⋆ PASS — the cleanest split verdict of the arc: the certified $56/e_7$ envelope GENUINELY REPAIRS two of D1231's three interface obstructions (nativ	P0 moiré remark Tr_{56} (Rev32.3)
A1552	D1233	s1140, s1145	House adjudication of the ChatGPT D1233 return (THE ONE OBJECT: the route-ledger round) — ⋆⋆ PASS — the round asked for a survey and got an architecture. The lane returned nine fully-typed routes AND the round's principal conceptual result: the six faces of	— (route ledger; App. T pointer)

A	D	s	Headline result	Carried by
A1553	D1234	s1146, s1147	House adjudication of the ChatGPT D1234 return (THE STAGED SOLDER: build R1) — ⋆⋆ PASS — THE STAGED CELL IS FINISHED. After two hundred sessions marked STAGED, the Albert-to-shell solder exists: a rank-27, determinant-4096, all-entries-±1 basis map B: $J_3(\mathbb{O}$_s	App. T §carrier arc (Rev32.2)
A1554	D1235	s1088, s1131	ADJUDICATION of the D1235 return (BOX-TOP UPDATE + PRE-REWRITE TRIAGE) — ⋆⋆ PASS	Rev33 checklist (ledgers); App. T
A1555	D1236	—	ADJUDICATION of the D1236 return (FACE LEDGER + DOOR CENSUS) — ⋆⋆ PASS	App. T six-face ledger
A1556	D1237	—	ADJUDICATION of the D1237 return (ℓ_odd BUILD-OR-OBSTRUCT + FACE PROBES) — ⋆⋆ PASS	App. T (odd covector no-go)
A1557	D1238	s1146	ADJUDICATION of the D1238 return (COVARIANT LEG + FACES) — ⋆⋆⋆ PASS (the round built its object)	App. T (covariant leg)
A1558	D1239	s1148	ADJUDICATION of the D1239 return (THE MOS→G_6 CHANNEL) — ⋆⋆ PASS (the channel exists; the premise does not follow)	App. T (MOS channel)
A1559	D1240	s1149, s1150	ADJUDICATION of the D1240 return (THE SOLDER Ξ + THE FORCING TABLE) — ⋆⋆ PASS (the solder exists; nothing	App. T (solder Ξ; forcing table)
A1560	D1241	s1149, s1150, s1151	ADJUDICATION of the D1241 return (THE MICROSCOPIC LEG) — ⋆⋆ PASS (obstructed at the record-support tier with	App. T (microscopic leg; premise boxes)
A1561	D1242	s1152, s1153	ADJUDICATION of the D1242 return (FACE 2, THE BASEPOINT) — ⋆⋆ PASS (the intrinsic branch is EMPTY over the	App. T (face 2 basepoint)
A1562	D1243	s1154, s1155	ADJUDICATION of the D1243 return (REV33 MECHANICAL PATCH LEDGER + APPENDIX-T CARRIER-ARC FOLD DRAFT) — ⋆⋆ PASS	Rev32.2 patch ledger (ledgers); App. T fold
Rev32.6 addition (S300–S302): the Rev32.4 cold round, the θ_{23} octant, the ladder arc, Track-2 v1.3				
A1563	cold round	—	Rev32.4 COLD HOSTILE REVIEW (ChatGPT, full scope; 8/8 verified, 6 confirmed-undisclosed incl. the θ_{23} "Fixed by" cell vs Paper 3 §5) — ⋆⋆ PASS 72	Rev32.5 fold (S300.10–.27)
A1564	cold round	—	Rev32.4 COLD HOSTILE REVIEW (Gemini, 19 pp; G1 y_t/W01 cross-component leak NEW) — ⋆ PASS-PARTIAL 88	Rev32.5 fold
A1565	cold round	—	Rev32.4 COLD HOSTILE REVIEW (Grok; D4 "99/100 with AX6" + D9 "20 predictions" NEW) — ⋆⋆ PASS 84; round verdict C5	Rev32.5 fold

A	D	s	Headline result	Carried by
A1566	D1244	s1156, s1157 (lane); s501 retyped	THE θ_{23} OCTANT: WHO FIXES IT? — ROUTE (a): `s501` is a candidate map, not a selector; two equivariant registrations; `s1107` quantifier reversal + $9 \notin 1..8$ menu — $\star\star$ PASS 86	P3 §re-tier (physical θ_{23}, $\lvert V_{cb}\rvert$ LOADED-CORR.; tally 2/8); App. N Part N.E
A1567	D1246	s1158 (lane)	COMPLEX-RATIO LADDER: (b) ABSENT, TYPED — 29×39 census, 0 phase-bearing ratios; $\Omega = \phi - \frac{1}{2}$ is the golden polynomial on the $(1,3)$ eigenvalue; missing object $Q_L = S_L e^{\alpha J_L}$ — $\star\star$ PASS 88	App. N Part N.E; App. I (one number three ways); P0 (three-layer picture)
A1568	D1248	s1159 v2 (house)	THE LOSS CONDITION AS A CENSUS: $C(L_{\text{vac}}) = \mathfrak{so}(4,4) \oplus \mathbb{R}^2$ (30), $C(H) = \mathfrak{e}_{6(6)} \oplus \mathfrak{so}(1,1)$ (79), $K \notin C(H)$, loss condition MET; lane check found App. G's compact Z_2 — $\star\star$	App. N Part N.E (fence, lane-amended text)
A1569	D1249	—	TRACK-2 v1.3 on the Rev32.5 basis (ChatGPT; two-turn PLAN→GO→BUILD; 270 files; ledger sidecar SL-01–SL-30 with six minus-exponent fields corrected) — $\star\star$ PASS 90 at intake	SOURCE_LEDGER ids + re-signed fields (App. X)
A1570	D1250	—	MACHINERY AUDIT (Gemini, text drop) — $\star$ PASS-PARTIAL 78	App. X engine/kernel sentence
A1571	D1250	—	MACHINERY AUDIT (Grok): workbook provenance stamps still v5.0.6/Rev24; README_s1159 quotes the v1 sha; 22 of 330 cited serials shipped, undisclosed — $\star\star$ PASS-PARTIAL 82	Engine re-stamp (S303); App. X
A1572	D1249	s1162 (house)	HOUSE HOSTILE REVIEW of Track-2 v1.3 + the author lane's self-verification: F1 the "exact" $\lvert V_{us}\rvert$ 0.225247 was a house hand calculation (closed form 0.225256); F2 stale `s1104` certificate; F3 presence-only resolvers; F4 27/27 vs 26/26 — PASS WITH REQUIRED ERRATA, 84	P3/P6/App. B ($\lvert V_{us}\rvert$ from `s1162`); P5/P2/App. X (`s1162` certificate); App. X resolver clause

7 Era 7: A1573–A1708 (S303–S340; added Rev32.9)

The notebook's last fold before this one was at the Rev32.6 seal, and the whole Registrar–Premises–Textbook–Graph build ran after it: the Library v2 cold rounds, the Registrar parse and read rounds, the premise rounds and the paper hostile reviews that fed the Rev32.8 and Rev32.9 queues. None

of it had a row here (REV32.9 row S313.LB). The rows below are *generated*, never typed, by `scripts/appy_era7_rows_S341.py` from the filed adjudications under `Coalition/assessments/`: the headline is each file's first heading, the dispatch is its first D-number, and the kernels are its first three distinct kernel serials. The **Carried by** column is a keyword classification of the file, stated here as such: "Registrar" means the round's content lives in the Registrar and premise records (their pages carry the reads), not in this suite; "REV32.8/32.9 queue" means a paper hostile review whose confirmed findings were queued as paper fixes and applied at Rev32.8 or Rev32.9; "—" means the generator traced no carrier, not that none exists. Five serials have no adjudication file of their own and are printed as such (§14). Every "⋆" is the house grade at intake, not a promotion of anything printed.

A	D	s	Headline result	Carried by
A1573	D1251	s1162, s1105	ChatGPT REV32.6 COLD RETURN: house adjudication (S303, 2026-09-05)	Rev32.7 changelog (ledgers)
A1574	—	s502, s1107	Gemini REV32.6 COLD RETURN: house adjudication (S303, 2026-09-05)	Rev32.7 changelog (ledgers)
A1575	—	s1162, s1104	Grok REV32.6 COLD RETURN: house adjudication (S303, 2026-09-05)	Rev32.7 changelog (ledgers)
A1576	D1252	s1162	ChatGPT TRACK-2 PUBLICATION JUDGMENT + INDEPENDENT CHECKS: house adjudication (S303, 2026-09-05)	Track 2 (not this suite)
A1577	D1251	s1162, s1104	ChatGPT return to D1251 v2 (Library v2 cold LIST round). House intake S304 2026-09-05 · claude-fable-5-1…	Registrar / Library v2 (not print)
A1578	D1251	s1162	Gemini return to D1251 v2 (Library v2 cold LIST round). House intake S304 2026-09-05 · claude-fable-5-1…	Registrar / Library v2 (not print)
A1579	D1251	—	Grok return to D1251 v2 (Library v2 cold LIST round). House intake S304 2026-09-05 · claude-fable-5-1…	Registrar / Library v2 (not print)
A1580	D1252	s1104, s1162	ChatGPT return to D1252 (Library redesign consultation). House intake S304 2026-09-05 · claude-fable-5-1…	Registrar / Library v2 (not print)
A1581	—	—	*reserved; no adjudication file under* `Coalition/assessments` *at S341*	—
A1582	—	—	*reserved; no adjudication file under* `Coalition/assessments` *at S341*	—
A1583	D1255	s588	Grok return to D1255 (Registrar / Library v2 hostile review THROUGH THE LIVE SITE; hubs H06+H07+H08+H10, 150…	REV32.8/32.9 queue (paper fixes)
A1584	D1253	—	HOUSE sample review of the D1253 Library-v2 generation round (ChatGPT, 24/24 parts). Filed S309 2026-09-06 ·…	Registrar / Library v2 (not print)
A1585	D1256	s1163	CHATGPT D1256 PROOF-LIBRARY REVIEW / CHAPTER MAP / PILOT — HOUSE ADJUDICATION	Registrar / Library v2 (not print)

A	D	s	Headline result	Carried by
A1586	D1257	—	# ⋆⋆ SECOND CORRECTION — appended S310 (later the same session, after the lane sent site ideas)	Track 2 (not this suite)
A1587	D1258	s1164, s253, s255	# ⋆⋆ CORRECTION — §6 CLOSED, appended S310 (later the same session; the PI dropped the files)	Registrar / premises (not print)
A1588	D1259	s1165	CHATGPT D1259: PRIOR-ART PASS + FIVE ALGEBRA-SIDE ARTICLES — HOUSE ADJUDICATION	Registrar (library v2), not print
A1589	D1260	—	*partial intake only* (`_TODAY/notes/S311/A1589_INTAKE_S311.md`); no adjudication file	S311 rows of the Rev32.9 queue
A1590	D1261	—	D1261 DESIGN REVIEW (ChatGPT). VERDICT ⋆⋆ PASS, SCORE 95.	—
A1591	D1262	—	INTAKE OF D1262 (ChatGPT): the tendon round	Registrar (library v2), not print
A1592	D1263	—	INTAKE OF D1263 (GROK): the R37 second-party round	Registrar (library v2), not print
A1593	D1264	—	INTAKE OF D1264 (Grok): the cold live-site review	Track 2 (not this suite)
A1594	D1265	—	INTAKE OF D1265 (ChatGPT): FETCH BLOCKED, REVIEW NOT EXECUTED	Registrar / premises (not print)
A1595	D1266	—	D1266 SYNC ROUND (ChatGPT). Intaken S315, 2026-09-08.	Track 2 (not this suite)
A1596	D1263	—	GROK, UNSOLICITED CONCURRENT LIVE READ OF THE S315 DEPLOY. Intaken S315, 2026-09-08.	Registrar (library v2), not print
A1597	—	—	GROK, THIRD UNSOLICITED LIVE READ (post-S315e deploy). Intaken S315, 2026-09-08.	Registrar (library v2), not print
A1598	D1267	s304	D1267 COMPREHENSIVE LIVE READ (Grok). Intaken S315, 2026-09-08.	Registrar (library v2), not print
A1599	D1267	—	GROK UNSOLICITED LIVE READ OF THE S315i DEPLOY	Registrar (library v2), not print
A1600	D1267	—	GROK LIVE READ OF THE S316d DEPLOY (unsolicited, self-typed CONCURRENT)	Registrar (library v2), not print
A1601	D1269	—	GROK, D1269 ADVERSARIAL RE-CHECK OF THE S316 SITE UPDATE	website (not this suite)
A1602	D1268	—	CHATGPT, D1268: THE REGISTRAR PREMISES TEXTBOOK LINKAGE AS A CATEGORY	Registrar / Library v2 (not print)

A	D	s	Headline result	Carried by
A1603	D1270	—	CHATGPT, D1270: THE v2.1 CONTRACT DERIVATION, AND THE FILE THAT UNBLOCKED THE FOUNDING ARTICLE	Track 2 (not this suite)
A1604	D1262	—	GROK, R45 ADOPTION: THE RULING ACCEPTED, THEN ATTACKED AT THE ONE PLACE IT CAN STILL BETRAY THE ARITHMETIC	Track 2 (not this suite)
A1605	D1270	—	CHATGPT, R46 REGENERATION AND COMPLETE PACKET RECOVERY	—
A1606	D1271	—	GROK RETURN TO D1271 (pre-flight access probe, S318a)	Registrar / premises (not print)
A1607	D1271	—	CHATGPT RETURN TO D1271 (pre-flight access probe, S318a)	Registrar / premises (not print)
A1608	D1271	—	⋆ TOMBSTONED VOID. PI RULING, S318 (2026-09-09).	—
A1609	D1272	—	GROK RETURN TO D1272 (R49d tier-1 calibration, 13 records / 17 uses)	Registrar (library v2), not print
A1610	D1272	—	CHATGPT RETURN TO D1272 (R49d tier-1 calibration, 13 records / 17 uses)	Registrar (library v2), not print
A1611	D1272	—	⋆ TOMBSTONED VOID. PI RULING, S319 (2026-09-09).	—
A1612	D1272	—	THE HOUSE'S OWN RETURN TO D1272 (R49d tier-1 calibration, 13 records / 17 uses)	Registrar (library v2), not print
A1613	D1273	—	GROK RETURN TO D1273 (content + access review of live cut S319a)	website (not this suite)
A1614	D1273	—	CHATGPT RETURN TO D1273 (content + access review of live cut S319a)	website (not this suite)
A1615	D1274	—	GROK LANE, D1274 BATCH 1 of 14. 24 RECORDS · 60 SOURCE USES. FILED S320, 2026-09-09.	Registrar (library v2), not print
A1616	D1274	—	CHATGPT LANE, D1274 BATCH 1 of 14. 24 RECORDS · 60 SOURCE USES. FILED S320, 2026-09-09.	Registrar (library v2), not print
A1617	D1274	—	⋆ TOMBSTONED VOID. FILED S321 (2026-09-09).	—
A1618	D1275	—	⋆ TOMBSTONED VOID. PI RULING, S321 (2026-09-09).	—
A1619	D1275	—	CHATGPT LANE, D1275 BATCH 1 of 14. 24 RECORDS · 60 SOURCE USES. FILED S321, 2026-09-09.	Registrar (library v2), not print

A	D	s	Headline result	Carried by
A1620	D1277	—	GROK LANE, D1277 BATCH 1. 24 RECORDS · 60 SOURCE USES. FILED S321, 2026-09-09.	Registrar (library v2), not print
A1621	D1277	—	CHATGPT LANE, D1277 BATCH 1. 24 RECORDS · 60 SOURCE USES. FILED S321, 2026-09-09.	Registrar (library v2), not print
A1622	D1278	—	CHATGPT LANE, D1278 (HOSTILE REVIEW OF THE SIMPLIFICATION STRATEGY). FILED S323.	REV32.8/32.9 queue (paper fixes)
A1623	D1280	—	GROK LANE, D1280 (R58 SEEDED ROUND). ⋆⋆ INTAKE OPEN — 14 OF 60 RECEIVED.	Registrar (library v2), not print
A1624	D1280	—	CHATGPT LANE, D1280 (R58 SEEDED ROUND). ⋆⋆ 60 OF 60. FILED S323, 2026-09-09.	Registrar (library v2), not print
A1625	D1279	—	GEMINI LANE, D1279 (WHOLE-ESTATE STRUCTURAL READ). FILED S323, 2026-09-09.	Registrar / premises (not print)
A1626	D1274	—	GROK, D1280c. THE 20 OUTSTANDING USES. RETURNED AND VERIFIED, S324 (2026-09-10).	Registrar (library v2), not print
A1627	D1281	—	GROK, D1281. THE S324a CUT, READ LIVE. Filed S324, 2026-09-10.	Registrar (library v2), not print
A1628	D1282	—	GROK on D1282: S325c host verification (W.6). Assessed S327 (2026-09-10).	—
A1629	D1283	—	*tombstone*: D1283 was composed and never sent (stale once cut S327c existed)	—
A1630	D1284	—	GROK on D1284: paired read round 1, the Grok half (R67 §3). Assessed S327 (2026-09-10).	Registrar (library v2), not print
A1631	D1285	s1047, s1048	GEMINI on D1285: paired read round 1, the Gemini half (R67 §3). Assessed S327 (2026-09-10).	Registrar (library v2), not print
A1632	D1286	s1051	CHATGPT on D1286: hostile review of read round 1 (NO READ CREDIT — drafting lane). Assessed S327 (2026-09-10).	REV32.8/32.9 queue (paper fixes)
A1633	D1287	s1162	House adjudication of the ChatGPT D1287 return (THE 22 SCORECARD NOTES) — S327, 2026-09-11	Registrar (library v2), not print
A1634	D1288	—	House adjudication of the ChatGPT D1288 return (THE 24 PREMISE NOTES) — S327, 2026-09-11	Registrar (library v2), not print
A1635	D1289	—	GROK on D1289: paired read round 2, the Grok half (R67 §3). Assessed S327 (2026-09-11).	Registrar (library v2), not print

A	D	s	Headline result	Carried by
A1636	D1290	—	GEMINI on D1290: paired read round 2, the Gemini half (R67 §3). Assessed S327 (2026-09-11).	Registrar (library v2), not print
A1637	D1291	—	CHATGPT on D1291: hostile review of round 2 (25 records, 58 uses). Adjudicated by the house, S327 (2026-09-11).	REV32.8/32.9 queue (paper fixes)
A1638	D1292	—	GROK on D1292: the follow-up read of the two records the house never sent (LIB2-001, LIB2-028). Assessed S327…	Registrar (library v2), not print
A1639	D1293	—	GROK on D1293: paired read round 3, the Grok half (R67 §3). Assessed S328 (2026-09-11).	Registrar (library v2), not print
A1640	D1294	—	GEMINI on D1294: paired read round 3, the Gemini half (R67 §3). Assessed S328 (2026-09-11).	—
A1641	D1295	s740, s741	CHATGPT on D1295: hostile review of round 3 (LIB2-053…072), review only, no read credit (R67 §3)	REV32.8/32.9 queue (paper fixes)
A1642	D1271	s740, s743	GROK, PI-SOLICITED: SITE DESCRIPTION · HOSTILE · POSITIVE · INSIGHTS/RECOMMENDATIONS ·…	Track 2 (not this suite)
A1643	D1296	—	GROK: REPLY TO D1296 (the house's answer to A1642) · plus the PI's "finish a thought?" exchange	Registrar / premises (not print)
A1644	D1297	—	GROK: REPLY TO D1297 (the H206 correction + the KK answer). Assessed S328 (2026-09-11).	—
A1645	D1298	—	GROK · D1298 · PAIRED READ ROUND 4 (R67 §3) · filed and banked S329 (2026-09-11)	Registrar (library v2), not print
A1646	D1299	—	GEMINI · D1299 · PAIRED READ ROUND 4 (R67 §3) · filed and banked S329 (2026-09-11)	Registrar (library v2), not print
A1647	D1300	s1152, s261	CHATGPT · D1300 · HOSTILE REVIEW OF READ ROUND 4 · filed S329 (2026-09-11). NO READ CREDIT…	REV32.8/32.9 queue (paper fixes)
A1648	—	s1169, s1126	HOUSE ASSESSMENT · s1169 · a bounded repair attempt on Paper 3 §2.2 "$\beta = \pi/8$" (A1641 F02…	Registrar / premises (not print)
A1649	—	s1170, s1166, s1152	HOUSE ASSESSMENT · s1170 · LIB2-305 W1 census: a registered u d energy under the e flip?	Registrar (library v2), not print
A1650	D1301	—	GROK, D1301, PAIRED READ ROUND 5 (batch 5). Filed S330, 2026-09-12.	Registrar (library v2), not print
A1651	D1302	—	GEMINI, D1302, PAIRED READ ROUND 5 (batch 5). Filed S330, 2026-09-12.	Registrar (library v2), not print

A	D	s	Headline result	Carried by
A1652	D1303	s1171, s243	CHATGPT, D1303, HOSTILE REVIEW OF READ ROUND 5 (batch 5). Filed S330, 2026-09-12.	REV32.8/32.9 queue (paper fixes)
A1653	D1304	s1172, s239, s240	GROK, D1304, PROCESS CONVERGENCE ROUND on the A1652-F01 handling. Filed S330, 2026-09-12.	Registrar (library v2), not print
A1654	D1305	—	GEMINI, D1305, PROCESS CONVERGENCE ROUND on the A1652-F01 handling. Filed S330, 2026-09-12.	Registrar (library v2), not print
A1655	D1306	s1173, s243	CHATGPT, D1306, PROCESS CONVERGENCE ROUND. Filed S330, 2026-09-12. NO READ CREDIT.	—
A1656	D1307	—	GROK, D1307: WINDOW CHARACTERIZATION. House intake S332, 2026-09-12 (claude-opus-5, Cowork).	—
A1657	D1308	—	GROK, D1308: W6 FINISHED, AND THE REMEDY TESTED ON ITSELF. House intake S332, 2026-09-12 (claude-opus-5, Cowork).	—
A1658	D1309	—	# ⋆ READ FIRST — PARTIALLY SUPERSEDED BY A1659 (S334, same day).	Registrar (library v2), not print
A1659	D1309	s1162	# ⋆ READ FIRST — EXTENDED BY A1660 (S334, same day). Grok's part 4 added 2 more disagreements and they	Registrar (library v2), not print
A1660	D1309	—	GROK, answering D1309 PART 4. House assessment, S334, 2026-09-13.	Registrar (library v2), not print
A1661	D1309	—	GROK, answering D1309 PART 3. House assessment, S334, 2026-09-13.	Registrar (library v2), not print
A1662	D1309	—	GROK, answering D1309 PART 2. House assessment, S334, 2026-09-13.	Registrar (library v2), not print
A1663	D1309	—	GROK, D1309 PART 1. THE PAIR IS COMPLETE. House assessment, S334, 2026-09-13.	Registrar (library v2), not print
A1664	D1312	—	(GEMINI / D1312) + A1665 (GROK / D1311) — WHAT THE LANES ACTUALLY READ. S334, 2026-09-13.	Registrar (library v2), not print
A1665	D1311	—	GROK read-object return, filed jointly with A1664 (`A1664_A1665_READ_OBJECT_S334.md`)	Registrar (library v2), not print
A1666	D1313	—	GROK · D1313 · READ ROUND 6 (R67 §3, R112). Filed S337, 2026-09-15.	Registrar (library v2), not print

A	D	s	Headline result	Carried by
A1667	D1314	—	GEMINI · D1314 · READ ROUND 6 (R67 §3, R112). Filed S337, 2026-09-15.	Registrar (library v2), not print
A1668	D1315	—	CHATGPT · D1315 · HOSTILE REVIEW OF READ ROUND 6. Filed S337, 2026-09-15. NO READ CREDIT (drafting…	REV32.8/32.9 queue (paper fixes)
A1669	D1316	—	GROK · D1316 · READ ROUND 7 (R67 §3, R112, R113). Filed S337, 2026-09-15.	Registrar (library v2), not print
A1670	D1317	—	GEMINI · D1317 · READ ROUND 7 (R67 §3, R112, R113). Filed S337, 2026-09-15.	Registrar (library v2), not print
A1671	D1318	—	CHATGPT · D1318 · HOSTILE REVIEW OF READ ROUND 7. Filed S337, 2026-09-15. NO READ CREDIT (drafting…	REV32.8/32.9 queue (paper fixes)
A1672	D1319	—	GROK · D1319 · READ ROUND 8 (R67 §3, R112, R113). Filed S338, 2026-09-15.	Registrar (library v2), not print
A1673	D1320	—	GEMINI · D1320 · READ ROUND 8 (R67 §3, R112, R113). Filed S338, 2026-09-15.	Registrar (library v2), not print
A1674	D1321	—	CHATGPT · D1321 · DRAFTER REVIEW, ROUND 8. Filed S338, 2026-09-15.	REV32.8/32.9 queue (paper fixes)
A1675	D1322	—	GROK · D1322 · READ ROUND 9 (R67 §3, R112). Filed S338, 2026-09-15.	Registrar (library v2), not print
A1676	D1323	—	GEMINI · D1323 · READ ROUND 9 (R67 §3, R112). Filed S338, 2026-09-15.	Registrar (library v2), not print
A1677	D1324	—	CHATGPT · D1324 · DRAFTER REVIEW, ROUND 9. Filed S338, 2026-09-15.	REV32.8/32.9 queue (paper fixes)
A1678	D1325	—	GROK · D1325 · READ ROUND 10 (R67 §3, R112). Filed S338, 2026-09-15.	Registrar (library v2), not print
A1679	D1326	—	GEMINI · D1326 · READ ROUND 10 (R67 §3, R112). Filed S338, 2026-09-15.	—
A1680	D1327	—	CHATGPT · D1327 · DRAFTER REVIEW, ROUND 10. Filed S338, 2026-09-15.	REV32.8/32.9 queue (paper fixes)
A1681	D1328	—	GROK return to D1328 (read round 11: batch 11 + R118 D-2 re-ask). Filed S339, 2026-09-16.	—
A1682	D1329	—	GEMINI return to D1329 (read round 11: batch 11 + R118 D-2 re-ask). Filed S339, 2026-09-16.	Registrar (library v2), not print

A	D	s	Headline result	Carried by
A1683	D1330	—	CHATGPT hostile review of read round 11 (D1330). NO READ CREDIT. Filed S339, 2026-09-16.	REV32.8/32.9 queue (paper fixes)
A1684	D1331	—	GROK return to D1331 (read round 12: batch 12, 25 RETIRED records). Filed S339, 2026-09-16.	Registrar (library v2), not print
A1685	D1332	—	GEMINI return to D1332 (read round 12: batch 12, 25 RETIRED records). Filed S339, 2026-09-16.	Registrar (library v2), not print
A1686	D1333	—	CHATGPT hostile review of read round 12 (D1333). NO READ CREDIT. Filed S339, 2026-09-16.	REV32.8/32.9 queue (paper fixes)
A1687	D1334	—	GROK return to D1334 (read round 13: batch 13, 24 RETIRED records, LIB2-262–285). Filed S339, 2026-09-16.	Registrar (library v2), not print
A1688	D1335	—	GEMINI return to D1335 (read round 13: batch 13, 24 RETIRED records, LIB2-262–285). Filed S339, 2026-09-16.	Registrar (library v2), not print
A1689	D1336	—	CHATGPT hostile review of read round 13 (D1336). NO READ CREDIT. Filed S339, 2026-09-16.	REV32.8/32.9 queue (paper fixes)
A1690	D1337	—	GROK return to D1337 (read round 14, the last batch: 14 RETIRED records). Filed S339, 2026-09-16.	Registrar (library v2), not print
A1691	D1338	—	GEMINI return to D1338 (read round 14, the last batch: 14 RETIRED records, LIB2-286–298 + 345). Filed S339,...	Registrar (library v2), not print
A1692	D1339	—	CHATGPT hostile review of read round 14 (D1339, the last batch). NO READ CREDIT. Filed S339, 2026-09-16.	REV32.8/32.9 queue (paper fixes)
A1693	D1340	—	GROK return to D1340 (round 15, the T-R round, R124). Filed S339, 2026-09-16.	Registrar (library v2), not print
A1694	D1341	—	GEMINI return to D1341 (round 15, the T-R round, R124). Filed S339, 2026-09-16.	Registrar (library v2), not print
A1695	D1342	—	CHATGPT hostile review of round 15 (D1342, T-R). Filed S339, 2026-09-16. NO READ CREDIT (drafting lane).	REV32.8/32.9 queue (paper fixes)
A1696	D1343	—	GROK return to D1343 (round 16, the R125 follow-up). Filed S339, 2026-09-16.	—
A1697	D1344	—	GEMINI return to D1344 (round 16, the R125 follow-up). Filed S339, 2026-09-16.	—
A1698	D1345	s1162	CHATGPT hostile review of round 16 (D1345, the R125 follow-up). Filed S339, 2026-09-16. NO READ CREDIT.	REV32.8/32.9 queue (paper fixes)

A	D	s	Headline result	Carried by
A1699	D1346	—	GROK read, round 17 (D1346, T-R R124/R125; LIB2-293 after the R126 re-word). Filed + banked S340, 2026-09-16.	Registrar (library v2), not print
A1700	D1347	—	GEMINI read, round 17 (D1347, T-R R124/R125; LIB2-293 after the R126 re-word). Filed + banked S340, 2026-09-16.	Registrar (library v2), not print
A1701	D1348	s1162	CHATGPT hostile review of round 17 (D1348, the R126 follow-up on LIB2-293). Filed S340, 2026-09-16. NO READ CREDIT.	REV32.8/32.9 queue (paper fixes)
A1702	D1349	—	GROK read, round 18 (D1349 ×4 parts, premise uses R127; 34 uses on 27 records). Filed + banked S340,...	Registrar (library v2), not print
A1703	D1350	—	GEMINI read, round 18 (D1350, premise uses R127; 34 uses on 27 records). Filed + banked S340, 2026-09-16.	Registrar (library v2), not print
A1704	D1351	—	CHATGPT hostile review of round 18 (D1351, premise uses R127; 34 uses on 27 records). Filed S340, 2026-09-16. NO...	REV32.8/32.9 queue (paper fixes)
A1705	D1352	—	GROK read, round 19 (D1352 ×2, R128 D-3 re-seal re-read; 25 uses on 14 records). Filed + banked S340,...	Registrar (library v2), not print
A1706	D1353	—	GEMINI read, round 19 (D1353, R128 D-3 re-seal re-read; 25 uses on 14 RETIRED records, Rev32.8 text). Filed +...	Registrar (library v2), not print
A1707	D1354	—	CHATGPT hostile review of round 19 (D1354, R128 D-3 re-seal re-read; 25 uses on 14 records). Filed S340,...	REV32.8/32.9 queue (paper fixes)
A1708	D1355	—	GEMINI read, round 20 (D1355, the R129 D-7 re-ask after H346; 11 uses on 7 records, Rev32.8 text). Filed + banked...	Registrar (library v2), not print

8 The four orphans

Four rounds in this era had, before S254, *no* representation in any tracker, ledger or index — not merely no tex citation. Their content is in the suite; the rounds themselves were unfindable.

A	Round	Why it matters that this one was adrift
1414	D1114	It carries the 13.907-untied / 11.907-tied freedom headline — the number Rev28 has now **withdrawn pending typed rebuild**. The single document explaining where that headline came from was the one document nothing pointed at.
1416	D1116	The parent-action round: the first two exact equations on C. Appendix S prints their consequences.
1417	D1117	The mixed-tensor round: the placement-charge table and the $V_{\pm 1}$ exclusion, both load-bearing in the Appendix S charge ledger.
1418	D1118	The orientation-odd phase: the V-side quadrature, and the *only* blind-adjudicated round of the hexad — exactly the kind of evidence a reviewer would want to find.

Closed at the Rev29 fold. The four orphan rounds A1414 and A1416–A1418 now carry full index rows in the programme's dispatch ledger, retrofitted at S258 from each assessment's own header and verdict. The gap this section documents — rounds cited in the suite but absent from the index — no longer exists for any round in Eras 1–5.

9 Era 8: A1709–A1724 (S341–S348; added Rev33.0)

Era 7 was folded at the Rev32.9 seal and then shipped *unchanged* through Rev32.10 and Rev32.11 — byte-identical in all three sealed trees. Sixteen rounds ran in that gap, including the whole Track-2 remediation arc that Rev32.11 exists to answer, and none of them had a row here. That is the third time this notebook has been found behind its own frontier: Era 6 was added at Rev32.5 "after the notebook was found sixty-two rounds behind", Era 7 at Rev32.9 "after the notebook was found to carry none of the Registrar build", and each time the finding was made by a reader, not by an instrument. **A gate now measures it** (`scripts/appendix_y_frontier_gate_S349.py`, wired into the house state check at S349): it compares this ledger's last serial against the newest *sealed* tree's and against the frontier on disk, and it fails when the ledger has not moved since the last seal *and* rounds are missing — so an ordinary between-seal gap stays silent and only "about to ship stale again" is red. Seven firing controls, all of which fire.

The rows below are *generated, never typed*, by `scripts/appy_era8_rows_S349.py` from the filed adjudications under `Coalition/assessments/`, on the same extraction as Era 7 — imported from that script rather than copied, so the rule has one home. The **Carried by** classification is this era's own: Era 7's prints "REV32.8/32.9 queue", which is false here, because these rounds' confirmed findings landed in Rev32.10, Rev32.11 and Rev33.0 (whose queue was open when this era was generated and is sealed with this edition). "—" means the generator traced no carrier, not that none exists. Sixteen serials, sixteen adjudication files, *no reservations.*

The round this era exists to record is one the house lost. A1724 (D1369 + D1370, ChatGPT Track-2 lane, S347) is filed with the verdict *"the round closes clean, and the house lost it on points"*: every deliverable arrived, every manifest verified from a fresh extract, the lane's executable reproduced byte-identically here — and **three of the four findings the house had dismissed at A1723 are real defects in our sealed text**. Two of those dismissals were house method errors: a grep matched to one spelling, and a verification aimed at the wrong page. The lane withdrew its own weakest claim unprompted and corrected two errors in its own memo against its own interest. A laboratory notebook that records only the rounds the house won is not a notebook.

A	D	s	Headline result	Carried by
A1709	D1356	—	GROK read, round 21 (D1356; REV32.10 S341.3 re-read after the re-anchor and R130; 62 uses on 34 records). Filed +...	Registrar (library v2), not print
A1710	D1357	—	GEMINI read, round 21 (D1357; REV32.10 S341.3 re-read after the re-anchor and R130; 62 uses on 34 records). Filed...	Registrar (library v2), not print
A1711	D1358	s1171	CHATGPT hostile review of round 21 (D1358; no read credit). Filed S342, 2026-09-17.	REV32.10–32.12 queue (paper fixes)
A1712	D1359	—	GROK read, round 22 (D1359; REV32.10 S341.3 re-read; 59 uses on 43 records). Filed + banked S342, 2026-09-17.	Registrar (library v2), not print
A1713	D1360	—	GEMINI read, round 22 (D1360; REV32.10 S341.3 re-read; 59 uses on 43 records). Filed + banked S342, 2026-09-17.	Registrar (library v2), not print
A1714	D1361	—	CHATGPT hostile review of round 22 (D1361; no read credit). Filed S342, 2026-09-17.	REV32.10–32.12 queue (paper fixes)
A1715	D1362	—	GROK read, round 23 (D1362; the R132 re-read; 12 uses on 4 records). Filed + banked S342, 2026-09-17.	Registrar (library v2), not print
A1716	D1363	—	GEMINI read, round 23 (D1363; the R132 re-read; 12 uses on 4 records). Filed + banked S342, 2026-09-17.	Registrar (library v2), not print
A1717	D1364	—	CHATGPT hostile review of read round 23 (D1364). Filed + house-verified S342, 2026-09-17. NO READ CREDIT (drafting...	REV32.10–32.12 queue (paper fixes)
A1718	D1365	—	GROK read, round 24 (D1365; the R137 re-read of LIB2-113; 4 uses on 1 record). Filed + banked S343, 2026-09-17.	Registrar (library v2), not print
A1719	D1366	—	GEMINI read, round 24 (D1366; the R137 re-read of LIB2-113; 4 uses on 1 record). Filed + banked S343, 2026-09-17.	Registrar (library v2), not print
A1720	D1367	s243	CHATGPT hostile review of round 24 (D1367; LIB2-113 under R137; no read credit). Filed S343, 2026-09-17.	REV32.10–32.12 queue (paper fixes)

A	D	s	Headline result	Carried by
A1721	—	s1178, s919	GROK (SuperGrok), unsolicited: "Long Jack Sarfatti Super Grok Analysis" — the OSS $J_3(\ )$-fibre $\times\ldots$	—
A1722	—	s1179, s1180, s1152	GROK (SuperGrok), PI-run chain P0–P9: one action (L_A), one projector, one mass matrix, one a_2 for the…	—
A1723	D1368	s909	D1368 return (ChatGPT, Track-2 v1.3 → v1.4 on the Rev32.10 basis)	Track 2 (not this suite)
A1724	D1369	s1189, s909	the D1369 + D1370 remediation round (ChatGPT, Track-2 lane), assessed as ONE round	Track 2 (not this suite)

Note added S362. The two paragraphs below were written at S356, when these eight rounds were to wait for the next fold. By PI call they did not: they are Era 9, §10, in this same release. The paragraphs are kept as written because they describe the gate correctly; only the plan changed.

Boundary, stated so the next reader knows where to start. This era stops at **A1724 / D1370**, the last round of S348 — the last *wrapped* session before this fold. **Eight rounds sit beyond it and belong to the next era: A1725–A1732 against D1371–D1378**, all filed during S349 — the three-lane boundary-methodology round (A1725/A1726/A1727), its Grok supplement (A1728), the three no-register returns (A1729/A1730/A1731), and the Grok cautionary case (A1732). The count is *derived from the filed adjudications on disk*, not carried: an earlier statement of this boundary named "A1725–A1727 and D1371–D1374" and added that D1374 was "staged and not yet sent". Both were true when written and neither is true now — D1374 was sent and its return is assessed at A1728 — and they are recorded here rather than removed, because a boundary paragraph that quietly re-counts itself is the defect this era exists to document.

And the silence between now and then is by design. The frontier gate (`scripts/appendix_y_frontier_gate_S349.py`) will **not** call for these eight until this ledger has sat through a seal unchanged: an ordinary between-seal gap is exactly the condition the gate is built to stay quiet through, and only "about to ship stale again" is red. So a reader who finds no Era 9 here is looking at a gate working, not at a notebook falling behind — which is the reading the three previous findings were made of. **Absence of a row is not currency; it is a deferral with a named trigger.**

10 Era 9: A1725–A1732 (S349; added Rev33.0)

Eight rounds, all filed in one session (S349), make up an arc about method rather than physics: *should the programme publish a register of its own boundary statements?* It ran as a cold three-lane round on the question (A1725/A1726/A1727 on D1371/D1372/D1373), a Grok supplement specifying the proposed index (A1728, D1374), three returns on the case against publishing any register (A1729/A1730/A1731 on D1375/D1376/D1377), and a Grok return specifying the cautionary case (A1732, D1378). **No paper number moved in this era.** Its findings were carried by rulings: R154 (the three standing items of the first round) and R155 (*no published register*, adopted under a declared reopener, the reader-performance audit), with R161 recording that audit as deferred rather than declined. The internal detector machinery was explicitly kept; its product is repaired manuscripts, not a publication.

The rows below are *generated, never typed*, by `scripts/appy_era9_rows_S362.py`, through the Era 8 row builder and so on the same extraction as Eras 7 and 8. The **Carried by** classification is this era's

own and is *derived*: every ruling file that cites the round's A-number or its D-number, except R151, which governs this notebook's eras and cites these serials only to defer them. "(no tex)" is printed only where every cited ruling says so in its own text; "—" means no ruling cites the round. Eight serials, eight adjudication files, *no reservations*.

What this era records is a coldness the house could not see for itself. The first round and the no-register round were both sent as *fresh threads*, one new chat per lane, so that three independent answers could be compared. A1731 (ChatGPT) opened with an unprompted declaration that saved programme context was present in its session, so its return should not count as memory-free; the round is therefore **two cold lanes and one warm**, and it is reported that way, not as three lanes agreeing. One round later A1732 (Grok) reported that its "fresh" thread was the same conversation that had just produced A1729, so A1732 is **not independent of A1729** and may not be cited as outside corroboration of the no-register arc. The transferable finding is the house's, and both times a lane supplied it: a new thread buys thread isolation, not memory-freedom, and the sender of a dispatch can *request* a fresh thread and record what the lane reports, but cannot assert it. Two further house defects are recorded in the adjudications themselves: the D1371–D1373 dispatches printed by-kind counts summing to 350 beside a census of 283 statements without saying the labels overlap (A1727 caught it; 53 statements carry more than one label), and D1374 asked twelve questions in three independent blocks of a lane known to truncate, and only the first block arrived, cut mid-sentence (A1728). Sent again as one block, the third arrived whole (A1732).

A	D	s	Headline result	Carried by
A1725	D1371	—	GROK return on D1371 (boundary-declaration methodology). S349, 2026-09-18.	Ruling R154 (no tex)
A1726	D1372	—	GEMINI return on D1372 (boundary-declaration methodology). S349, 2026-09-18.	Ruling R154 (no tex)
A1727	D1373	—	CHATGPT return on D1373, AND THE ROUND CLOSE. S349, 2026-09-18.	Rulings R154, R155 (no tex)
A1728	D1374	—	GROK, D1374 SUPPLEMENT: THE CONTRACT IS SPECIFIED; S2 AND S3 NEVER ARRIVED	Ruling R155 (no tex)
A1729	D1375	—	GROK, D1375: THE CASE AGAINST PUBLISHING ANY REGISTER. COMPLETE, AND STRONG.	Rulings R155, R161
A1730	D1376	—	GEMINI, D1376: SECOND LANE FOR OPTION 1 — BUT THE AGREEMENT CONCENTRATES ON ONE LIMB	Ruling R155 (no tex)
A1731	D1377	—	CHATGPT, D1377: THIRD LANE FOR OPTION 1 — AND IT DECLARED ITS OWN COLDNESS BREACH	Ruling R155 (no tex)
A1732	D1378	—	GROK, D1378: THE CAUTIONARY CASE SPECIFIED. COMPLETE, USABLE, AND NOT COLD.	—

Boundary. This era stops at **A1732 / D1378**, the frontier on disk when it was written (S362): the

next free numbers were A1733 and D1379. It was added at the Rev33.0 fold by PI call rather than left for the next one; the Era 8 boundary paragraph above recorded the opposite plan, and is kept as written with a note.

11 Era 10: A1733–A1735 (S366; added Rev33.1)

Three rounds, all filed in one session (S366), make up an arc about *what a check certifies*. An unsolicited site-read critique arrived from Grok (A1733, no dispatch): twenty checkable reads of the served site, eighteen confirmed, two partly located, none wrong; twelve proposals and ten charges, routed to the problem board as eight new asks and adopted nowhere. Two ChatGPT rounds then took the house's first mass-sector observation-map kernels apart: D1379 reproduced `s1203` exactly and showed three of its gates not to carry the load their names claim (A1734); the successor `s1204` was built to the lane's own contract, and D1380 reproduced it exactly on a third host and graded that contract one clause met, six partly met, with four real defects found at its boundary (A1735). **No paper number moved in this era.** Its findings were carried by kernels and queue rows, not by rulings.

The rows below are *generated, never typed*, by `scripts/appy_era10_rows_S367.py`, through the Era 8 row builder and so on the same extraction as Eras 7–9. The classifier is this era's own and is *derived* from each object: **D** is the dispatch the adjudication's header points to ("—" for an unsolicited return); **s** is the kernel the round receipted, matched by content hash to a kernel file on disk; **Carried by** lists the queue rows that attribute a finding to the round. The Era 9 classifier was run first and is not used: on these three rounds it read a dispatch the unsolicited return only cites, two rulings that cite those serials for other reasons, and a kernel serial named only as a future one — the "first serial in the text" shape that one classifier per era exists to prevent.

What this era records is a check that certifies itself. Three times in one kernel a gate was satisfied by the object it was meant to test: a "one function" test passed because the function inserted its own name into the register it read; a provenance lint tested the magnitude of a constant rather than where it came from; and a page trace validated a dictionary rather than the page. None of the three was found by a check failing. Each was found by a *mutation*: the lane changed the object, and the gate stayed quiet. The successor kernel fixed those three and the next round found the next layer the same way. Both ChatGPT rounds were sent as continuing threads by choice, and the lane was grading a kernel built to its own specification; no coldness is claimed for either, and neither is cited as independent corroboration. The transferable finding is the method: a gate whose quiet has never been seen to break is not yet known to be a gate.

A	D	s	Headline result	Carried by
A1733	—	—	GROK, UNSOLICITED: "Super Grok Thoughts on Rev33.0" — A SITE-READ CRITIQUE WITH TWELVE PROPOSALS, TEN CHARGES…	—
A1734	D1379	s1203	CHATGPT, D1379: s1203 REPRODUCED EXACTLY, AND THREE OF ITS GATES SHOWN NOT TO CARRY THE LOAD THEIR NAMES CLAIM.…	Queue S366.9, S366.13

A	D	s	Headline result	Carried by
A1735	D1380	s1204	CHATGPT, D1380: s1204 REPRODUCED EXACTLY; CONTRACT VERDICT 1 MET / 6 PARTLY MET, WEAKEST CLAUSE 7; THE BOUNDARY…	Queue S366.22

Boundary. This era stops at **A1735 / D1380**, the frontier on disk when it was written (S367): the next free numbers were A1736 and D1381. It is written into the working notebook for the Rev33.1 fold.

12 Era 11: A1736–A1738 (S367; added Rev33.1)

Three returns, all filed in one session (S367). A scheduled literature scan arrived from ChatGPT (A1736, no dispatch): ten items citing eleven preprints, all eleven located as cited; their content claims were *not* verified at source, because the archive refused the house's fetches, so every theorem and number in the scan stands as the lane's claim. One cheap audit and two board candidates were routed, the items that bear on decay rates and lifetimes were held, and nothing was adopted. Two lanes then read the $N-1$ blind formula census, `s1206`, under ruling R186. Gemini (D1381, A1737) declared a warm thread and did not run the kernel; its four objections were graded by measurement — two refuted, one a misreading, one kept as a sensitivity — and its proposed print sentence was rejected as an over-claim. ChatGPT (D1382, A1738) ran it and matched a result hash it had never been shown, the first independent reproduction of the census, and in the same return demonstrated that the enumerator was not exhaustive, that one row was scored in display units, and that root powers silently took an absolute value. The census was repaired (`s1206` v3) before its Appendix X text was written. **No existing paper number moved in this era.**

The rows below are *generated, never typed*, by the Era 10 generator (`scripts/appy_era10_rows_S367.py`) on its own derived classifier, unchanged.

What this era records is a reproduction that could not be faked, and a condition that was not met. The census result hash was withheld from everything the ChatGPT lane received, so a matching hash is evidence that the kernel was run; the Gemini lane, which could not run it, said so and echoed the house value as the house value. Neither lane was cold: both declared a warm thread or saved memory, so the one *cold* read R186 asks for has still not happened, and neither return is cited as independent corroboration of the census's reading. What the executing lane contributed is the part a warm lane can contribute honestly: the kernel ran on a third host, and its defects were found by running it.

A	D	s	Headline result	Carried by
A1736	—	—	CHATGPT, SCHEDULED LITERATURE SCAN (Definition A, week of 2026-09-19…25): 10 WORKS, 11 PREPRINTS — ALL 11…	Queue S367.6, S367.9
A1737	D1381	s1206	GEMINI, D1381: WARM, NOT RUN — THE COLD READ R186-5 REQUIRES DID NOT HAPPEN. A2 GRADED BY MEASUREMENT: TWO…	—

A	D	s	Headline result	Carried by
A1738	D1382	s1206	CHATGPT, D1382: THE FIRST INDEPENDENT REPRODUCTION OF THE CENSUS (HASH MATCHES A VALUE IT WAS NEVER SHOWN) — AND…	Queue S367.5, S367.15

Boundary. This era stops at **A1738 / D1382**, the frontier on disk when it was written (S369, at the Rev33.1 seal): the next free numbers were A1739 and D1383. It was added at the seal because the frontier gate (S348.6) found the narrative ledger unchanged since the snapshot while three assessments had been filed.

13 Era 12: A1739–A1792 (S371–S395; added Rev33.2)

Fifty-four rounds, filed between S371 and S395, and the house work that ran beside them. The era has three arcs.

Repairing the Rev33.1 print (S371–S377). ChatGPT's Track-2 rounds (D1383, D1384) found that the Rev33.0 R_2 retype had not reached four print sites, and that a printed neutrino tension rested on a width that was not on the source ledger; both were house-confirmed and repaired, and Paper 5's ratio row now scores $+3.43\sigma$ on ledger rows. Rounds on the magic numbers, the charm quark, the neutrino ratio row and the mass sector followed (D1385, D1387–D1389). The look-elsewhere specification Gemini froze for Paper 9 (D1386) became the census `s1209`, printed at Rev33.2. Three PI-initiated dives (A1745–A1747) were graded. The charge-and-colour rounds (D1390–D1396) confirmed the overturn of the `s1214` frame and gave the B-frame statement over all $O(4,4)$ frames that Appendix X now prints.

The gauging and its clock (S378–S389). A continuing ChatGPT thread mapped what the registered gauging does. The rank-13 end gaugings are twisted circle gaugings and the full gauging is not (D1403, D1414; Appendix G). Mass is the modulus of one charge only on a rank-one sheet (D1404, D1405). The clock generator is not a positive Hamiltonian, and an antiunitary reading of the time bridge had been mis-booked (D1412; Appendix H). The origin's tree quartic is indefinite (D1408), and the house then showed that the golden point is a tree-level Morse–Bott minimum on a flat branch that reaches that origin (`s1283`, `s1361`; Appendix I). Grok and Gemini ran a Wyler census in fresh threads (D1420), and an unsolicited Wyler input was graded (A1783).

The winding interface and the jets (S390–S395). Four PI-initiated threads were graded (A1786–A1789): the Möbius gluing is the O mapping torus, and two pasted constructions failed their own identities. The last three rounds (D1424–D1426) placed the gauging's non-geometry in a locally non-geometric R slot against every $SL(8)$ section (Appendix G), and closed one question about second jets before finding the two shipped second jets obstructed at third order.

House-led work. Beside the rounds the house sealed the Φ_{mass} model (`s1207`), mapped every row that reads Λ_{G_2} (`s1211`, the basis of the registered ± 0.5 MeV), typed the scheme-bearing cells (`s1336`, `s1337`: the measured-side $y_t(M_Z) = 0.970 \pm 0.002$), and made the kernels whose records depended on the host reproducible on any host (`s1318`, `s1319`, `s1353`, `s1357`; an erratum now sits beside each kernel they correct). At the Rev33.2 seal (S396) it tested the census's rank-one reading against comparators of one scheme and scale and found that it does not survive (`s1360`).

Numbers that moved in this era (all printed at Rev33.2): $y_t(M_Z) = 0.970 \pm 0.002$ (SL-57); Paper 5's ratio row at $+3.43\sigma$ (SL-01, SL-02); the ^{6}He sharp, $0.0039\% \to 0.0005\%$ (a mis-transcribed

world value, house defect S372.2); $\Lambda_{G_2} = 260 \pm 0.5$ MeV (SL-60), with row 20's d_{cmp} at +3.69 once the band is folded in; $\chi_c = 1.618 \pm 0.382$ under `s1182`'s normalisation (SL-61).

The rows below are *generated, never typed*, by the Era 12 generator (`scripts/appy_era12_rows_S396.py`). It runs on its own derived classifier: run with Era 10's, every dispatch and kernel field came back empty, because from A1750 a round has two files — the house's withheld forecasts and the adjudication — and its headers no longer name a dispatch or a kernel. Era 12 reads the dispatch from the adjudication's file name and the kernels from the result hashes the adjudication cites.

What this era records is a practice: forecasts written before the send. From A1750 every round carried a house-withheld file, written before the dispatch was composed and kept off every send, holding the house's blind values and its forecasts; each was graded against the lane's printed return at intake. A lane value that matches a withheld value is therefore evidence that the lane computed it, and a forecast the house got wrong is on the record beside the one it got right. The practice did not catch the house's own transcription errors — the ^{6}He world value was mis-copied in the house's kernel, not by a lane — and those are printed as house defects.

A	D	s	Headline result	Carried by
A1739	D1383	—	D1383 return (ChatGPT, Track-2 v1.4 → v1.5 on the Rev33.1 basis), both turns assessed as one round	Queue S371.3
A1740	D1384	—	D1384 return (ChatGPT, Track-2 series thread): S371.2 rework (A), S371.1 retype wording (B), Track-2 v1.5.1 (C)	Queue S371.1
A1741	D1385	—	CHATGPT, D1385 MAGIC 20/28 AS A SPECTRAL GAP: ⋆ FROZEN NEGATIVE, HOUSE-CONFIRMED. The cyclic-return /…	—
A1742	D1386	—	GEMINI, D1386 PAPER 9 LOOK-ELSEWHERE SPEC: RECEIVED COMPLETE, WARM. B1 IS FROZEN (sha 332d12c6…). ONE…	—
A1743	D1387	—	GROK, D1387 CHARM BRAINSTORM: RECEIVED COMPLETE (3/3 sentinels, ITEMS 11/11). TEN IDEAS; EIGHT OF THEM ARE BRANCH (a).	—
A1744	D1386	—	GEMINI, REPLY TO THE D1386 RESULT FYI (s1209): AN ACKNOWLEDGEMENT WITH NO NEW RESULT. 10 read-backs graded against…	—
A1745	—	—	GEMINI, PI-INITIATED DIVE "SPECTRAL DEFORMATION / COSET MODULI / VECTOR-LIKE CHAIN HOPPING": NINE IDEAS + TWO…	—
A1746	—	—	GROK, PI-INITIATED "BRAINSTORM ON MASS SECTOR SOLUTIONS": AN ACCURATE RE-STATEMENT OF THE D1387 CHARM BRIEF, NOT…	—

A	D	s	Headline result	Carried by
A1747	—	—	GROK, PI-INITIATED NEUTRINO-SECTOR "FIX LIST" (10 fixes + a run-first order): WRITTEN WITHOUT THE PROGRAMME'S…	—
A1748	D1388	—	GROK, D1388 (the neutrino ratio row $R_2{}^2 = 0.029180$): ROUND CLOSED — P1, P2, P3 RECEIVED COMPLETE (5/5…	Queue S371.2
A1749	D1389	—	CHATGPT, D1389 (the mass sector: ledger + room to conjecture): ROUND CLOSED — TURN 1 (6/6) + TURN 2 (15/15). 14…	—
A1750	D1390	s1214, s1215	CHATGPT, D1390 TURN 1 (Part A complete · Part B precommit): ⋆ THE s1214 FRAME OVERTURN IS CONFIRMED…	Queue S373.4, S373.7, S374.9
A1751	D1391	—	CHATGPT RETURN TO D1391 (magic 28, the named object) — TURN 1 FILED S374 (2026-09-29); TURN 2 FILED S375 —…	—
A1752	D1392	s1212	GEMINI RETURN TO D1392 (magic 28: is the torus parity I a shell grading?) — ASSESSED S374 (2026-09-29)	—
A1753	D1393	s1221, s1224	CHATGPT, D1393 TURN 1 = STOPPED AT PART A (a correct stop under the dispatch's overbroad criterion): ⋆…	—
A1754	D1394	—	ChatGPT return to D1394 (charge & colour solution-path assessment). Assessed S377 2026-09-30 by the house.	—
A1755	D1395	—	ChatGPT return to D1395 (hostile read of the Q1(a) census + C15 frame check; next steps). Assessed S377 2026-09-30.	—
A1756	D1396	—	ChatGPT return to D1396 (the PI's charge-length / false-vacuum collapse-cycle / two-ways-in-time pitch). Assessed…	—
A1757	D1397	—	ChatGPT return to D1397 (what do the programme's involutions flip — time, charge, mass?). Assessed S378…	—
A1758	D1398	—	ChatGPT return to D1398 (BUILD on frozen arrays: the ray charge-clock intertwiner). Assessed S378 2026-09-30…	—
A1759	D1399	—	ChatGPT return to D1399 (BUILD on frozen arrays: the SU(8)-frame charge-clock intertwiner K56). Assessed S378…	—
A1760	D1400	—	ChatGPT return to D1400 (think piece + build with the complex 56-bein: ray reflections, null cones, true vacuum)…	Queue S378.10
A1761	D1401	—	ChatGPT return to D1401 (BUILD on frozen arrays + think piece: the lift of J0·P, the one internal clock flip…	—

A	D	s	Headline result	Carried by
A1762	D1402	—	ChatGPT return to D1402 (BUILD: the interaction-level intertwiner for the parity lift _P of J0·P). Assessed…	—
A1763	D1403	s1234	CHATGPT return on D1403 (the graded gauging: the line, the ends, the balanced point) — HOUSE ASSESSMENT — S380…	—
A1764	D1404	s1238	CHATGPT return on D1404 (mass as the modulus of one charge: theorem, null lattice, 5D / (4,4) / triality, next…	Queue S380.18
A1765	D1405	s1242	CHATGPT return on D1405 (the charge form B on the torus-fixed Minkowski branch F0: derivation chain, signature, a…	—
A1766	D1406	s1243	CHATGPT return on D1406 (the product on the three real eights; what ties it to the charges; the coupling form;…	—
A1767	D1407	s1246	ChatGPT return on D1407 (S382, 2026-10-02): THE REDUCTION HOLDS; V < 0 PROVED ON THE PSD STRATA AND ON TWO…	—
A1768	D1408	s1249	ChatGPT return on D1408 (S382, 2026-10-02): THE TREE QUARTIC IS INDEFINITE AT o AND AT F08 — o IS NOT A TREE…	—
A1769	D1409	s1251	HOUSE ASSESSMENT OF THE D1409 RETURN (ChatGPT, CONTINUING thread): CROSS-POLLINATION ALONG THE RUNGS. S383…	—
A1770	D1410	s1252	HOUSE ASSESSMENT OF THE D1410 RETURN (ChatGPT, CONTINUING thread): THE SOLDER TEST AND ENDS TO CONES. S383…	—
A1771	D1411	s1254	HOUSE ASSESSMENT OF THE D1411 RETURN (ChatGPT, CONTINUING thread): WHAT SELECTS = $\sigma\cdot$? S384…	—
A1772	D1412	s1255	HOUSE ASSESSMENT OF THE D1412 RETURN (ChatGPT, CONTINUING thread): THE PI's TWO HYPOTHESES. S384 (2026-10-03),…	—
A1773	D1413	s1259	HOUSE ASSESSMENT OF THE D1413 RETURN (ChatGPT, CONTINUING thread): THE PI's SUPERPOSITION OF SIGNATURES — WHAT…	—
A1774	D1414	s1262	HOUSE ASSESSMENT OF THE D1414 RETURN (ChatGPT, CONTINUING thread): THE 5D → 4D TWIST. S385 (2026-10-03),…	Queue S385.19
A1775	D1415	s1265, s1267	HOUSE ASSESSMENT OF THE D1415 RETURN (ChatGPT, CONTINUING thread): BLIND CONFIRMATION OF s1265 — THREE TWIST…	—
A1776	D1416	s1258, s1280	HOUSE ASSESSMENT OF THE D1416 RETURN (ChatGPT, CONTINUING thread): THE OPEN LEG OF THE LIB2-195 FENCE — ONE…	Queue S387.15

A	D	s	Headline result	Carried by
A1777	D1417	s1279, s1281, s1282	HOUSE ASSESSMENT OF THE D1417 RETURN (ChatGPT, CONTINUING thread): WEAVE THE VACUUM ARC — THE MASS FORMULA IS…	—
A1778	D1418	s1287	HOUSE ASSESSMENT OF THE D1418 RETURN (ChatGPT, CONTINUING thread; S388, 2026-10-04)	—
A1779	D1419	s1288, s1289	HOUSE ASSESSMENT OF THE D1419 RETURN (ChatGPT, CONTINUING thread; S388, 2026-10-04)	—
A1780	D1420	s1294	HOUSE ASSESSMENT OF THE D1420 RETURN (GROK, FRESH THREAD; S389, 2026-10-04)	—
A1781	D1420	s1294	HOUSE ASSESSMENT OF THE D1420 RETURN (GEMINI, FRESH THREAD; S389, 2026-10-04)	—
A1782	D1421	s1290, s1295	HOUSE ASSESSMENT OF THE D1421 RETURN (ChatGPT, CONTINUING thread; S389, 2026-10-04)	—
A1783	—	s1298	HOUSE ASSESSMENT OF AN UNSOLICITED ChatGPT INPUT: “WYLER / CORPUS LINKAGE CHECK” (S389, 2026-10-04)	—
A1784	D1422	s1301	HOUSE ASSESSMENT OF THE D1422 RETURN (ChatGPT, CONTINUING thread; S389, 2026-10-04)	—
A1785	D1423	s1307	HOUSE ASSESSMENT OF THE D1423 RETURN (ChatGPT, CONTINUING thread): TWO OPEN LEGS — WINDING / JOINT O·C56…	—
A1786	—	—	HOUSE ASSESSMENT OF A PI-INITIATED SIDE QUEST (another Claude session; two pastes). S390 (2026-10-04), Cowork QB.	—
A1787	—	s1308	HOUSE ASSESSMENT OF A PI-INITIATED GEMINI THOUGHT: “Spin(4,4) chirality × Ḿobius strip × torus…	—
A1788	—	s1316	HOUSE ASSESSMENT OF A PI-INITIATED GEMINI + GROK PASTE: “BPS Zeta mass object thread” (QB brief “Rev32.5+”, a…	—
A1789	—	s1316, s1317	HOUSE ASSESSMENT OF A PI-INITIATED GROK / GEMINI FOLLOW-UP: “RESEARCH DISPATCH: EXECUTION OF OPEN TICKETS”…	—
A1790	D1424	s1331	D1424 return (ChatGPT, CONTINUING): THE SHARP GRADING BOUND, A CLASS OBSTRUCTION FOR SECTION-VIOLATING FRAMES, AND…	—
A1791	D1425	s1352	D1425 return (ChatGPT, CONTINUING): THE SECOND JET CLOSES, INCLUDING AT DETERMINANT ONE; NO SECTION-OBEYING CGI…	—

A	D	s	Headline result	Carried by
A1792	D1426	s1358	D1426 return (ChatGPT, CONTINUING): BOTH SHIPPED SECOND JETS ARE OBSTRUCTED AT THIRD ORDER; NEW SECOND JETS EXTEND…	—

Boundary. This era stops at **A1792 / D1426**, the frontier on disk when it was written (S396, for the Rev33.2 seal): the next free numbers were A1793 and D1427. Every number in the era has an adjudication file; A1780 and A1781 share one withheld file (one dispatch, two lanes).

14 Reserved numbers: nothing is missing

Two numbers in this range have no assessment file, and both are clean reservations rather than losses: **A1377** was reserved for D1080, an unsent dispatch subset, and **A1415** was reserved against the occupancy bank and left untouched. Both reservations are recorded in the neighbouring assessments. Era 5 (A1450–A1473) contains no reservations at all — twenty-four consecutive numbers, twenty-four assessment files, each with a matched verifier kernel and results file. *Era 7 (Rev32.9):* five serials in A1573–A1708 have no adjudication file of their own — **A1581** and **A1582** (reserved for the D1253 and D1254 returns), **A1589** (a partial intake note only), **A1629** (a tombstone: its dispatch was never sent) and **A1665** (filed jointly with A1664); each is printed as such in §7. The sequence A1358–A1473 is therefore complete at 114 rounds: 116 numbers less the two reservations. *Era 12 (Rev33.2):* A1739–A1792 has no reservations; A1780 and A1781 (one dispatch, D1420, sent to two lanes) share a withheld file and have an adjudication each.

15 Reverse index: printed object → certifying round

The table above answers "what did round N produce?". This one answers the question a reader actually asks: "where does *this* claim come from?"

Printed object	Certified by	Where it prints
the relative vertex sign -1 (amputated, F_4-fenced)	A1395	App. R
the rank-7 vertex readout; 10=10 dead	A1396	App. R
$\Pi_{5\leftarrow 6}$ (loaded); no equivariant transfer	A1396, A1400	App. R, T
the odd-sector quantization theorem; $z \in \{\pm i\}$	A1407	App. S, M
$v_{\mathrm{EW}}/\sqrt{\sigma} = \varphi^{105/8}$ as a framed registration	A1369, A1381	App. O
forcing 105/8 refuted (no depth selector)	A1370	App. O, N
the support floor $\{3, 11, 17\}$	A1421	App. M
minimal height refuted-as-canonical	A1421	App. S
$[j_A] \in \mathbb{RP}^2$; null axis $(5, -18, 1)$	A1420, A1428	App. S
the D1125 loaded finite effective action	A1429, A1430	App. S
the transgression obstruction theorem	A1430	App. S
P_{ann}; $q = \pm 1$ categorically absent	A1431	App. S

continued overleaf

Printed object	**Certified by**	**Where it prints**
the operator-ray collapse; non-factorization	A1431	App. S
0.0331562% dead (twice, off-support)	A1431, A1432	App. S, X
0.2744490308% as a legal miss	A1432	App. S, X
the affine support theorem; $\varphi/2 = \sin(3\pi/10)$	A1432	App. S
AX-AFF-ONSET (integer marking at $n^* = 7$)	A1432	App. S
the non-selection theorem	A1433	App. S
AX-MG, two independent clauses	A1434, A1435	App. S
lift degeneracy ($w = c\varphi^{-m}$ only)	A1435	App. S, T, X
the observation-functor faithfulness wall	A1436, A1438	App. T, X
the withdrawal of the bits+dims headline	A1414, A1436	App. X
the two-W separation (W_{sig} vs S_{shell})	A1437	App. T
$g_{+-} = \Gamma_J = W^2$ at zero continuous parameters	A1438	App. T
the non-gluing obstruction; $\mathfrak{B}_W$	A1438	App. T
the $\mathfrak{so}(5)$ Cartan closure	A1439	App. T
$27 \downarrow \mathfrak{so}(5) = 6 \cdot 1 \oplus 5 \oplus 16$	A1440	App. T
triality = the imaginary unit; signature = chirality	A1440	App. T
the solder no-go (SOLDER-BRANCH)	A1440, A1441	App. T
the spin–charge no-go (colour-protective)	A1441	App. T
the conformal-mode obstruction	A1441	App. T
the reflection-positive state at finite regulator	A1442	App. T
B_{MM} inertia $(3, 3, 4)$; the positive cone	A1442	App. T
AX-COT and its independence theorem	A1443, A1444	App. T
the no-linear-projector theorem	A1444	App. T
$G_{\mathrm{eff}} = \mathrm{SM}/\mathbb{Z}_6$ under local faithfulness	A1444	App. T
the χ source–response wall; $\mathfrak{N}_{\mathrm{mass}}$	A1444	App. T, X
the A1444-G2 erratum (wrong-carrier D_5 signs)	A1446	App. T (erratum box)
$o = -D_\varphi$ exact; the order line	A1446	App. T
the minimal dilation U_0	A1446	App. T
g_χ derived away; $\chi^2 = 1/4$ *inside* the attachment	A1446, A1447	App. T
$K_{\mathrm{rel}} = D_\varphi$ (the log leg a theorem)	A1447	App. T
$\mathcal{O}_{\mathrm{MOS}}$; the exact POVM; OS evenization	A1447	App. T
$\Pi^{\mathrm{MOS}}_{5\leftarrow 6}$ rank 3, $\neq$ microscopic	A1448	App. T
$M_N = \Lambda\sqrt{\varphi^{2N} + 1/4}$ (one-particle)	A1448	App. T
the record process; $\rho^* = \varphi/2$	A1448	App. T
X_{actual} splits (type derived, value = data)	A1449	App. T, L
the ^{28}Si typed-obstruction re-adjudication	A1425	Paper 9 §typed
Era 5 — the post-Rev27 derivation dive		
$U_{\mathrm{micro}} \in SO(6)$; three forced dark V_4 characters	A1450, A1451	App. O
the tiered ledger $\dim \mathrm{Hom} = 0/(3, 3, 3)/9$	A1450	App. X

continued overleaf

Printed object	**Certified by**	**Where it prints**
$\{S, H\} = 0$ from the branch-Real mouth pairing	A1452	App. O
$\ker M^2_{\text{full}} = \ker \Pi \otimes F$; the mass–export ordering functor	A1454	App. O
mass-first ordering (ratified-conditional; premises printed)	A1455	App. O
the Riesz-protected graded quadrature, generic	A1457	App. O, K
$\frac{1}{4}$ tangent/normal protection refuted	A1457	App. O
the bulk-local normal-blind counterterm theorem	A1458	App. F
the smooth-circle normal-blindness theorem	A1459	App. F
AX-LEP-PRIM (named, *not* adopted)	A1459, A1460	App. S, X
the L_{eff} kill-test: $1 - 4\varphi^{-2} = 4\varphi - 7 < 0$	A1461	App. I, X
$a_X = -2a_T$ (universal branch); $a_M := 2a_T + a_X$	A1462	P6
$\mathfrak{L}_{\text{MRSS}}$, the unique minimal chiral lift	A1463	App. P
the NS partner tower at $\lvert p_5 \rvert R = \frac{1}{2}$	A1463	App. P
$Q_Y \cong \mathbb{R}^3$; the BPS kink wall	A1464	App. P
the kink direction is independent (counter-model pair)	A1464	App. P
the conditional primitive-X theorem (norm-80, $\perp Y$)	A1465	App. P, X
falsifier $y_d = y_e \iff a_C = a_X$	A1465	App. P
the centralizer theorem: $X = 2(1, 1, 1, 1, 1)$	A1466	App. P, X
the X-semion weld, representation tier	A1466	App. P, S
the radial-shadow theorem	A1466	App. P
the central C_4^F functor forced; $\det T_X = -\frac{27}{16} A(A - B)^2$	A1467	App. T
no Fibonacci fiber functor into Hilb_{fd}	A1468	App. Q, T
the pointed operator's golden spectrum $\{\varphi, 1, \varphi^{-1}\}$	A1468	App. Q, T
$M^2 = M + I - P_m$, the unique minimal pointed completion	A1469	App. Q
$O_\varphi^\top \lvert M \rvert O_\varphi = J_{\text{vac}}$ exactly; the walls merge	A1469	App. Q, O
GEN-SEPARABILITY refuted and retired as a price	A1470	App. O, X
$[D_F, \mathcal{A}_{\text{gen}}] = 0$ forced; census $9 \to 3$	A1471	P3, App. X
mixing as a controlled defect; the home for $V_{\text{CKM}} = I$	A1471	P3
doublet–triplet from the MRSS Higgs parity	A1471, A1472	P4
the boundary price 6 microscopic / 3 visible	A1471, A1473	App. X, F
ν DIRAC-LOADED; charge controls 0 vs 10	A1472	P5, App. X
the A654 supersession arrow	Rev29 fold	P3
the priced-clause register (26 at A1473, 27 after A1474)	A1473, A1474	App. X

continued overleaf

Printed object	Certified by	Where it prints
the promotion gate S_{promo}	A1473	App. X, O
the boundary law (six finite counterterms)	A1473	App. X, F
$\Delta_{\min} = \frac{1}{2}\varphi^{-4}\varepsilon^2\|K\|^2$; $6 = 3 + 3$	A1473	App. X

16 The casualty register

Eight rounds in this era corrected the *house*, not the lane. They are listed because the protocol's credibility rests on them being findable.

Round	What the house got wrong, and what caught it
A1395	Two house overclaims on the parent vertex refuted; the wall re-counted honestly (3 continuous data + orientation fibre).
A1401	The house's "primitive quartic" expectation refuted: no primitive quartic invariant exists at that tier.
A1426	One house prediction corrected in the cascade/equivariance round.
A1427	*Two* house casualties in one round: the blanket-blindness claim refuted, and a second wording correction.
A1434	The D1130 dispatch asserted "the continuum already died spectrally." It had not — D1129 *constructed* the spectral continuum and refuted only *selection*. A standing pre-send grep rule against banked results was adopted in consequence.
A1436	The 17-dimensional decomposition behind the freedom headline is internally inconsistent as typed. Confirmed at primary source by house grep, not conceded on assertion. The headline is withdrawn at Rev28.
A1437	The single label "W" denoted two categorically different operators. Split suite-wide.
A1446	A banked, house-signed row priced the chamber lift from a commutant computed on the *wrong carrier* (the permutation module, not the chamber carrier). The finite-sign residue is retired and replaced by the minimal-dilation theorem. The erratum is printed in Appendix T rather than absorbed.

Protocol failures are logged too, and so are their resolutions. Nine receipt variances (rounds A1437–A1444) ran on informal-thread returns without a formally filed dispatch; two carry protocol-breach marks. Seven consecutive bundles omitted their own dependencies, cured only at the last round of that era. Three riders (an ω countersign, a mouth-flip atlas comparison, a catalecticant computation) went unanswered for thirteen rounds, and one owed bundle (`258f9982`) was never uploaded — so the rows depending on it are graded HOUSE-PRIMARY, single-lane, and say so.

What Era 5 settled. The enforcement backlog was retyped house-side at S255: the house now owns verification logistics and builds its own kernel when a return arrives without one, so the rider and owed-bundle debts close as enforcement items — `258f9982` was never delivered and is no longer owed. A separate rider, on the framework's neutrino typing, ran *five* implicit rounds unanswered before landing in full at A1472. Era 5 also ran under a temporary dispensation in which the lane chose its own object; three rounds executed without the gate freeze their item required (A1464 by scheduling, A1466 by knowing PI waiver, A1455 by running ahead of its own criteria). Each is priced in its row, and in the A1455 case the mathematics was banked while the *verdict* was withheld and sent to the PI. The era closed by inventing a fix for the underlying problem: receipt-proof by dispatch-hash echo, first achieved at A1473 and now standard, which replaces a lane's assertion that it received the dispatch with a cryptographic demonstration. None of this changes a verification result; all of it is part of the record.

The sealed table's own defect (A1474, S258)

The house is not exempt from this register, and the most recent entry is its own.

For the ω round the house computed its answers in advance and sealed them, so that the lane's return could be adjudicated blind. Five of the six sealed verdicts were confirmed independently. The sixth was wrong — not in its arithmetic, but in its reasoning. The house had written that if ω selects no dynamics then it "adds nothing the promotion gate does not already own". The lane pointed out that this conflates two different claims: ω contributing no dynamics selector does not entail that the promotion gate owns what is left. It does not. S_{promo} constrains the finite Dirac operator; it says nothing about a polarization or a Hamiltonian on the Freudenthal system.

The house verdict is withdrawn. The consequence is not cosmetic: had the sealed reasoning stood, a real open question would have been absorbed into an existing wall and never printed. Instead it is clause 27 of the priced register. The protocol caught a house error precisely because the house committed first and in writing.

A later catch, owed since Rev30: interface-type erasure (A1482)

One row of this register was ratified by the PI and then never printed. It is printed here, eight seals late, with that delay stated rather than quietly closed.

At S265 the house put $\Psi_{abcd}\,\mathrm{Tr}(T^aT^bT^cT^d)$ into the demands of dispatch D1176. The ChatGPT lane refused it as **ill-typed**, and was right: Ψ's indices live on the six-real fundamental colour carrier, the T^a on the eight-dimensional adjoint, and there is no canonical map between the two. The expression erases an interface. The house withdrew it in D1176 item 3 and recorded the correction at A1482 T6.

What makes it a casualty rather than a typo is the same session's other traffic: the house charged the lane with precisely this defect *three times* while committing it once itself. The asymmetry is the finding. A programme that audits its correspondents for a class of error is obliged to audit itself for

the same class in the same session, and this one did not.

The row was ratified for print at S265 under the precedent set when A638/A639 were admitted above — house casualties get printed — and marked owed at the Rev30 fold. Rev30, Rev31, Rev31.1, Rev31.2 and the whole Rev32 series sealed without it. It was recovered by the S264–S352 master sweep at S353 (`Trackers/LOST_SHEEP_MASTER_SWEEP_S353.md`, row S353-6), which also established why no instrument had caught it: the ratification named a destination file, `appendix_y_casualty_register.tex`, **that has never existed**, and the queue row was ticked on the strength of the PI's ratification rather than on the print. **A tick is not a discharge, and a locator that resolves to nothing is not a home** — the two rules this single row breaks are the two the register discipline was built on.

Extending the register backwards: the first catch (A638/A639)

The register above begins in Era 2. It should begin earlier, and at the Rev29 fold the programme admits the earliest entry to the published record.

At S43, a lane returned a verification checklist it had not run — a fabricated confirmation, presented as completed work. The house caught it by checking the disk rather than reading the claim (A638), and the correction was recorded the same session (A639). That catch is the origin of VERIFY-DON'T-RELAY, the rule under which every row in this appendix was produced: a lane's assertion that something verified is not evidence that it verified, and the house re-runs.

It appeared in no casualty register for two hundred sessions. It appears here because a programme that prints its later catches and suppresses its first one is telling a flattering story about when it learned to be careful. The rule has a date, and the date is the day the programme was fooled.

17 The Era-5 end-state adjudication

The closing round of the dive (A1473) delivered a 23-row ledger stating, for each round of the era, what its results ended up typed as once every later round had had its say. The house cross-checked every banked results hash in that ledger against its own copies: **10 of 10 present hashes match**. The adjudication below is the house's own reading of the end state, printed alongside the lane's, as the fold task required.

- **Where the two ledgers agree** — twenty-one of twenty-three rows. Both record the same tier, the same conditionality, and the same open prices.
- **Where they differed** — the lane's ledger spanned nineteen rows read literally from "SD14 onward" while the house dispatch had asked for twenty-three. The discrepancy was a *house* drafting error in the range specification, not a lane omission. The lane resolved it by returning both ledgers rather than picking one. The house owns the error here, in print.
- **The SD14 archive gap** — one early round of the era has no bundle in the inbox and was flagged three times across the arc. It stays retired. Rows depending on it are graded house-primary and say so.

Merged supersession audit

Three separate supersession audits ran during Eras 4 and 5. Merged, and with each arrow's direction stated once:

- A1451 → A1450 claim 9: *two* $Spin(5)$ form factors, not one.
- A1472 → A1471: Higgs-parity uniqueness is *conditional* on the loaded low-energy scalar spectrum, not unconditional.
- A1471 → A1463-era shorthand: the endpoint shorthand is retired; the boundary price is 6 microscopic and 3 visible.
- A1446 → A1444-G2: the "8 orthogonal / 4 orientation-preserving D_5 signs" row is retired as a wrong-carrier count; the minimal-dilation theorem prints instead.
- The Rev29 fold → A654: the zero-free-parameter CKM claim of S46 is SUPERSEDED; see the arrow printed in Paper 3.
- Retired axioms, not superseded but withdrawn: LOG-HODGE NATURALITY, AX-RIESZ-CLIFFORD-MASS, AX-KUBO-MASS, AX-MASS-LABEL, AX-MAP-HISTORY as the actuality law, and GEN-SEPARABILITY as a price. None may be cited as live.

18 What this appendix does not do

It does not restate the physics: every row points at the component that carries it. It does not upgrade a tier — where the ledger and a printed section disagree on wording, the printed section governs and the discrepancy is a defect to be fixed, not resolved here. It is a *locator* at section granularity, not a per-claim citation index; building the latter for Eras 1–5 is the natural next pass, and this appendix is the scaffold for it. And its per-round core table covers A1358–A1473; the A1300–A1357 head band and the A1475–A1500 freeze-first era are carried in their own sections (added at Rev30), Era 6 (A1501–A1572) in its own section (added Rev32.5, after the notebook was found sixty-two rounds behind its frontier), Era 7 (A1573–A1708) in its own generated section (added Rev32.9, after the notebook was found to carry none of the Registrar build), while rounds before the Rev25 freeze are recorded in the Rev25-era ledgers and in `Papers/PAPERS_HISTORY.md`.

19 The Registrar at seal (generated, 2026-10-07)

A dated snapshot, not live status. Registrar master `735316e3244997c9`, sealed with Rev33.2_S398 on 2026-10-07: 357 records (IN_REVIEW 81, PUBLISHED 276). Each row gives the record, its primary assertion (with the number of further assertions), where its primary source use sits in this edition, the record class, and its review state *at this seal*. The live state of any record is its own page, linked from the inline tags beside the sentences it quotes. Generated by `scripts/seal_registrar_table.py`; never typed.

Record	Assertion	Component / section	p.	Class	State
LIB2-001	`AS-LIB2-001-primary` +2	Appendix E, §4, Theorem 2: admissible-gap case count	173	CLAIM	PUBLISHED
LIB2-002	`AS-LIB2-002-primary` +3	Paper 3, §5.4, What survives, and it is not nothing	65	CLAIM	PUBLISHED
LIB2-003	`AS-LIB2-003-primary`	Appendix X, Honest precision summary: Atmospheric angle…	351	COMPARATOR	PUBLISHED
LIB2-004	`AS-LIB2-004-primary` +2	Paper 3, §5.4, Current replacement attachment	65	RETIRED	PUBLISHED
LIB2-005	`AS-LIB2-005-primary`	Appendix T, §15.5, Intrinsic-basepoint placeholder	340	OPEN	IN_REVIEW

Record	Assertion	Component / section	p.	Class	State
LIB2-006	AS-LIB2-006-primary +3	Appendix F, §1, Scope and provenance: V1–V6	183	CLAIM	PUBLISHED
LIB2-007	AS-LIB2-007-primary	Paper 4, §4.2, Ghost-free reading: identity-anchored...	74	CLAIM	PUBLISHED
LIB2-008	AS-LIB2-008-primary +2	Appendix F, §5.3, Jordan-54 Lie-triple test	189	CLAIM	PUBLISHED
LIB2-009	AS-LIB2-009-primary	Appendix F, §5.3, Missing-16 Lie-triple test	190	CLAIM	PUBLISHED
LIB2-010	AS-LIB2-010-primary +4	Appendix F, §5.3, Generated-algebra identification	190	CLAIM	IN_REVIEW
LIB2-011	AS-LIB2-011-primary	Appendix X, P0-1 audit, The result: definition of LJ	375	CLAIM	PUBLISHED
LIB2-012	AS-LIB2-012-primary	Appendix T, §15.1, The solder and transported pointing	337	CLAIM	PUBLISHED
LIB2-013	AS-LIB2-013-primary	Appendix T, §15.1, Dated amendments to Table 1: face 3	338	CLAIM	PUBLISHED
LIB2-014	AS-LIB2-014-primary	Appendix E, §7, The dark attachment: provenance upgrade	177	CLAIM	PUBLISHED
LIB2-015	AS-LIB2-015-primary +2	Appendix H, §11, Theorem 11.1: frozen-word setup	209	CLAIM	PUBLISHED
LIB2-016	AS-LIB2-016-primary +2	Appendix X, Carrier fence beside clause 27: three tens	377	CLAIM	PUBLISHED
LIB2-017	AS-LIB2-017-primary +5	Appendix S, §9.2, Theorem 9.2: declared-class negative	317	CLAIM	PUBLISHED
LIB2-018	AS-LIB2-018-primary +2	Appendix E, §3, Remark 2: status of H2	173	CLAIM	PUBLISHED
LIB2-019	AS-LIB2-019-primary	Appendix E, §3, Theorem 1: symmetry-form selector	172	CLAIM	PUBLISHED
LIB2-020	AS-LIB2-020-primary	Appendix E, §4, Theorem 2: arithmetic selector	173	CLAIM	PUBLISHED
LIB2-021	AS-LIB2-021-primary +2	Appendix E, §7, Closure declaration: minimum and Hessian	179	CLAIM	PUBLISHED
LIB2-022	AS-LIB2-022-primary +3	Appendix E, §7, Boundary datum: three faces	177	CLAIM	PUBLISHED
LIB2-023	AS-LIB2-023-primary	Appendix E, §7, Dark attachment: detector eigenspace	178	CLAIM	PUBLISHED
LIB2-024	AS-LIB2-024-primary	Appendix E, §7, Dark attachment: coefficient identity	177	CLAIM	PUBLISHED
LIB2-025	AS-LIB2-025-primary +2	Appendix E, §7, Forced interior split	178	CLAIM	PUBLISHED
LIB2-026	AS-LIB2-026-primary +4	Appendix S, §9.1, Theorem 9.1: critical point and Hessian	317	CLAIM	PUBLISHED
LIB2-027	AS-LIB2-027-primary	Appendix S, §9.4, Theorem 9.4: moduli space	318	CLAIM	PUBLISHED
LIB2-028	AS-LIB2-028-primary +3	Appendix S, §9.4, Priced axiom 9.5: adopted relations	318	CLAIM	PUBLISHED
LIB2-029	AS-LIB2-029-primary +2	Paper 0, Executive State Map: occupation-selector row	6	CLAIM	PUBLISHED
LIB2-030	AS-LIB2-030-primary	Appendix E, §8, S284 rulings: target-triviality	179	CLAIM	PUBLISHED
LIB2-031	AS-LIB2-031-primary +2	Appendix E, §8, S284 rulings: H2 rank and price	179	CLAIM	PUBLISHED
LIB2-032	AS-LIB2-032-primary +2	Appendix E, §8, C0 independence theorem	180	CLAIM	PUBLISHED
LIB2-033	AS-LIB2-033-primary	Appendix E, §8, F-SEL(c): scoped closure	181	CLAIM	PUBLISHED
LIB2-034	AS-LIB2-034-primary	Appendix E, §8, Casimir-naturality: conclusion	180	CLAIM	PUBLISHED
LIB2-035	AS-LIB2-035-primary +3	Appendix E, §8, Corrected non-degenerate positivity...	180	CLAIM	PUBLISHED
LIB2-036	AS-LIB2-036-primary	Appendix E, §6(i), Gram-only selector obstruction	174	CLAIM	PUBLISHED
LIB2-037	AS-LIB2-037-primary	Appendix E, §6(ii), Frame-free diagonal-invariant...	175	CLAIM	PUBLISHED
LIB2-038	AS-LIB2-038-primary +2	Appendix E, §6(iii), Transported-selector source isotropy	175	CLAIM	PUBLISHED
LIB2-039	AS-LIB2-039-primary	Appendix E, §6(iv), Fixed-grade detection obstruction	175	CLAIM	PUBLISHED
LIB2-040	AS-LIB2-040-primary +2	Appendix E, §6(v), Even-action sign obstruction	175	CLAIM	PUBLISHED
LIB2-041	AS-LIB2-041-primary +4	Appendix E, §6(vi), Frame-free orbit obstruction	176	CLAIM	PUBLISHED
LIB2-042	AS-LIB2-042-primary +2	Appendix E, §6(vi), Norm-only Hessian obstruction	176	CLAIM	PUBLISHED

Record	Assertion	Component / section	p.	Class	State
LIB2-043	AS-LIB2-043-primary	Appendix E, §6(vii), Unshifted Kempf–Ness rank bound	176	CLAIM	PUBLISHED
LIB2-044	AS-LIB2-044-primary	Appendix E, §6(vii), Beat-transitivity obstruction	176	CLAIM	PUBLISHED
LIB2-045	AS-LIB2-045-primary +2	Appendix E, §7, Boundary-source obstruction	177	CLAIM	PUBLISHED
LIB2-046	AS-LIB2-046-primary	Appendix E, §5, Corollary 1: spectral identity	174	CLAIM	PUBLISHED
LIB2-047	AS-LIB2-047-primary	Paper 1, §3, Roman-surface paragraph: vacuum adjoint	26	CLAIM	PUBLISHED
LIB2-048	AS-LIB2-048-primary	Appendix B, §B.5, Structural support (a): DET-7	161	CLAIM	PUBLISHED
LIB2-049	AS-LIB2-049-primary	Appendix A, §A.9, Exact dimension ratio	156	CLAIM	PUBLISHED
LIB2-050	AS-LIB2-050-primary	Paper 3, Abstract, Rev29 update: Route-B weight	49	CLAIM	PUBLISHED
LIB2-051	AS-LIB2-051-primary	Appendix B, §B.5, Structural support (b): C1 weight	161	CLAIM	PUBLISHED
LIB2-052	AS-LIB2-052-primary	Appendix B, §B.5, Structural support (c): generator…	161	CLAIM	PUBLISHED
LIB2-053	AS-LIB2-053-primary +2	Appendix K, §4, Theorem 4.3: reflected trace weight	248	CLAIM	PUBLISHED
LIB2-054	AS-LIB2-054-primary +3	Appendix P, §2, Theorem 2.1: tick grading	288	CLAIM	PUBLISHED
LIB2-055	AS-LIB2-055-primary +2	Appendix P, §2, Theorem 2.2: cylinder-condition count	288	CLAIM	PUBLISHED
LIB2-056	AS-LIB2-056-primary +2	Appendix P, §2, Theorem 2.4: census and cocycle	289	CLAIM	PUBLISHED
LIB2-057	AS-LIB2-057-primary +2	Paper 8, §2, PSL(2,7) class-algebra structure constants	123	CLAIM	PUBLISHED
LIB2-058	AS-LIB2-058-primary	Appendix I, §6, Proposition 6.1: scaling setup	221	CLAIM	PUBLISHED
LIB2-059	AS-LIB2-059-primary +2	Appendix I, §6, Remark 6.4(iii): cubic relevance	223	CLAIM	PUBLISHED
LIB2-060	AS-LIB2-060-primary	Paper 5, §2, Solar-angle expression	84	CLAIM	PUBLISHED
LIB2-061	AS-LIB2-061-primary +2	Paper 5, §2, Reactor-angle expression	84	CLAIM	PUBLISHED
LIB2-062	AS-LIB2-062-primary	Paper 5, §3, Phase expression	89	CLAIM	IN_REVIEW
LIB2-063	AS-LIB2-063-primary	Paper 3, §2, Route-B readout	49	CLAIM	PUBLISHED
LIB2-064	AS-LIB2-064-primary	Paper 3, §2, Measured-angle diagnostic	50	CLAIM	IN_REVIEW
LIB2-065	AS-LIB2-065-primary +2	Paper 3, §2, First-row unitarity readout	50	CLAIM	PUBLISHED
LIB2-066	AS-LIB2-066-primary +4	Appendix B, §B.6, Vcb comparator row	164	CLAIM	PUBLISHED
LIB2-067	AS-LIB2-067-primary	Appendix B, §B.6, Vub hierarchy and displayed comparison	164	CLAIM	PUBLISHED
LIB2-068	AS-LIB2-068-primary	Paper 3, Candidate Construction 2.1, retained formula	54	CLAIM	PUBLISHED
LIB2-069	AS-LIB2-069-primary	Paper 3, §2.1, Global-fit Dirac-phase comparator	51	CLAIM	PUBLISHED
LIB2-070	AS-LIB2-070-primary	Paper 3, §2.1, Direct UT-angle comparator	51	CLAIM	PUBLISHED
LIB2-071	AS-LIB2-071-primary	Paper 3, §2.1, Unitarity of the standard parametrisation	51	CLAIM	PUBLISHED
LIB2-072	AS-LIB2-072-primary +3	Paper 3, §2.2, Internal consistency note	56	CLAIM	PUBLISHED
LIB2-073	AS-LIB2-073-primary +2	Paper 3, §5.2, Universal-weight control	64	CLAIM	PUBLISHED
LIB2-074	AS-LIB2-074-primary	Paper 3, §3.2, PMNS construction	57	CLAIM	PUBLISHED
LIB2-075	AS-LIB2-075-primary +4	Paper 3, Candidate Construction 2.1, Zero-correction control	55	CLAIM	IN_REVIEW
LIB2-076	AS-LIB2-076-primary	Appendix F, §2, Proof step L4: adjoint multiplicities	183	CLAIM	PUBLISHED
LIB2-077	AS-LIB2-077-primary +4	Appendix F, §3, Leg A: invariant-space dimension and…	185	CLAIM	PUBLISHED
LIB2-078	AS-LIB2-078-primary +2	Appendix F, §7, Theorem 7.1: invariant projectors	191	CLAIM	PUBLISHED
LIB2-079	AS-LIB2-079-primary	Appendix F, §5.2, Dyonic completion at the origin	188	CLAIM	PUBLISHED
LIB2-080	AS-LIB2-080-primary +3	Appendix F, §5.2, Remark 5.2(b2): singlet count	188	CLAIM	PUBLISHED
LIB2-081	AS-LIB2-081-primary	Appendix F, §4, Theorem 4.1: fixed package	185	CLAIM	PUBLISHED
LIB2-082	AS-LIB2-082-primary	Appendix F, §4, Theorem 4.1(2): extra Cartan	185	CLAIM	PUBLISHED

Record	Assertion	Component / section	p.	Class	State
LIB2-083	AS-LIB2-083-primary	Appendix F, §4.1, State-by-state centre check	187	CLAIM	PUBLISHED
LIB2-084	AS-LIB2-084-primary +2	Appendix P, §2, Theorem 2.3: signed-trace pattern	288	CLAIM	PUBLISHED
LIB2-085	AS-LIB2-085-primary	Appendix P, §2, Theorem 2.5: scope	289	CLAIM	PUBLISHED
LIB2-086	AS-LIB2-086-primary +2	Appendix I, §6, Remark 6.4(ii): colour isotropy	222	CLAIM	PUBLISHED
LIB2-087	AS-LIB2-087-primary +3	Appendix A, §A.7.5, Rev32.1 Schur correction	153	CLAIM	PUBLISHED
LIB2-088	AS-LIB2-088-primary +2	Appendix A, §A.7.8(ii), Conditional Artin application	155	CLAIM	PUBLISHED
LIB2-089	AS-LIB2-089-primary	Appendix S, §9.3, Quadratic bridge census: P to B	317	CLAIM	PUBLISHED
LIB2-090	AS-LIB2-090-primary	Appendix S, §9.3, Quadratic bridge census: B to P	317	CLAIM	PUBLISHED
LIB2-091	AS-LIB2-091-primary +2	Appendix F, §8, Declared connection class	191	CLAIM	PUBLISHED
LIB2-092	AS-LIB2-092-primary +2	Appendix N, §1.2, Leg 2: golden numerology density	268	CLAIM	PUBLISHED
LIB2-093	AS-LIB2-093-primary +2	Appendix N, §1.3, Leg 3: mode-locking comparison	268	CLAIM	PUBLISHED
LIB2-094	AS-LIB2-094-primary +2	Appendix N, §1.4, Leg 4: continuous offset and…	269	CLAIM	PUBLISHED
LIB2-095	AS-LIB2-095-primary +3	Appendix N, §1.5, Leg 5: absent tunnelling solution	269	CLAIM	PUBLISHED
LIB2-096	AS-LIB2-096-primary	Appendix N, §2, Proposition 2.1: compact-colour…	270	CLAIM	PUBLISHED
LIB2-097	AS-LIB2-097-primary +2	Appendix N, §2, Carrier truncation: cross-translator…	271	CLAIM	PUBLISHED
LIB2-098	AS-LIB2-098-primary +4	Appendix N, §3, Proposition 3.1: package obstruction	271	CLAIM	PUBLISHED
LIB2-099	AS-LIB2-099-primary +4	Appendix N, §4, Local Row-1 exclusion	272	CLAIM	PUBLISHED
LIB2-100	AS-LIB2-100-primary +2	Appendix N, §6, Octant-registration fence	274	CLAIM	PUBLISHED
LIB2-101	AS-LIB2-101-primary	Appendix N, §6, Complex-ratio ladder census	274	CLAIM	PUBLISHED
LIB2-102	AS-LIB2-102-primary +3	Appendix N, §6, Jordan-scaling centralizer census	274	CLAIM	PUBLISHED
LIB2-103	AS-LIB2-103-primary +3	Appendix N, §6, Grading centralizer census	274	CLAIM	PUBLISHED
LIB2-104	AS-LIB2-104-primary	Appendix Q, §3(i), Bare-quartic gradient	296	CLAIM	PUBLISHED
LIB2-105	AS-LIB2-105-primary +2	Appendix Q, §3(ii), Polarization obstruction	296	CLAIM	PUBLISHED
LIB2-106	AS-LIB2-106-primary +2	Appendix O, §5, Remark 5.2: single-chi test	280	CLAIM	PUBLISHED
LIB2-107	AS-LIB2-107-primary +3	Appendix I, §8, Lemma 8.3: signed-residue range	226	CLAIM	PUBLISHED
LIB2-108	AS-LIB2-108-primary	Paper 1, §6, Status under the explicit B-map	29	CLAIM	PUBLISHED
LIB2-109	AS-LIB2-109-primary +4	Paper 1, §8, Local anomaly sums	30	CLAIM	PUBLISHED
LIB2-110	AS-LIB2-110-primary +2	Appendix K, §2.1, Theorem 2.1: trilinear	243	CLAIM	PUBLISHED
LIB2-111	AS-LIB2-111-primary	Appendix K, §1, Positive Yukawa statement	242	CLAIM	PUBLISHED
LIB2-112	AS-LIB2-112-primary +2	Appendix K, §3.2, Half-norm mirror leakage	244	CLAIM	PUBLISHED
LIB2-113	AS-LIB2-113-primary	Appendix K, §3.3, Reynolds-forced equal weights	245	CLAIM	PUBLISHED
LIB2-114	AS-LIB2-114-primary	Appendix K, §3.4, Candidate-compensator sweep	245	CLAIM	PUBLISHED
LIB2-115	AS-LIB2-115-primary +2	Appendix K, §3.5, Incidence rank	246	CLAIM	PUBLISHED
LIB2-116	AS-LIB2-116-primary	Appendix K, §3.5, Running setup	246	CLAIM	PUBLISHED
LIB2-117	AS-LIB2-117-primary +3	Appendix K, §3.6, Edge-uniform metrics	246	CLAIM	PUBLISHED
LIB2-118	AS-LIB2-118-primary	Appendix K, §1, Negative conclusion	242	CLAIM	PUBLISHED
LIB2-119	AS-LIB2-119-primary	Appendix A, §A.6, W01: internal carrier result	147	CLAIM	PUBLISHED
LIB2-120	AS-LIB2-120-primary +2	Appendix A, §A.6, W01: declared attachment	147	CLAIM	PUBLISHED

Record	Assertion	Component / section	p.	Class	State
LIB2-121	AS-LIB2-121-primary +2	Appendix X, Table 4, Top row	360	CLAIM	PUBLISHED
LIB2-122	AS-LIB2-122-primary +3	Paper 2, §3.2, Ratio typing	34	CLAIM	PUBLISHED
LIB2-123	AS-LIB2-123-primary +2	Paper 2, §3.2, Required coefficient	35	CLAIM	PUBLISHED
LIB2-124	AS-LIB2-124-primary +2	Appendix X, Table 4, Named operational anchors	361	CLAIM	PUBLISHED
LIB2-125	AS-LIB2-125-primary +2	Paper 2, §3.2, Tau anchor	34	CLAIM	PUBLISHED
LIB2-126	AS-LIB2-126-primary +3	Paper 2, §3.2, Step (3): physical mass	35	CLAIM	PUBLISHED
LIB2-127	AS-LIB2-127-primary	Paper 2, §3.2, External Koide premise	36	CLAIM	PUBLISHED
LIB2-128	AS-LIB2-128-primary	Paper 2, §3.4, Equation (19): constituent relation	36	CLAIM	PUBLISHED
LIB2-129	AS-LIB2-129-primary	Paper 2, §3.4, Casimir and geometric-mean prescription	36	CLAIM	PUBLISHED
LIB2-130	AS-LIB2-130-primary +2	Paper 2, §3.6, Bottom comparison and status rule	38	CLAIM	PUBLISHED
LIB2-131	AS-LIB2-131-primary +2	Paper 2, §3.6, Charm offset and comparator	38	CLAIM	PUBLISHED
LIB2-132	AS-LIB2-132-primary	Paper 2, §3.6, Heavy-quark RG/matching audit	39	CLAIM	PUBLISHED
LIB2-133	AS-LIB2-133-primary	Appendix O, §5, Outer-mirror operator	279	CLAIM	PUBLISHED
LIB2-134	AS-LIB2-134-primary +2	Paper 5, §2, Literal mass reading	85	CLAIM	PUBLISHED
LIB2-135	AS-LIB2-135-primary +2	Appendix I, §6, Proposition 6.3: trace and...	222	CLAIM	PUBLISHED
LIB2-136	AS-LIB2-136-primary	Appendix I, §6, Proposition 6.2: baryonic tensor	222	CLAIM	PUBLISHED
LIB2-137	AS-LIB2-137-primary +2	Appendix I, §6, Remark 6.4(i): spectral background	222	CLAIM	PUBLISHED
LIB2-138	AS-LIB2-138-primary +3	Appendix X, §4.3, Collapse to the scalar line	379	CLAIM	PUBLISHED
LIB2-139	AS-LIB2-139-primary	Paper 2, §3.5, Declared ultraviolet boundary	37	CLAIM	IN_REVIEW
LIB2-140	AS-LIB2-140-primary +3	Paper 5, §2, Two readings, one family: selected rung	85	CLAIM	IN_REVIEW
LIB2-141	AS-LIB2-141-primary +3	Paper 5, §2, Post-hoc depth rule	85	CLAIM	IN_REVIEW
LIB2-142	AS-LIB2-142-primary +3	Paper 5, §2, Boundary corollaries	86	CLAIM	IN_REVIEW
LIB2-143	AS-LIB2-143-primary +3	Paper 9, §2.1, The exhibited functor	132	CLAIM	IN_REVIEW
LIB2-144	AS-LIB2-144-primary +4	Paper 9, §1, Weight and interface distinction	131	CLAIM	PUBLISHED
LIB2-145	AS-LIB2-145-primary +6	Paper 9, §3, Small-nucleus freedom audit	133	CLAIM	PUBLISHED
LIB2-146	AS-LIB2-146-primary +4	Paper 9, §4, Theorem 4.1: A = 5 gap	133	CLAIM	PUBLISHED
LIB2-147	AS-LIB2-147-primary +3	Paper 9, §4, Theorem 4.2: leading two-alpha relation	133	CLAIM	PUBLISHED
LIB2-148	AS-LIB2-148-primary	Paper 9, §6, Equation (3): trace ladder	134	CLAIM	PUBLISHED
LIB2-149	AS-LIB2-149-primary +3	Paper 9, §6, Pentagon residual trace	134	CLAIM	PUBLISHED
LIB2-150	AS-LIB2-150-primary	Paper 9, §6, Remark 6.1: pentagon vault value	135	CLAIM	PUBLISHED
LIB2-151	AS-LIB2-151-primary	Paper 9, §6, Remark 6.1: ninth-weight vault	135	CLAIM	PUBLISHED
LIB2-152	AS-LIB2-152-primary +2	Paper 9, §7, Frozen Hoyle bar	135	CLAIM	PUBLISHED
LIB2-153	AS-LIB2-153-primary +3	Paper 9, §8, Ledger aggregate	136	CLAIM	PUBLISHED
LIB2-154	AS-LIB2-154-primary +5	Paper 9, §9, Original bar and residual	138	CLAIM	PUBLISHED
LIB2-155	AS-LIB2-155-primary +3	Paper 9, §9, Charge-magnitude defects	139	CLAIM	PUBLISHED
LIB2-156	AS-LIB2-156-primary +3	Paper 9, §9, Direct annular second-moment attachment	137	CLAIM	PUBLISHED
LIB2-157	AS-LIB2-157-primary	Paper 9, §9, Ribbon full-twist obstruction	137	CLAIM	PUBLISHED
LIB2-158	AS-LIB2-158-primary +2	Paper 9, §9, Perturbative annular disposition	138	CLAIM	PUBLISHED

Record	Assertion	Component / section	p.	Class	State
LIB2-159	AS-LIB2-159-primary +5	Paper 9, §9, Separate readout campaign	138	CLAIM	PUBLISHED
LIB2-160	AS-LIB2-160-primary +4	Paper 9, §9, Isospin coefficient	139	CLAIM	PUBLISHED
LIB2-161	AS-LIB2-161-primary +4	Paper 9, §9, Registered up–down operation	139	CLAIM	PUBLISHED
LIB2-162	AS-LIB2-162-primary +3	Paper 9, §7, Breathing-kernel typing	135	CLAIM	PUBLISHED
LIB2-163	AS-LIB2-163-primary +2	Paper 9, §7, Banked baseline ruling	135	CLAIM	IN_REVIEW
LIB2-164	AS-LIB2-164-primary +2	Paper 9, Table 1, Glueball pair	136	CLAIM	PUBLISHED
LIB2-165	AS-LIB2-165-primary +2	Appendix E, §8, First finite OS subgate	180	CLAIM	PUBLISHED
LIB2-166	AS-LIB2-166-primary	Appendix E, §8, Toy resolvent checkpoint	180	CLAIM	PUBLISHED
LIB2-167	AS-LIB2-167-primary +4	Appendix X, §3.5, No-energy-order fence	377	CLAIM	PUBLISHED
LIB2-168	AS-LIB2-168-primary +2	Appendix T, §5, Named datum 5.4: candidate action	324	CLAIM	PUBLISHED
LIB2-169	AS-LIB2-169-primary	Appendix T, §6, Theorem 6.7: conformal-mode obstruction	325	CLAIM	IN_REVIEW
LIB2-170	AS-LIB2-170-primary +3	Appendix T, §7, Named datum 7.1: finite state	325	CLAIM	PUBLISHED
LIB2-171	AS-LIB2-171-primary	Appendix T, §7, Theorem 7.3: representation and...	326	CLAIM	PUBLISHED
LIB2-172	AS-LIB2-172-primary +2	Appendix T, §7, Proposition 7.4: positive completion cone	326	CLAIM	PUBLISHED
LIB2-173	AS-LIB2-173-primary	Appendix T, §10, Theorem 10.1: half-log score	331	CLAIM	PUBLISHED
LIB2-174	AS-LIB2-174-primary +2	Appendix T, §10, Named datum 10.7: composite and premises	331	CLAIM	PUBLISHED
LIB2-175	AS-LIB2-175-primary +4	Appendix T, §11, Theorem 11.2: partial isometry	332	CLAIM	PUBLISHED
LIB2-176	AS-LIB2-176-primary +3	Appendix T, §12, Theorem 12.1: record isometry	333	CLAIM	PUBLISHED
LIB2-177	AS-LIB2-177-primary +2	Appendix T, §12, Actuality status split	333	CLAIM	PUBLISHED
LIB2-178	AS-LIB2-178-primary +2	Appendix T, §15.1, Dated face-2 amendment	338	CLAIM	PUBLISHED
LIB2-179	AS-LIB2-179-primary	Appendix T, §15.2, Invariant scalar-row attempt	338	CLAIM	PUBLISHED
LIB2-180	AS-LIB2-180-primary +2	Appendix T, §15.2, Grading character	338	CLAIM	PUBLISHED
LIB2-181	AS-LIB2-181-primary +3	Appendix T, §15.2, Source-intertwiner space and...	338	CLAIM	PUBLISHED
LIB2-182	AS-LIB2-182-primary +3	Appendix T, §15.3, Record-quotient channel existence	339	CLAIM	PUBLISHED
LIB2-183	AS-LIB2-183-primary +2	Appendix T, §15.3, Reversal-covariant score solder	339	CLAIM	PUBLISHED
LIB2-184	AS-LIB2-184-primary +4	Appendix T, §15.3, Normalization-census result	339	CLAIM	PUBLISHED
LIB2-185	AS-LIB2-185-primary +2	Appendix T, §15.3, Ambient extension and tier-local...	339	CLAIM	PUBLISHED
LIB2-186	AS-LIB2-186-primary +3	Appendix T, §15.4, Grade-four response premise	339	CLAIM	PUBLISHED
LIB2-187	AS-LIB2-187-primary +3	Appendix T, §15.4, Unit-response isometry alternative	340	CLAIM	PUBLISHED
LIB2-188	AS-LIB2-188-primary	Appendix T, §7, Proposition 7.5: free determinant	326	CLAIM	PUBLISHED
LIB2-189	AS-LIB2-189-primary +2	Appendix T, §8, Proposition 8.5: nonlinear reflection...	328	CLAIM	PUBLISHED
LIB2-190	AS-LIB2-190-primary	Appendix T, §8, Proposition 8.5: positive-path...	328	CLAIM	IN_REVIEW
LIB2-191	AS-LIB2-191-primary	Appendix T, §8, Theorem 8.6: linear-projector obstruction	328	CLAIM	PUBLISHED
LIB2-192	AS-LIB2-192-primary +2	Appendix T, §8, Theorem 8.6: nonlinear simplicity measure	328	CLAIM	IN_REVIEW
LIB2-193	AS-LIB2-193-primary +6	Appendix T, §11, Proposition 11.3: one-particle...	332	CLAIM	PUBLISHED

Record	Assertion	Component / section	p.	Class	State
LIB2-194	AS-LIB2-194-primary +2	Appendix T, §14.1, Isotropy for every carrier map	335	CLAIM	PUBLISHED
LIB2-195	AS-LIB2-195-primary +4	Appendix T, §14.2, Braid representation and centre	335	CLAIM	PUBLISHED
LIB2-196	AS-LIB2-196-primary	Appendix Q, §1, Proposition 1.1: complete classical…	295	CLAIM	PUBLISHED
LIB2-197	AS-LIB2-197-primary +2	Paper 1, §9, Theorem 9.1: congruence closure	30	CLAIM	PUBLISHED
LIB2-198	AS-LIB2-198-primary +2	Paper 4, §12, A1472 correction: loaded scalar spectrum	82	CLAIM	PUBLISHED
LIB2-199	AS-LIB2-199-primary +2	Paper 4, §7, Structural rather than completed…	78	CLAIM	PUBLISHED
LIB2-200	AS-LIB2-200-primary +2	Paper 4, §7, AX6 polarization postulate	78	CLAIM	PUBLISHED
LIB2-201	AS-LIB2-201-primary +4	Paper 8, §4, Moment-ledger model and parking decision	127	CLAIM	PUBLISHED
LIB2-202	AS-LIB2-202-primary	Paper 8, §4, Time-silence dimensional obstruction	127	CLAIM	PUBLISHED
LIB2-203	AS-LIB2-203-primary +2	Paper 8, §4, External time-axis splice	127	CLAIM	PUBLISHED
LIB2-204	AS-LIB2-204-primary +2	Paper 5, §2, Named splitting data beside the literal…	85	COMPARATOR	PUBLISHED
LIB2-205	AS-LIB2-205-primary	Paper 5, §2, Named atmospheric splitting beside the…	85	COMPARATOR	PUBLISHED
LIB2-206	AS-LIB2-206-primary	—, —	—	COMPARATOR	IN_REVIEW
LIB2-207	AS-LIB2-207-primary +2	Paper 5, §2, Cosmology comparator in the…	86	COMPARATOR	PUBLISHED
LIB2-208	AS-LIB2-208-primary +2	Paper 5, §2, Frequentist-constrained cosmology bound	86	COMPARATOR	PUBLISHED
LIB2-209	AS-LIB2-209-primary	Paper 4, §7, Three distinct electroweak scheme objects	77	COMPARATOR	PUBLISHED
LIB2-210	AS-LIB2-210-primary	—, —	—	COMPARATOR	IN_REVIEW
LIB2-211	AS-LIB2-211-primary	—, —	—	COMPARATOR	IN_REVIEW
LIB2-212	AS-LIB2-212-primary	Paper 3, §2.1, Comparator refresh: global-fit Dirac phase	51	COMPARATOR	PUBLISHED
LIB2-213	AS-LIB2-213-primary	Paper 3, Candidate Construction 2.1, Experimental…	55	COMPARATOR	PUBLISHED
LIB2-214	AS-LIB2-214-primary	Paper 3, §2, Named comparator for the corrected…	50	COMPARATOR	PUBLISHED
LIB2-215	AS-LIB2-215-primary	Paper 3, §2, Named Vub comparator	49	COMPARATOR	PUBLISHED
LIB2-216	AS-LIB2-216-primary	Paper 3, §2.1, Comparator refresh: Vcb	51	COMPARATOR	PUBLISHED
LIB2-217	AS-LIB2-217-primary	Paper 3, §2, Vud comparator supplied in the suite	50	COMPARATOR	PUBLISHED
LIB2-218	AS-LIB2-218-primary	Paper 2, §3.6, Charm MS-bar comparator and…	38	COMPARATOR	PUBLISHED
LIB2-219	AS-LIB2-219-primary	Paper 2, §3.6, Bottom MS-bar comparator and…	38	COMPARATOR	PUBLISHED
LIB2-220	AS-LIB2-220-primary +2	Appendix X, Honest precision summary, top-quark comparator	351	COMPARATOR	PUBLISHED
LIB2-221	AS-LIB2-221-primary	Paper 2, §3.2, Equation (16): external tau anchor	34	COMPARATOR	PUBLISHED
LIB2-222	AS-LIB2-222-primary	Paper 3, §2.1, Comparator refresh: two presentations…	51	COMPARATOR	PUBLISHED
LIB2-223	AS-LIB2-223-primary	Paper 3, §2.1, Comparator refresh: direct UT angle	51	COMPARATOR	PUBLISHED
LIB2-224	AS-LIB2-224-primary	Paper 4, §7, Three distinct electroweak scheme…	77	COMPARATOR	PUBLISHED
LIB2-225	AS-LIB2-225-primary	Paper 4, §7, Three distinct electroweak scheme…	77	COMPARATOR	PUBLISHED
LIB2-226	AS-LIB2-226-primary	Paper 5, §2, Precision and structural status: named…	84	COMPARATOR	PUBLISHED
LIB2-227	AS-LIB2-227-primary	Paper 5, §2.1, Framework versus NuFIT comparison:…	87	COMPARATOR	PUBLISHED
LIB2-228	AS-LIB2-228-primary +2	Paper 5, §2.1, Reactor comparison box: degree…	88	COMPARATOR	PUBLISHED
LIB2-229	AS-LIB2-229-primary +2	Appendix G, §6, Fences and language, item (4):…	198	COMPARATOR	PUBLISHED

Record	Assertion	Component / section	p.	Class	State
LIB2-230	AS-LIB2-230-primary +2	Paper 3, §2.1, Comparator refresh: historical Vcb value	51	COMPARATOR	PUBLISHED
LIB2-231	AS-LIB2-231-primary +2	Paper 3, §2.1, Comparator refresh: historical Vub value	51	COMPARATOR	PUBLISHED
LIB2-232	AS-LIB2-232-primary	Paper 3, Candidate Construction 2.1, Experimental…	55	COMPARATOR	PUBLISHED
LIB2-233	AS-LIB2-233-primary	Paper 0, Claim-language convention, S112 enforcement rule	8	RETIRED	PUBLISHED
LIB2-234	AS-LIB2-234-primary	Paper 0, Claim-language convention, S112 enforcement rule	8	RETIRED	PUBLISHED
LIB2-235	AS-LIB2-235-primary +2	Paper 3, §5.4, Current physical-row tally	64	RETIRED	PUBLISHED
LIB2-236	AS-LIB2-236-primary	Appendix N, §1.1, Retirement of the bounded-domain…	268	RETIRED	PUBLISHED
LIB2-237	AS-LIB2-237-primary	Paper 3, §2, Hand-calculated correction withdrawn	50	RETIRED	PUBLISHED
LIB2-238	AS-LIB2-238-primary	Paper 3, §2, Rounded-angle value and its former use	50	RETIRED	PUBLISHED
LIB2-239	AS-LIB2-239-primary +2	Paper 3, §2, Route-selection provenance	50	RETIRED	PUBLISHED
LIB2-240	AS-LIB2-240-primary	Paper 7, §7, Atmospheric-angle precedent	121	RETIRED	PUBLISHED
LIB2-241	AS-LIB2-241-primary +3	Appendix N, §6, Retired octant-selection assertions	274	RETIRED	PUBLISHED
LIB2-242	AS-LIB2-242-primary	Paper 5, §3, Retained leptonic-phase readout	89	RETIRED	IN_REVIEW
LIB2-243	AS-LIB2-243-primary	Paper 5, §3, Retained leptonic-phase readout	89	RETIRED	IN_REVIEW
LIB2-244	AS-LIB2-244-primary +3	Paper 5, §3, Retained leptonic-phase readout	89	RETIRED	IN_REVIEW
LIB2-245	AS-LIB2-245-primary	Paper 8, §2, Retired physical use of the…	123	RETIRED	IN_REVIEW
LIB2-246	AS-LIB2-246-primary	Paper 8, §4, Historical baryon/meson mass ratio	126	RETIRED	PUBLISHED
LIB2-247	AS-LIB2-247-primary	Paper 2, §3.2, Former proof/parameter classification…	34	RETIRED	PUBLISHED
LIB2-248	AS-LIB2-248-primary	Appendix O, §5, Remark 5.2: withdrawn universal…	280	RETIRED	PUBLISHED
LIB2-249	AS-LIB2-249-primary	Paper 4, §7, Two objects formerly named AX6	78	RETIRED	PUBLISHED
LIB2-250	AS-LIB2-250-primary	Appendix A, §A.7, Schur gauge-protection withdrawal	153	RETIRED	PUBLISHED
LIB2-251	AS-LIB2-251-primary +2	Appendix A, §A.7.8, Generic cross-block containment gap	155	RETIRED	PUBLISHED
LIB2-252	AS-LIB2-252-primary	Appendix B, The T-bridge does not canonicalize the…	162	RETIRED	PUBLISHED
LIB2-253	AS-LIB2-253-primary	Appendix B, §B.6, Current phase provenance and…	162	RETIRED	PUBLISHED
LIB2-254	AS-LIB2-254-primary	Appendix E, Selection frontier F-SEL(c), withdrawal (a)	180	RETIRED	PUBLISHED
LIB2-255	AS-LIB2-255-primary	Appendix T, §2, Theorem 2.1: two distinct operators	321	RETIRED	PUBLISHED
LIB2-256	AS-LIB2-256-primary	Appendix T, §3, Named datum 3.5: corrected object	322	RETIRED	PUBLISHED
LIB2-257	AS-LIB2-257-primary	Appendix T, §3, Remark 3.8: distinct structures	322	RETIRED	PUBLISHED
LIB2-258	AS-LIB2-258-primary	Appendix T, §9 preamble, S253 erratum: wrong-carrier…	329	RETIRED	PUBLISHED
LIB2-259	AS-LIB2-259-primary	Appendix X, §1.3, Typed freedom ledger replacing the…	367	RETIRED	PUBLISHED
LIB2-260	AS-LIB2-260-primary +2	Appendix S, §8.1, Consequence for the earlier parent…	312	RETIRED	PUBLISHED
LIB2-261	AS-LIB2-261-primary	Appendix S, §4, Paired-source replacement	308	RETIRED	PUBLISHED
LIB2-262	AS-LIB2-262-primary +2	Appendix S, §5, Relative-lift dependence of the…	310	RETIRED	PUBLISHED
LIB2-263	AS-LIB2-263-primary	Appendix S, §7, Casualty #7: depth-exclusion cut	311	RETIRED	PUBLISHED
LIB2-264	AS-LIB2-264-primary +2	Appendix X, §3.3, Promotion-gate construction correction	374	RETIRED	PUBLISHED
LIB2-265	AS-LIB2-265-primary	Paper 2, §3.2, Authoritative charged-lepton attribution	36	RETIRED	PUBLISHED
LIB2-266	AS-LIB2-266-primary	Paper 2, §3.2, Authoritative charged-lepton attribution	36	RETIRED	PUBLISHED
LIB2-267	AS-LIB2-267-primary +2	Paper 2, §3.2, External Koide attribution	36	RETIRED	PUBLISHED

Record	Assertion	Component / section	p.	Class	State
LIB2-268	AS-LIB2-268-primary +2	Paper 2, §3.2, External tau role in the current chain	36	RETIRED	PUBLISHED
LIB2-269	AS-LIB2-269-primary	Paper 2, §3.2, External tau role in the current chain	36	RETIRED	PUBLISHED
LIB2-270	AS-LIB2-270-primary	Paper 2, §3.6, Current bottom boundary value and offset	38	RETIRED	PUBLISHED
LIB2-271	AS-LIB2-271-primary	Paper 2, §3.6, Current bottom boundary value and offset	38	RETIRED	PUBLISHED
LIB2-272	AS-LIB2-272-primary	Paper 2, §3.6, Current bottom boundary value and offset	38	RETIRED	PUBLISHED
LIB2-273	AS-LIB2-273-primary	Paper 2, §3.6, Current charm boundary value and offset	38	RETIRED	PUBLISHED
LIB2-274	AS-LIB2-274-primary	Paper 2, §3.6, Current charm boundary value and offset	38	RETIRED	PUBLISHED
LIB2-275	AS-LIB2-275-primary	Paper 2, §3.6, Current charm boundary value and offset	38	RETIRED	PUBLISHED
LIB2-276	AS-LIB2-276-primary	Paper 2, §3.6, Current charm boundary value and offset	38	RETIRED	PUBLISHED
LIB2-277	AS-LIB2-277-primary	Paper 7, §7, D4 closed: old ansatz superseded	121	RETIRED	IN_REVIEW
LIB2-278	AS-LIB2-278-primary	Paper 5, §3, Retained leptonic-phase readout	89	RETIRED	IN_REVIEW
LIB2-279	AS-LIB2-279-primary	Paper 5, §3, Retained leptonic-phase readout	89	RETIRED	IN_REVIEW
LIB2-280	AS-LIB2-280-primary	Paper 5, §3, Retained leptonic-phase readout	89	RETIRED	IN_REVIEW
LIB2-281	AS-LIB2-281-primary	Paper 5, §2, Retained reactor-angle expression	84	RETIRED	PUBLISHED
LIB2-282	AS-LIB2-282-primary	Paper 5, §2, Retained reactor-angle expression	84	RETIRED	PUBLISHED
LIB2-283	AS-LIB2-283-primary	Paper 5, §2, Current PMNS angle provenance	84	RETIRED	PUBLISHED
LIB2-284	AS-LIB2-284-primary	Paper 7, §7, Atmospheric-angle precedent	121	RETIRED	PUBLISHED
LIB2-285	AS-LIB2-285-primary	Appendix B, §B.6, Current CKM magnitude provenance	164	RETIRED	PUBLISHED
LIB2-286	AS-LIB2-286-primary	Appendix B, §B.6, Retained Vcb formula	164	RETIRED	PUBLISHED
LIB2-287	AS-LIB2-287-primary	Appendix B, §B.6, Retained Vub hierarchy relation	164	RETIRED	PUBLISHED
LIB2-288	AS-LIB2-288-primary	Appendix B, §B.6, Retained Vcb formula	164	RETIRED	PUBLISHED
LIB2-289	AS-LIB2-289-primary	Paper 4, §7, Current weak-angle target and precision…	78	RETIRED	PUBLISHED
LIB2-290	AS-LIB2-290-primary	Paper 4, §7, Current weak-angle target and precision…	78	RETIRED	PUBLISHED
LIB2-291	AS-LIB2-291-primary	Paper 4, §7, Current weak-angle target and precision…	78	RETIRED	PUBLISHED
LIB2-292	AS-LIB2-292-primary	Paper 4, §7, Current weak-angle target and precision…	78	RETIRED	PUBLISHED
LIB2-293	AS-LIB2-293-primary	Paper 3, §2, Kernel-line exact-framework Cabibbo readout	50	RETIRED	PUBLISHED
LIB2-294	AS-LIB2-294-primary	Paper 3, §2, Kernel-line exact-framework Cabibbo readout	50	RETIRED	PUBLISHED
LIB2-295	AS-LIB2-295-primary	Paper 3, §2, Kernel-line exact-framework Cabibbo readout	50	RETIRED	PUBLISHED
LIB2-296	AS-LIB2-296-primary +2	Appendix I, §6, Remark 6.4: vacuum normalization…	223	RETIRED	PUBLISHED
LIB2-297	AS-LIB2-297-primary	Appendix N, Part N.B, Proposition 2.1: compact-colour…	270	RETIRED	PUBLISHED
LIB2-298	AS-LIB2-298-primary	Paper 0, Generation-replication boundary	18	RETIRED	PUBLISHED
LIB2-299	AS-LIB2-299-primary	Appendix X, Registered input: physical one-input condition	358	OPEN	IN_REVIEW
LIB2-300	AS-LIB2-300-primary	Paper 9, Posture: missing continuum object	130	OPEN	IN_REVIEW
LIB2-301	AS-LIB2-301-primary	Appendix T, §8, Theorem 8.6: remaining Einstein object	328	OPEN	IN_REVIEW
LIB2-302	AS-LIB2-302-primary	Appendix T, §8, Theorem 8.11: named mass-response map	329	OPEN	IN_REVIEW
LIB2-303	AS-LIB2-303-primary	Paper 9, §9, Reopen class of the six-to-seven-cell…	139	OPEN	IN_REVIEW
LIB2-304	AS-LIB2-304-primary	Paper 9, §9, Coherence mechanism is not yet a solution	139	OPEN	IN_REVIEW
LIB2-305	AS-LIB2-305-primary	Paper 9, §9, Isospin sector: unpriced neutron–proton…	139	OPEN	IN_REVIEW
LIB2-306	AS-LIB2-306-primary	Paper 9, §9, Open breathing-accounting audit	139	OPEN	IN_REVIEW
LIB2-307	AS-LIB2-307-primary	Appendix T, §7, Proposition 7.5: interacting chiral…	326	OPEN	IN_REVIEW

Record	Assertion	Component / section	p.	Class	State
LIB2-308	AS-LIB2-308-primary	Appendix N, Part N.E, Atmospheric-octant reopen condition	274	OPEN	IN_REVIEW
LIB2-309	AS-LIB2-309-primary	Appendix N, Part N.E, Typed missing complex-ratio operator	274	OPEN	IN_REVIEW
LIB2-310	AS-LIB2-310-primary	Paper 4, §7, Running-scale and radiative-shift boundary	78	OPEN	IN_REVIEW
LIB2-311	AS-LIB2-311-primary	Appendix I, §8, Amplitude-bridge boundary: surviving…	225	OPEN	IN_REVIEW
LIB2-312	AS-LIB2-312-primary	Appendix X, Honest precision summary: missing conversion…	351	OPEN	IN_REVIEW
LIB2-313	AS-LIB2-313-primary	Appendix X, Scheme-label declaration: bottom bridge is…	352	OPEN	IN_REVIEW
LIB2-314	AS-LIB2-314-primary	Paper 2, §3.6, Missing constituent-to-MS conversion	38	OPEN	IN_REVIEW
LIB2-315	AS-LIB2-315-primary	Appendix B, Lemma B.5 , Conditional readout status	161	OPEN	IN_REVIEW
LIB2-316	AS-LIB2-316-primary	Paper 3, Candidate Construction 2.1, Operator-based…	55	OPEN	IN_REVIEW
LIB2-317	AS-LIB2-317-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-318	AS-LIB2-318-metadata +1	Paper 6, End of the current B-frame discussion; distinct…	106	NOT_AN_ENTRY	IN_REVIEW
LIB2-319	AS-LIB2-319-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-320	AS-LIB2-320-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-321	AS-LIB2-321-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-322	AS-LIB2-322-metadata +1	Paper 0, Two-axis provenance: physical attachment is…	9	NOT_AN_ENTRY	IN_REVIEW
LIB2-323	AS-LIB2-323-metadata +1	Appendix E, Three PI rulings printed beside the closure,…	179	NOT_AN_ENTRY	IN_REVIEW
LIB2-324	AS-LIB2-324-metadata +1	Appendix E, Selection-frontier closure and remaining…	181	NOT_AN_ENTRY	IN_REVIEW
LIB2-325	AS-LIB2-325-metadata +1	Appendix T, §15.5, Intrinsic-basepoint placeholder	340	NOT_AN_ENTRY	IN_REVIEW
LIB2-326	AS-LIB2-326-metadata +1	Appendix X, Source-ledger and release-artifact note	348	NOT_AN_ENTRY	IN_REVIEW
LIB2-327	AS-LIB2-327-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-328	AS-LIB2-328-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-329	AS-LIB2-329-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-330	AS-LIB2-330-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-331	AS-LIB2-331-metadata +1	Appendix X, P0-1 audit, The result: real maximal isotropy	375	NOT_AN_ENTRY	IN_REVIEW
LIB2-332	AS-LIB2-332-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-333	AS-LIB2-333-metadata +1	Paper 1, §3, Roman singular data, not the…	26	NOT_AN_ENTRY	IN_REVIEW
LIB2-334	AS-LIB2-334-metadata +1	Paper 1, §3, Antipodal correspondence and missing…	26	NOT_AN_ENTRY	IN_REVIEW
LIB2-335	AS-LIB2-335-metadata +1	Appendix I, §6, Hadronic scope fence: legacy…	223	NOT_AN_ENTRY	IN_REVIEW
LIB2-336	AS-LIB2-336-metadata +1	Appendix E, §3, Remark 2: conditional role of DET-7	173	NOT_AN_ENTRY	IN_REVIEW
LIB2-337	AS-LIB2-337-metadata +1	Appendix A, §A.2, Theorem 0.1:…	143	NOT_AN_ENTRY	IN_REVIEW
LIB2-338	AS-LIB2-338-metadata +1	Appendix T, §4, Remark 4.2: priced AdS branch	323	NOT_AN_ENTRY	IN_REVIEW
LIB2-339	AS-LIB2-339-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-340	AS-LIB2-340-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-341	AS-LIB2-341-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW

Record	Assertion	Component / section	p.	Class	State
LIB2-342	AS-LIB2-342-metadata +1	Paper 0, Abstract continuation: charge/gauge terminal	2	NOT_AN_ENTRY	IN_REVIEW
LIB2-343	AS-LIB2-343-metadata +1	Paper 4, §7, Undetermined matching scale	78	NOT_AN_ENTRY	IN_REVIEW
LIB2-344	AS-LIB2-344-primary +2	Appendix X, Conditional generation-count update, s496/s498	362	CLAIM	PUBLISHED
LIB2-345	AS-LIB2-345-primary	Paper 5, §1, Current comparison basis	84	RETIRED	PUBLISHED
LIB2-346	AS-LIB2-346-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-347	AS-LIB2-347-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-348	AS-LIB2-348-metadata +1	Appendix A, §A.7.7, Remark 0.13: distinct…	154	NOT_AN_ENTRY	IN_REVIEW
LIB2-349	AS-LIB2-349-metadata +1	Paper 0, §2, Current operational AX3 label	13	NOT_AN_ENTRY	IN_REVIEW
LIB2-350	AS-LIB2-350-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-351	AS-LIB2-351-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-352	AS-LIB2-352-metadata +1	Paper 8, §4, Historical hadronic source pointers	126	NOT_AN_ENTRY	IN_REVIEW
LIB2-353	AS-LIB2-353-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW
LIB2-354	AS-LIB2-354-metadata +1	Appendix I, §6, Hadronic scope fence: legacy…	223	NOT_AN_ENTRY	IN_REVIEW
LIB2-355	AS-LIB2-355-metadata +3	Paper 8, §3 note, Deliberately library-resident…	125	NOT_AN_ENTRY	IN_REVIEW
LIB2-356	AS-LIB2-356-primary +2	Paper 3, §5.4, Internal mismatch corollary	64	CLAIM	PUBLISHED
LIB2-357	AS-LIB2-357-metadata +1	—, —	—	NOT_AN_ENTRY	IN_REVIEW

Why this belongs in the suite rather than in a private notebook. A theory programme that runs a hundred and fourteen adversarial rounds and prints only the survivors is indistinguishable, from outside, from one that ran a hundred and fourteen and printed only the flattering ones. The value of this appendix is that it makes the denominator visible: the negatives, the refuted house predictions, the withdrawn headline, the retired axioms, the protocol failures, and the two numbers that died twice. Engine unchanged; no observable moves; nothing here is promoted.